PRINCIPLES OF SEDIMENT TRANSPORT
IN
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PRINCIPLES OF SEDIMENT TRANSPORT
IN
RIVERS, ESTUARIES AND COASTAL SEAS

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For those who like sediments
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### APPENDICES A: TRANSPOR-program; Computation of sediment transport in current and in wave direction

- **B: Sand transport in closed conduits**
- **C: Side-wall roughness correction method of Vanoni-Brooks**
- **D: Pollution aspects of sediments**
PREFACE

This book reflects the results of basic research and practical experience in sediment transport and morphology in rivers, estuaries and coastal seas all over the world during a period of about 15 years.

The purpose of this book is to give a unified view of sediment (sand and mud) transport over a wide range of conditions; from quasi-steady river flow to the violent wave-breaking processes in the surf zone of coastal seas. It was not the intention of the author to give a complete overview of the overwhelming amount of literature available. On the contrary, the emphasis is laid on the description and the application of those theories and formulae which have proven to give realistic results based on the author's experience.

The application of refined theories consisting of many complicated equations is often not justified, given the uncertainties of the input data like current velocity, wave height, bed material composition, bed forms and roughness. An elegant way to overcome this problem is to represent the refined model by a much more simple parameter model or computer data base model.

Chapter 2 presents an overview of the near-bed fluid velocities and shear stresses, both being the driving agents of the sediment particle motions. The current boundary layer as well as the wave boundary layer are discussed. Basic wave properties are presented. Mass transport by non-breaking and breaking waves is summarized.

Chapter 3 covers the fluid and sediment properties like density, porosity, shape, size and settling characteristics of the sediment particles.

Chapter 4 presents the processes of initiation of motion and suspension in terms of the critical velocities and bed-shear stresses. Special attention is given to the design of stable channels which is an important aspect of irrigation projects.

Chapter 5 covers the characteristics of bed forms generated by currents alone, by waves and by combined currents and waves. Bed form classification diagrams are given and the shape and dimensions of the bed forms are discussed.

Chapter 6 presents information of the effective (grain and form) roughness of a sediment bed. Methods based on bed-form parameters as well as methods based on integral parameters (velocity, depth, sediment size) are discussed.

Chapter 7 deals with bed load and suspended load transport in steady flow. Both deterministic and stochastic approaches are discussed. The classical diffusion theory is used to describe the distribution of the sediment concentrations over the depth. Special attention is given to stratification effects of high-concentration suspensions.

The Chapters 8 and 9 cover the field of sediment transport by waves and by combined currents and waves. The various transport processes related to breaking and non-breaking waves are identified; high-frequency and low-frequency phenomena are discussed. Emphasis is put on data analysis of concentrations and transport rates measured in flumes, tunnels and in nature.

Chapter 10 is related to the transport of sediments in non-steady and non-uniform conditions. The basic principles of erosion and deposition of sediment particles are presented.
Chapter 11 presents detailed information of the transport of cohesive sediment materials (mud). Basic phenomena like cohesion, flocculation, settling, deposition, consolidation and erosion which take place in a continuous cycle, are discussed.

Chapter 12 deals with the mathematical modelling of sediment transport and morphology. Three-, two- and one-dimensional models of flow, waves, sediment transport and morphology are presented.

Sediment transport cannot be studied without proper knowledge of measuring instruments. The accuracy of the data is strongly related to the type of instrument applied. Chapter 13 presents a detailed overview of the available measuring principles, statistics, methods and instruments. Simple mechanical and sophisticated optical, acoustical and nuclear instruments are discussed.

The book ends with four appendices. Appendix A presents the TRANSPOR-program (available on diskette) which is the sediment transport model of the author. Appendix B deals with sand transport in closed conduits (pipelines). Appendix C presents a method to eliminate side wall roughness which is necessary for narrow flumes and channels. Appendix D is related to pollution aspects of sediments.

Many calculation examples of available methods and formulae are presented throughout the book, which may help the reader to find a way through the many available equations. A 3.5"-diskette (TRANSPOR-program) for computing sediment concentrations, transport rates and bed-form dimensions in a current alone and in combined currents and waves is available to help the reader to solve practical problems.

The present book has been written with a view to morphology of sediment beds. This latter field of work will be described in a forthcoming book: "Principles of Morphology in Rivers, Estuaries and Coastal Seas".

The author hopes that the present book and the TRANSPOR-diskette will serve as a useful tool for students and graduates in civil engineering, earth sciences, physical geography and oceanography.

Leo C. van Rijn
Oldemarkt
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1. INTRODUCTION

1.1 Definitions

Sediment is fragmental material, primarily formed by the physical and chemical desintegration of rocks from the earth's crust. Such particles range in size from large boulders to colloidal size fragments and vary in shape from rounded to angular. They also vary in specific gravity and mineral composition, the predominant material being quartz.

Once the sediment particles are detached, they may either be transported by gravity, wind or/and water.

When the transporting agent is water, it is called fluvial or marine sediment transport. The process of moving and removing from their original source or resting place is called erosion. In a channel the water flow erodes the available material in the banks and/or the stream bed until the flow is "loaded" with as much sediment particles as the energy of the stream will allow it to carry.

Usually, three modes of particle motion are distinguished:

- rolling and/or sliding particle motion,
- saltating or hopping particle motion,
- suspended particle motion.

When the value of the bed-shear velocity just exceeds the critical value for initiation of motion, the bed material particles will be rolling and/or sliding in continuous contact with the bed. For increasing values of the bed-shear velocity the particles will be moving along the bed by more or less regular jumps, which are called saltations.

When the value of the bed-shear velocity begins to exceed the fall velocity of the particles, the sediment particles can be lifted to a level at which the upward turbulent forces will be of comparable or higher order than the submerged weight of the particles and as a result the particles may go into suspension.

Usually, the transport of particles by rolling, sliding and saltating is called bed-load transport, while the suspended particles are transport as suspended load transport.

The suspended load may also include the fine silt particles brought into suspension from the catchment area rather than from the streambed material (bed material load) and is called the wash load. A grain size of 50 μm is frequently used to make the separation between bed material load and wash load. Sometimes a value of 63 μm is used (USA). Another method of discrimination is given by Bagnold (1962), see Chapter 7. Based on energy considerations it can be shown that all particles with a fall velocity smaller than 1.6 \( \bar{u} I \) (\( \bar{u} = \) depth-averaged velocity, \( I = \) water surface gradient) can be transported in unlimited quantities, the latter being a typical feature of wash load transport.

Bed load and suspended load may occur simultaneously, but the transition zone between both modes of transport is not well-defined.

Sediment transport by flowing water is strongly linked to surface soil erosion due to rain. Water seeping into the ground can contribute to landslides (subsurface erosion) which may become major sources of sediments for rivers.

The whole process can be seen as a continuous cycle of:

soil erosion → sediment transport → sedimentation.
Soil erosion and sediment yield (in tonnes per km$^2$ per year) strongly depend on the local climatic (rainfall), soil, land (surface slope) and vegetation conditions. Values may vary from 50 to 500 tonnes per km$^2$ per year. Universal formulae are not available. Based on local data, regional formulae have been developed.

A detailed discussion of soil loss estimation has been given by Kirkby and Morgan (Wiley, New York, 1980).

The Yellow river in China is a dramatic example of a river carrying enormous quantities of sediments, primarily eroded in the Loess valleys of the middle reach, to the sea.

Proper land use and management can substantially reduce the problems related to sediment transport in rivers and estuaries. The design of works (terracing, debris dams, slope fixation) can also help to control surface erosion.

The following classification and definitions in accordance with the ISO-standards (ISO 4363) are given:

```
<table>
<thead>
<tr>
<th>Total load (origin)</th>
<th>Bed material load</th>
<th>Wash load moving as suspended load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>moving as bed load</td>
<td></td>
</tr>
<tr>
<td></td>
<td>moving as suspended load</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total load (transport)</td>
<td></td>
</tr>
</tbody>
</table>
```

Bed material : The material, the particle sizes of which are found in appreciable quantities in that part of the bed that is affected by transport.

Bed material load : The part of the total sediment transport which consists of the bed material and which rate of movement is governed by the transport capacity of the channel.

Suspended load : That part of the total sediment transport which is maintained in suspension by turbulence in the flowing water for considerable periods of time without contact with the stream bed. It moves with practically the same velocity as that of the flowing water.

Bed load : The sediment in almost continuous contact with the bed, carried forward by rolling, sliding or hopping.

Wash load : That part of the suspended load which is composed of particle sizes smaller than those found in appreciable quantities in the bed material. It is in near-permanent suspension and, therefore, is transported through the stream without deposition. The discharge of the wash load through a reach depends only on the rate with which these particles become available in the catchment area and not on the transport capacity of the flow.
1.2 History

Sediment transport problems related to irrigation channels were studied and solved by trial and error in ancient China, Mesopotamia, Egypt and in the Roman Empire.

One of the first researchers who studied theoretically and experimentally the effect of the flow on a loose sediment bed was DuBoat (1734-1809) in France. He determined the flow velocity causing erosion of the bed; various soil materials (clay to stones) were studied. DuBoat developed the shear-resistance concept.

Hagen (1797-1884) in Germany and DuPuit (1804-1866) in France gave qualitative descriptions of the motion of sediment particles along the bed and in suspension.

Brahms (1753) proposed an expression for the critical velocity of a bed consisting of stones. Tulla (1770-1828) studied fluvial processes in the Rhine river.

The first bed-load type formula based on depth and slope was presented by DuBoys (1847-1924) in France, who schematized the transport process as the movement of particles in a series of layers.

Around 1900 the first movable bed models were built by Fargue (1827-1910) in France and by Reynolds (1892-1912) in England. Basic research on sediment transport in laboratory flumes was started seriously by Engels (1854-1945) in Germany and by Gilbert (1843-1918) in the USA. The data of Gilbert presented in the famous publication: "Transportation of Debris by Running Water" (1914) are still used by many researchers for calibration of bed-load transport formulae.

Theoretical work was done by Forchheimer (1852-1933) and by Schoklitsch (1888-1969) in Germany. The latter (1914) developed an equation for the critical bed-shear stress (initiation of motion of a particle) resting on a longitudinal sloping bed. Similar work for a particle resting on a transverse sloping bed (river bank) was done by Leiner (1912).

Empirical field research related to the design of irrigation channels (regime formulae) was done by Kennedy in India (1895).

Shields (1936) made a significant contribution related to the (critical) bed-shear stress for initiation of motion of sediment particles. The curve proposed is well known as the "Shields" curve.

Recent work to be mentioned is that of Kalinske (1946), Vanoni (1946), Einstein (1950) and Bagnold (1966).
1.3 Symbols and units

The most important symbols used in this book are:

\[ \begin{align*}
    a & = \text{reference level} & (\text{m}) \\
    C & = \text{Chézy-coefficient} & (\text{m}^{0.5}\text{s}^{-1}) \\
    c & = \text{concentration} & (-) \\
    D_s & = \text{dimensionless particle parameter} & (-) \\
    d & = \text{particle diameter} & (\text{m}) \\
    F & = \text{force} & (\text{N}) \\
    f & = \text{friction factor} & (-) \\
    g & = \text{acceleration of gravity} & (\text{m s}^{-2}) \\
    H & = \text{wave height} & (\text{m}) \\
    h & = \text{flow depth} & (\text{m}) \\
    I & = \text{energy gradient} & (-) \\
    k_s & = \text{equivalent or effective bed roughness} & (\text{m}) \\
    Q & = \text{volumetric flow discharge} & (\text{m}^3\text{s}^{-1}) \\
    Q_i & = \text{volumetric total sediment discharge or transport rate} & (\text{m}^3\text{s}^{-1}) \\
    q & = \text{flow discharge per unit width} & (\text{m}^2\text{s}^{-1}) \\
    q_i & = \text{sediment discharge or transport rate per unit width} & (\text{m}^2\text{s}^{-1}) \\
    s & = \text{relative density} (\rho/\rho_p) & (-) \\
    T & = \text{dimensionless bed-shear stress parameter} & (-) \\
    t & = \text{time} & (\text{s}) \\
    U, V, W & = \text{instantaneous velocities in x, y, z direction} & (\text{m s}^{-1}) \\
    u, v, w & = \text{time-averaged velocities in x, y, z direction} & (\text{m s}^{-1}) \\
    U_\phi & = \text{peak orbital velocity near bed} & (\text{m s}^{-1}) \\
    \bar{u} & = \text{depth-averaged velocity} & (\text{m s}^{-1}) \\
    \bar{u}_i & = \text{cross-section averaged velocity} & (\text{m s}^{-1}) \\
    u_* & = \text{bed-shear velocity} & (\text{m s}^{-1}) \\
    w_\phi & = \text{particle fall velocity} & (\text{m s}^{-1}) \\
    Z & = \text{dimensionless suspension number} & (-) \\
    z & = \text{vertical coordinate} & (\text{m}) \\
    z_0 & = \text{zero-velocity level} & (\text{m}) \\
    \lambda & = \text{bed form length} & (\text{m}) \\
    \Delta & = \text{bed form height} & (\text{m}) \\
    \delta & = \text{thickness} & (\text{m}) \\
    \varepsilon & = \text{mixing coefficient} & (\text{m}^3\text{s}^{-1}) \\
    \tau_b & = \text{bed-shear stress} & (\text{N m}^{-2}) \\
    \rho & = \text{fluid density} & (\text{kg m}^{-3}) \\
    \rho_s & = \text{sediment density} & (\text{kg m}^{-3}) \\
    \theta & = \text{dimensionless particle mobility parameter} & (-) \\
    \psi & = \text{dimensionless particle mobility parameter} & (-) \\
    \phi & = \text{dimensionless transport rate} & (-) \\
    \nu & = \text{kinematic viscosity coefficient} & (\text{m}^2\text{s}^{-1}) \\
    \mu & = \text{dynamic viscosity coefficient} & (\text{kg m}^{-1}\text{s}^{-1}) \\
    \kappa & = \text{Von Karman constant} & (-)
\end{align*} \]

Instantaneous variables (velocity) are represented by Capital letters (U). Time-averaged variables are represented by normal letters (u).
The following subscripts are frequently used:

\[ h = \text{bed, bed-load, bottom} \]
\[ c = \text{current, current-related} \]
\[ cr = \text{critical, initiation of motion} \]
\[ cw = \text{current and waves} \]
\[ f = \text{fluid} \]
\[ m = \text{mean, median, mixture} \]
\[ s = \text{sediment, suspended load} \]
\[ t = \text{total, total load} \]
\[ w = \text{waves, wave-related} \]

Examples are:

\[ \tau_{b,c} = \text{current-related bed-shear stress} \] (N m\(^{-2}\))
\[ \tau_{b,cw} = \text{bed-shear stress for current and waves} \] (N m\(^{-2}\))
\[ \tau_{b,w} = \text{wave-related bed-shear stress} \] (N m\(^{-2}\))
\[ \tau_{b,cr} = \text{critical bed-shear stress} \] (N m\(^{-2}\))
\[ q_{b} = \text{volumetric bed-load transport rate per unit width} \] (m\(^{3}\)s\(^{-1}\))
\[ q_{s} = \text{volumetric suspended-load transport rate per unit width} \] (m\(^{3}\)s\(^{-1}\))
\[ q_{t} = \text{volumetric total load transport per unit width} \] (m\(^{3}\)s\(^{-1}\))
\[ q_{b,c} = \text{current-related (volumetric) bed load transport rate per unit width} \] (m\(^{3}\)s\(^{-1}\))

Units are according to the "Systeme International d'Unités" (SI-units), which has been adopted by the International Organization for Standardization (IOS).

Force is expressed in Newtons (N)
Mass is expressed in kilograms (kg)
Length is expressed in meters (m)
Time is expressed in seconds (s)

1.4 Characteristic parameters

The following dimensionless characteristic parameters are herein used:

- particle diameter, \( D_* \)
- particle mobility parameter, \( \theta \)
- excess bed-shear stress parameter, \( T \)
- suspension parameter, \( Z \)
- transport rate, \( \Phi \)

1. Particle parameter, \( D_* \)

The particle diameter \( D_* \) reflects the influence of gravity, density and viscosity and reads as:

\[ D_* = \left[ \frac{(s-1)g}{v^2} \right]^{1/3} d_{s0} \] \hspace{1cm} (1.1)

in which:
\( d_{s0} = \) median particle diameter of bed material
\( s = \) specific gravity \((= \rho_s/\rho)\)
\( v = \) kinematic viscosity coefficient
\( g = \) acceleration of gravity
2. *Particle mobility parameter* \( \theta \)

*Plane bed*

The particle mobility parameter \( \theta \) is the ratio of the hydrodynamic fluid (drag and lift) force and the submerged particle weight. The fluid force is proportional to \( \rho \, d^2 (u_*)^2 \) and the submerged particle weight is proportional to \( (\rho_s - \rho) g d^3 \), yielding a ratio of:

\[
\theta = \frac{(u_*)^2}{(s-1)g \, d_{50}} = \frac{\tau_b}{(\rho_s - \rho) g \, d_{50}} = \frac{hI}{(s-1) \, d_{50}} \quad (1.2)
\]

in which:
- \( \tau_b \) = overall time-averaged bed-shear stress due to currents and/or waves
- \( u_* \) = overall bed-shear velocity \( (\tau_b = \rho \, u_*) \)
- \( s \) = specific density \( (\rho_s / \rho) \)
- \( h \) = flow depth (m)
- \( I \) = energy gradient (-)

*Bed forms*

When bed forms are present, the grain-related or effective bed-shear stress \( (\tau'_b) \) in stead of the overall bed-shear stress \( (\tau_b) \) should be used to calculate the particle mobility parameter.

The effective bed-shear stress represents that part of the overall bed-shear stress acting on the bed material particles \( (\tau'_b = \mu \, \tau_b) \) with \( \mu = \) efficiency factor. Basically, it is assumed that the form drag dissipated in the eddies generated by the bed forms is not fully effective in the entrainment of sediment particles from the bed into the fluid.

3. *Excess bed-shear stress parameter* \( T \)

The excess bed-shear stress parameter \( T \), is defined as:

\[
T = \frac{\tau'_b - \tau_{b,cr}}{\tau_{b,cr}} \quad (1.3)
\]

in which:
- \( \tau_{b,cr} \) = critical time-averaged bed-shear stress according to Shields.

4. *Suspension parameter*, \( Z \)

The suspension parameter \( Z \) reflects the ratio of the downward gravity forces and the upward fluid forces acting on a suspended sediment particle in a current and reads as:

\[
Z = \frac{w_s}{\beta \, \kappa \, u_*} \quad (1.4)
\]

in which:
- \( w_s \) = particle fall velocity in clear fluid
- \( u_* \) = overall bed-shear velocity
- \( \kappa \) = Von Karman constant
- \( \beta \) = ratio of sediment and fluid mixing coefficient
5. Transport rate, $\phi$

The dimensionless transport $\phi$ usually is represented as:

$$\phi = \frac{q_t}{(s-1)^{0.5}g^{0.5}d_{50}^{1.5}}$$  \hspace{1cm} (1.5)

Another dimensionless expression is:

$$\phi = \frac{q_t}{w_s d_{50}}$$  \hspace{1cm} (1.6)

in which:

$q_t$ = volumetric total sediment transport rate (m$^2$/s)
$g$ = acceleration of gravity (m$^2$/s)
$d_{50}$ = median particle size of bed material (m)
$w_s$ = particle fall velocity of bed material (m/s)
$s$ = specific density ($\rho_s/\rho$)

The volumetric sediment transport rate ($q_t$) can also be made dimensionless with the specific flow discharge ($q$), yielding the discharge-weighted concentration.

$$c_t = \frac{q_t}{q}$$  \hspace{1cm} (1.7)

in which:

$q_t$ = volumetric total sediment transport rate (m$^2$/s)
$q$ = specific flow discharge (m$^2$/s)

Sometimes the transport rate is given as a submerged mass per unit time and width. The mass (above water) is equal to $\rho_s/\rho_s$ times the submerged mass.
2. FLUID VELOCITIES AND BED SHEAR STRESSES

2.1 Introduction

Sediment transport processes in currents and/or waves are dominant in the near-bed region. Therefore, it is of essential importance to know the magnitude and direction of the near-bed velocities and shear-stresses in the presence of bed forms (ripples, dunes) or in case of a flat bed.

Detailed information of fluid velocities and bed-shear stresses is presented by Van Rijn, 1990 ("Principles of Fluid Flow and Surface Waves in Rivers, Estuaries, Seas and Oceans").

In this Chapter a summary is given of experimental results and theoretical expressions. The following subjects are covered:
- currents,
- waves,
- currents and waves.

2.2 Currents

2.2.1 Current boundary layer

1. Equation of motion

The equation of motion for steady uniform (turbulent) flow is given by (see Fig. 2.2.1):

\[ \tau_z \Delta x \Delta y = \rho \ g \ (h-z) \Delta x \Delta y \sin\beta \]

or

\[ \tau_z = \rho \ g \ (h-z) \sin\beta \]

or using \( I = \sin\beta \)

\[ \tau_z = \rho \ g \ (h-z) \ I \]

For \( z = 0 \) this yields the bed-shear stress:

\[ \tau_b = \rho \ g \ h \ I \]

(2.2.1)

By definition \( \tau_b = \rho (u_* \cdot c)^2 \), yielding

\[ u_* \cdot c = (g \ h \ I)^{0.5} \]

(2.2.2)

2. Turbulent flow

Turbulence is a random fluctuating velocity field which interacts with and derives its energy from the mean flow field. A turbulent velocity field can only be described by statistical quantities such as root-mean-square values, amplitude distribution, correlations and spectra. The amplitudes are generally normally distributed so that the root-mean-square deviation gives a good estimate of the fluctuations.

According to the Reynolds’ procedure, the shear stress in a turbulent flow at height \( z \) in a steady uniform flow can be described as (Fig. 2.2.1):
\[ \tau_z = \rho v \frac{du}{dz} - \rho \overline{u'w'} \]  \hspace{1cm} (2.2.4)

in which:

- \( u = \) time-averaged fluid velocity at height \( z \)
- \( u' = \) turbulent fluid velocity fluctuation in horizontal direction
- \( w' = \) turbulent fluid velocity fluctuation in vertical direction
- \( \rho = \) fluid density
- \( v = \) kinematic viscosity coefficient

Although the time-averaged vertical velocity \( w \) is equal to zero (\( w = 0 \)), the vertical turbulent fluctuations are not equal to zero (\( w' \neq 0 \)). Consequently, the turbulence shear stress \( \tau_t = -\rho \overline{u'w'} \neq 0 \).

The turbulent shear stress \( \tau_t \) is dominant in the major part of the flow depth. In case of a smooth bottom the viscous shear stress \( \tau_v \) becomes dominant close to the bottom because the turbulent fluctuations \( u' \) and \( w' \) die out near the bottom and are equal to zero at the bottom (\( u' = w' \) at \( z = 0 \)). The layer where the viscous shear stress is dominant is called the viscous sublayer (\( \delta_v \)).

Above the viscous sublayer the flow is turbulent. The most important turbulent sublayer is the logarithmic sublayer. Between the viscous sublayer and the logarithmic sublayer there is a transition sublayer, sometimes called the buffer sublayer. Above the logarithmic sublayer there is an outer sublayer (see also Figure 2.2.2).

### 2.2.2 Hydraulic regimes

The effect of the bottom (or wall) roughness on the velocity distribution in a turbulent flow was first investigated for pipe flow by Nikuradse (1933). He used pipes covered with uniform sand grains at the inside and he measured velocity distributions at different Reynolds’ numbers (Re), pipe diameters (D) and grain sizes (d_0). Based on these experiments, Nikuradse introduced the concept of the equivalent sand grain roughness or Nikuradse roughness (\( k_s \)) as a standard for all other types of roughness elements (k).

The roughness elements mainly influence the velocity distribution close to the bottom, because the roughness elements generate eddies (with a size of the order of the roughness elements) which affect the turbulence structure and hence the velocities close to the bottom. Further away, the eddies will rapidly be absorbed in the general existing turbulence pattern. The type of flow regime can be related to the ratio of the Nikuradse roughness (\( k_s \)) and a length scale of the viscous sublayer (\( v/u_{*c} \)) in which \( v = \) kinematic viscosity coefficient (m²/s) and \( u_{*c} = \) current-related bed-shear stress (m/s).

Based on experimental results, it was found:

- **Hydraulically smooth flow**, for \( \frac{k_s}{v/u_{*c}} = \frac{u_{*c}}{v} \frac{k_s}{v} \leq 5 \) \hspace{1cm} (2.2.5)

  The roughness elements are much smaller than the thickness of the viscous sublayer and do not affect the velocity distribution (Fig. 2.2.3).

- **Hydraulically rough flow**, for \( \frac{k_s}{v/u_{*c}} = \frac{u_{*c}}{v} \frac{k_s}{v} \geq 70 \) \hspace{1cm} (2.2.6)

  A viscous sublayer does not exist and the velocity distribution is not dependent on the viscosity (\( v \)) of the fluid (Fig. 2.2.3).

- **Hydraulically transitional flow**, for \( 5 < \frac{u_{*c}}{v} \frac{k_s}{v} < 70 \) \hspace{1cm} (2.2.7)

  The velocity distribution is affected by viscosity as well as by the bottom roughness.
**Figure 2.2.1 Shear stress**

**Figure 2.2.2 Current boundary layer**

**Figure 2.2.3 Velocity distribution in smooth and rough flow**

**Figure 2.2.4 Mixing length and eddy viscosity distribution**
2.2.3 Velocity distribution over the depth

A general expression for the velocity distribution over the depth for smooth and rough flows is:

\[ u = \frac{u^*c}{\kappa} \ln \left( \frac{z}{z_0} \right) \]  \hspace{1cm} (2.2.8)

in which:

- \( u^*c \) = current-related bed-shear velocity (m/s)
- \( \kappa \) = constant of Von Karman (0.4)
- \( z_0 \) = zero-velocity level (\( u = 0 \) at \( z = z_0 \)), (m)
- \( z \) = vertical coordinate (m)

Smooth flow regime : \( z_0 = 0.11 \frac{v}{u^*c} \) for \( \frac{u^*c \kappa}{v} \leq 5 \)  \hspace{1cm} (2.2.9)

Rough flow regime : \( z_0 = 0.033 k_s \) for \( \frac{u^*c \kappa}{v} \geq 70 \)  \hspace{1cm} (2.2.10)

Transition regime : \( z_0 = 0.11 \frac{v}{u^*c} + 0.033 k_s \) for \( 5 < \frac{u^*c \kappa}{v} < 70 \)  \hspace{1cm} (2.2.11)

The zero velocity level should be interpreted as a computation parameter without physical meaning, because Eq. (2.2.8) is not valid close to the bottom. In case of a rough boundary the position of the zero-velocity level is unknown. Usually, the bottom plane (\( z = 0 \)) is defined as the plane that is formed after smoothing of the roughness elements. For spherical roughness elements this yields a value of about 0.75k above the underside of the spheres (k = sphere diameter). A realistic value of the zero-velocity level (\( z = z_0 \)) can be found by plotting the values of \( u/u^* \) against \( z-z'/z_0' \) on semi-logarithmic scale and varying \( z_0 \) until the best fit is obtained. This method yields values \( z_0' \approx 0.25k \) for sand and gravel particles.

Averaging of Eq. (2.2.8) over the depth, yields:

\[ \bar{u} = \frac{1}{h} \int_{z_0}^{h} \frac{u^*c}{\kappa} \ln \left( \frac{z}{z_0} \right) dz = \frac{u^*c}{\kappa} \left[ \frac{z_0}{h} - 1 + \ln \left( \frac{h}{z_0} \right) \right] \]  \hspace{1cm} (2.2.12)

Neglecting the \( z_0/h \)-parameter in Eq. (2.2.12), the depth-averaged flow velocity does occur at \( z = h/e \approx 0.37 \, h \), in which \( e \) is the base of the natural logarithm (\( e \approx 2.72 \)).

Applying Eq. (2.2.12) in Eq. (2.2.8), the velocity distribution can also be expressed as:

\[ u = \left[ \frac{\bar{u}}{z_0/h - 1 + \ln(h/z_0)} \right] \ln \left( \frac{z}{z_0} \right) \]  \hspace{1cm} (2.2.13)

2.2.4 Fluid mixing coefficient

A turbulent flow field has a diffusive character. Gradients of momentum and scalar quantities are rapidly diminished by this diffusive action.
The analogy of turbulent motion with the movements of molecules leads to the analogy given by Boussinesq and the introduction of an eddy-viscosity concept for the apparent turbulent shear stress \(-\rho u'w' = \rho \varepsilon_f \frac{\partial u}{\partial z}\) \((u'w')\) are velocity fluctuations in horizontal and vertical direction).

Applying a logarithmic velocity profile and assuming a linear shear stress distribution over the depth, a parabolic fluid mixing coefficient is obtained (Fig. 2.2.4):

\[ \varepsilon_{fc} = \kappa z(1-z/h)u_{*,c} \]  \hspace{1cm} (2.2.14)

in which:

\[ \varepsilon_{fc} = \text{fluid mixing coefficient related to current (m}^2/\text{s)} \]

Based on the experimental results of Coleman (1970), Van Rijn (1984) introduced a parabolic-constant mixing coefficient distribution; parabolic in the lower half of the depth and constant in the upper half of the depth:

\[ \varepsilon_{fc} = \kappa z(1-z/h)u_{*,c} \hspace{1cm} \text{for } z/h < 0.5 \]  \hspace{1cm} (2.2.15)

\[ \varepsilon_{fc} = 0.25 \kappa u_{*,c} h \hspace{1cm} \text{for } z/h \geq 0.5 \]  \hspace{1cm} (2.2.16)

The main reason for applying Eq. (2.2.16) is that it produces a more realistic concentration profile with finite concentrations at the water surface (Van Rijn, 1989).

### 2.2.5 Bed shear stress and bed friction

The overall time-averaged bed-shear stress is defined as:

\[ \tau_{h,c} = \rho g h I = \rho g \frac{\bar{u}^2}{C^2} = \frac{1}{8} \rho f_c \bar{u}^2 \]  \hspace{1cm} (2.2.17)

in which:

\( h \) = water depth (m)

\( I \) = energy line gradient (-)

\( \bar{u} \) = depth-averaged velocity (m/s)

\( C \) = Chézy-coefficient \((C^2 = 8g/f_c), (m^{0.5}/s)\)

\( f_c \) = friction factor of Darcy-Weisbach (-)

\( k_s \) = effective bed roughness height (m)

\( \rho \) = fluid density \((kg/m^3)\)

\( g \) = acceleration of gravity \((m/s^2)\)

The friction factor and Chézy-coefficient for laminar, hydraulic smooth, rough and transitional flow are presented below:

1. **Laminar flow**

The friction factor for laminar flow in a wide open channel is:

\[ f_c = \frac{64}{Re} \]  \hspace{1cm} (2.2.18)
in which:
\[ \text{Re} = \frac{\bar{u}h}{v} = \text{Reynolds number} \ (-) \]

2. Turbulent flow

The Chézy-coefficient (C) and the friction factor \( f_c \) can be derived from Eq. (2.2.12).
Chézy found empirically:

\[ \bar{u} = C \ (h \ I)^{0.5} \quad (2.2.19) \]

in which:
\( \bar{u} \) = depth-averaged velocity (m/s)
C = Chézy roughness coefficient (m\(^{0.5}\)/s)
h = water depth (m)
I = water surface slope or energy gradient (-)

Using \( u_{*c} = (ghI)^{0.5} \) in Eq. (2.2.12), yields

\[ C = \frac{g^{0.5}}{\kappa} \left[ -1 + \ln \left( \frac{h}{z_o} \right) \right] = \frac{2.3 \ g^{0.5}}{\kappa} \log \left( \frac{0.4 \ h}{z_o} \right) \quad (2.2.20) \]

or using \( \kappa = 0.4 \) and \( g = 9.81 \text{ m/s}^2 \):

\[ C = 18 \ \log \left( \frac{0.37 \ h}{z_o} \right) \quad (2.2.21) \]

Using Eqs. (2.2.9), (2.2.10) and (2.2.11), yields:

Hydraulic smooth flow : \[ C = 18 \ \log \left( \frac{12 \ h}{3.3 \ v/u_{*c}} \right) \quad (2.2.22a) \]

\[ C = 18 \ \log \left( \frac{11.4 \ h}{vC/\bar{u}} \right) \quad (2.2.22b) \]

Hydraulic rough flow : \[ C = 18 \ \log \left( \frac{12 \ h}{k_s} \right) \quad (2.2.23) \]

Transitional flow : \[ C = 18 \ \log \left( \frac{12 \ h}{k_s + 3.3 \ v/u_{*c}} \right) \quad (2.2.24a) \]

\[ C = 18 \ \log \left( \frac{12 \ h}{k_s + 1.05 \ vC/\bar{u}} \right) \quad (2.2.24b) \]

The friction factor follows from: \[ f_c = \frac{8g}{C^2} \quad (2.2.25) \]
yielding for hydraulic rough flow:

\[ f_c = 0.24 \left[ \log(12h/k_s) \right]^{-2} \quad (2.2.26) \]

Equation (2.2.23) can be approximated by the Strickler formula (in the range of \( C = 40 \) to \( 70 \) \( m^{0.5}/s \)):

\[ C = 25 \left( \frac{h}{k_s} \right)^{1/6} \quad (2.2.27) \]

Another widely used resistance equation is the Manning equation, which reads as:

\[ \bar{u} = \frac{h^{2/3} I^{1/2}}{n} \quad (2.2.28) \]

in which:

- \( n = 0.045 (k_s)^{1/6} \) = Manning coefficient
- \( h \) = waterdepth (m)
- \( k_s \) = effective bed roughness (m)

It is noted that the waterdepth \( (h) \) should be replaced by the hydraulic radius \( R \) for narrow channels.

### 2.3 Waves

#### 2.3.1 Near-bed orbital velocities

Applying linear wave theory, the peak value of the orbital excursion \( \hat{A}_\delta \) and velocity \( \hat{U}_\delta \) at the edge of the wave boundary layer can be expressed as:

\[ \hat{A}_\delta = \frac{H}{2 \sinh(kh)} \quad (2.3.1) \]

\[ \hat{U}_\delta = \omega \hat{A}_\delta = \frac{\pi H}{T \sinh(kh)} \quad (2.3.2) \]

in which:

- \( \omega = 2\pi/T \) = angular velocity \((s^{-1})\)
- \( k = 2\pi/L \) = wave number \((m^{-1})\)
- \( H \) = wave height (m)
- \( L = (g1^2/2\pi) \tanh(kh) \) = wave length (m)
- \( T \) = wave period (s)
- \( h \) = wave depth (m)

Based on field measurements in the near-bed region inside the surf zone (with a relative wave height of \( H_r/h \) in the range of 0.2 to 0.45) and outside the surf zone \( (H_r/h = 0.14 \) to 0.27), Van Heteren and Stive (1985) found that Eq. (2.3.2) applied to instantaneous water surface elevations produced a maximum over-prediction of 25% of measured horizontal rms-velocities outside the surf zone and an underprediction of 5% inside the surf zone.
Dean (1986) reports that linear wave theory provides a good prediction of near-bottom kinematics for a wide range of relative wave heights and wave steepnesses.

An estimate of the near-bed peak velocities under the crest and trough of asymmetrical waves in shallow water can be obtained by applying higher order wave theories:

According to second order wave theory of Stokes (which is valid for \( h \geq 0.01 \, gT^2 \)), the peak velocities under the wave crest and trough are:

\[
\hat{U}_{\delta,\text{crest}} = \frac{\omega \, H}{2 \, \sinh(kh)} + \frac{3 \, \omega \, k \, H^2}{16 \, \sinh^4(kh)} \quad (2.3.3a)
\]

\[
\hat{U}_{\delta,\text{trough}} = \frac{\omega \, H}{2 \, \sinh(kh)} - \frac{3 \, \omega \, k \, H^2}{16 \, \sinh^4(kh)} \quad (2.3.3b)
\]

This yields: \( \hat{U}_{\text{crest}} / \hat{U}_{\text{trough}} = 1.1 \) for high wind waves (\( H/h = 0.2, \, h = 10 \, m, \, T = 7 \, s \)) and 1.7 for high swell waves (\( H/h = 0.2, \, h = 10 \, m, \, T = 14 \, s \)). This latter value may not be realistic because it is outside the validity range.

Dean (1986), Koyama and Iwata (1986) and Rienecker and Fenton (1981) used a stream function method to represent the higher order terms of the unsteady Bernoulli equation. Using a suitable stream function, the non-linear free surface boundary condition can be applied in a number of free surface points along the wave profile in an iterative way. Klopman (1989) made a numerical program based on the Fourier approximation method of Rienecker and Fenton (1981).

Comparison of measured and computed peak velocities shows good agreement for the crest peak velocities. The computed trough peak velocities are however much too small in shallow water (\( h < 3 \, m \)), see Kroon and Van Rijn (1993).

Based on analysis of measurements in shallow water, Van Rijn proposes for shallow water (\( h \leq 0.01 \, gT^2 \)):

\[
\hat{U}_{\delta,\text{crest}} = \alpha \, \hat{U}_\delta \quad (2.3.4a)
\]

\[
\hat{U}_{\delta,\text{trough}} = (2 - \alpha) \, \hat{U}_\delta \quad (2.3.4b)
\]

with \( \alpha = 1 + 0.3(H_c/h) \) and \( \hat{U}_\delta \) according to linear wave theory.

Another problem is the representation of the wave spectrum in a real situation. Assuming that the higher waves contribute most to the sediment transport process, the significant wave height (\( H_s \) or \( H_{1/3} \)) in combination with the peak period (\( T_p \)) are herein assumed to be the characteristic wave parameters for the sediment transport process.
2.3.2 Wave boundary layer

The wave boundary layer is a thin layer forming the transition layer between the bed and the upper layer of irrotational oscillatory flow (Fig. 2.3.1). The thickness of this layer remains thin (0.01 to 0.1 m) in short period waves ($T < 12$ s) because the flow reverses before the layer can grow in vertical direction. The boundary layer thickness $\delta_w$ can be defined as the minimum distance between the wall and a level where the velocity equals the peak value of the free stream velocity ($\hat{U}_f$), see Fig. 2.3.1.

![Figure 2.3.1 Wave boundary layer](image1)

![Figure 2.3.2 Hydraulic regimes in oscillatory flow](image2)
In case of laminar flow the following values can be given:

Jonsson 1980 : \[ \delta_w = \frac{2\pi}{\beta} \] (2.3.5)

Manohar 1955 : \[ \delta_w = \frac{4.6}{\beta} \] (2.3.6)

in which:
\[ \beta = \frac{(\pi/vT)^{0.5}}{\nu} \] = length parameter of Stokes (m)
\[ \nu \] = kinematic viscosity coefficient (m²/s)
\[ T \] = oscillation period (s)

In case of turbulent flow Jonsson and Carlsen (1976) proposed:

\[ \left( \frac{30 \delta_w}{k_s} \right) \log \left( \frac{30 \delta_w}{k_s} \right) = 1.2 \left( \frac{\hat{A}_\delta}{k_s} \right) \] for \( 10 < \frac{\hat{A}_\delta}{k_s} < 500 \) (2.3.7)

Equation (2.3.7) can also be represented by:

\[ \frac{\delta_w}{\hat{A}_\delta} = 0.072 \left( \frac{\hat{A}_\delta}{k_s} \right)^{-0.25} \] (2.3.8)

Equation (2.3.7) is based on theoretical and experimental research. Artificial triangular roughness elements have been used in wave tunnel experiments. The \( k_s \)-value of these roughness elements was determined from water surface slope measurements in a steady uniform flow yielding \( k_s \)-values in the range of 2.5 to 4 times the maximum height of the elements.

The theoretical results of Fredsøe (1984) can be roughly (error ± 20%) approximated by:

\[ \frac{\delta_w}{\hat{A}_\delta} = 0.15 \left( \frac{\hat{A}_\delta}{k_s} \right)^{-0.25} \] (2.3.9)

Although the wave boundary layer thickness is rather small, the generated shear stresses and turbulence intensities are rather large and are important for the sediment transport processes.

2.3.3 Hydraulic regime in waves

Jonsson (1966) first presented a plot of the hydraulic regime in oscillatory flow in terms of the Reynolds number and relative roughness, as follows:

\[ \text{Hydraulic regime} = F \left( \frac{\bar{U}_\delta \hat{A}_\delta}{\nu}, \frac{\hat{A}_\delta}{k_s} \right) \] (2.3.10)

Figure 2.3.2 shows the graph of Jonsson (1966, 1980). The results of Kamphuis (1975) are also shown.
The onset of turbulence in the wave boundary layer has been studied by many researchers (Sleath (1974, 1988); Davies (1985)). The results of Sleath (1974, 1988) are probably the most accurate results. Sleath investigated the onset of turbulence in oscillatory flow over flat beds of sand, gravel and pebbles. Both horizontal and vertical velocity fluctuations were measured using a laser Doppler velocity meter. For practical purposes the following expression can be applied to determine the transition to fully developed turbulent flow in case of a flat bed:

\[
\frac{(\bar{U}_{0}\Delta t)}{\omega \nu} = 5770 \left( \frac{\dot{A}_{0}}{d_{50}} \right)^{0.45}
\]

in which: \(d_{50}\) = median particle size of bed material.

### 2.3.4 Velocity distribution in wave boundary layer

#### 1. Experimental results

Detailed knowledge of the flow velocities in the boundary layer above rippled and flat beds is essential to understand the sediment entrainment processes. Examples of velocity distributions at maximum flow for a laminar and a turbulent case are shown in Fig. 2.3.1. The fundamental difference is the vertical mixing effect in turbulent flow giving a more uniform profile. Turbulent flow is the most interesting case for sediment transport processes because the flow will be turbulent in case of a mobile rippled bed regime and a flat bed (sheet flow) regime.

Several experimental studies have been performed in wave tunnels with rippled beds. The most interesting data are presented by DuToit (1982). He used a laser-Doppler velocity meter to measure the instantaneous horizontal velocities above the crest and trough of a sand ripple (turbulent flow).

Figure 2.3.3 shows the horizontal velocity component above a sand ripple during half a cycle. The essential features are, as follows:

- the free stream is moving with maximum velocity from right to left and a well defined vortex fills almost half the trough on the downstream side of the ripple,
- the vortex increases in size as the free stream decelerates,
- the vortex fills almost the whole trough,
- the free stream reverses and the vortex is being ejected over the crest and a strong surge of fluid (sweeping through the trough and spilling over the crest) is building up,
- the flow has already started separating at the crest and a new vortex starts to form,
- a jet (associated with the separation) shoots out over the trough and has almost reached the trough.

Figure 2.3.4 shows the variation range of the peak velocity as a function of the height above the crest for various test conditions and ripple dimensions. Both measurements in oscillating u-tubes and oscillating tray rigs were used. The ripples were self-formed natural sand ripples with \(\dot{A}_{0}/\lambda = 0.7\) and \(\Delta/\lambda = 0.17\). As can be observed, the peak velocity direct above the crest can be about 1.5 to 1.6 times as large as the peak velocity outside the boundary layer.

Another informative experimental study related to oscillatory flow over a rippled bed has been performed by Sato et al. (1984). The experiments were performed in an oscillatory water tunnel. Velocities above symmetrical and asymmetrical ripples were measured with split hot-film sensors under conditions of both sinusoidal and asymmetrical oscillating flow. The artificial ripples, which consisted of cement mortar, were copied from self-generated sand ripples.
Figure 2.3.3 Horizontal velocity field above rippled bed (DuToit, 1982)

\[ \hat{U}_0 = \text{peak velocity outside boundary layer} \]
\[ \hat{U} = \text{peak velocity at height } z_1 \text{ in boundary layer} \]
\[ z_1 = \text{height above crest} \]
\[ \lambda = \text{ripple length, } \beta = (\omega/2v)^{0.5} \]

Figure 2.3.4 Vertical distribution of peak velocity above ripple crest (DuToit, 1982)
Figure 2.3.5 shows the spatial distribution of the equiphase mean velocity, Reynolds stresses and mean pressures in half a period. The flow is seen to be locally accelerated above a ripple crest and decelerated above a trough. The Reynolds stress is large in the region of the lee vortex. As the vortex is ejected upwards, the region of high turbulence moves with it and then diffuses.

Figure 2.3.6 shows the time history of the mean velocity horizontal (u) and vertical (w) components and the turbulent quantities (u')^2 and (w')^2 and u'w' in points at 0.03 m above the ripple crest and the ripple trough (period = 4 s). The plotted values represent results averaged over 30 periods. Coherent vortices were formed in the lee side of the ripple when the velocity of the main flow reached its maximum. These vortices continued to develop until they were ejected after flow reversal. The ejected vortices were transported over the ripple during the next half period. The values of (u')^2 and (w')^2 show two peaks in each half period, which correspond to the passage of two vortices created in the lee side of the nearest and the neighbouring ripples. Values of u'w' above a crest show a strong peak according to the passage of the first vortex but they donot show a peak with the passage of the second vortex. It appears that turbulence maintains a coherent structure just after the vortex ejection and that, as the vortex moves upwards, turbulence decays and diffuses away.

Finally, the flow visualization experiments of Kaneko and Honji (1979) and Honji et al. (1980) are reported. Steady laminar vortices were generated above ripples in case \( \lambda \gamma / \lambda < 1 \) (standing vortices). Large separation vortices were found for \( \lambda \gamma / \lambda > 1 \). Similar results were observed by Soulsby et al. (1983) in field conditions using an underwater camera.

2. Theoretical models

Neglecting convective accelerations and vertical fluid velocities, the basic equation of motion for a flat bed, reads as:

\[
\frac{\partial U}{\partial t} + \frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{1}{\rho} \frac{\partial \tau}{\partial z} = 0
\]  
(2.3.12)

in which:

- \( U \) = instantaneous fluid velocity at height \( z \)
- \( P \) = instantaneous fluid pressure at height \( z \)
- \( \tau \) = instantaneous fluid shear stress at height \( z \)

Various researchers have applied Eq. (2.3.12) to determine the U-velocity. The basic problem is to relate the shear stress to the gradient of the velocity (\( \tau = dU/dz \)) using a turbulence closure model. The boundary condition is \( U = U_0 \) at \( z = \delta_w \). The pressure inside the boundary layer is assumed to be equal to that just outside the boundary layer where the flow is irrotational, giving: \( \partial P/\partial x = \rho \partial U/\partial t \)

One of the first attempts to describe the turbulent boundary layer by a mathematical model was done by Kajiura (1968). He adopted the eddy-viscosity concept in a three-layer model. In the small inner layer the eddy viscosity \( \varepsilon \) was taken as a constant. In the overlap layer \( \varepsilon \) was assumed to vary linearly with the distance from the bed and in the outer layer \( \varepsilon \) was kept constant at a certain distance from the bed. By use of the equation of motion, Kajiura obtained a rather laborious analytical-numerical solution. Brevik (1981) has simplified the analysis by Kajiura by avoiding the inner layer, where \( \varepsilon \) was kept constant.

2.13
Figure 2.3.5 Spatial distribution of velocity, Reynolds shear stress and pressure above a rippled bed (Sato et al., 1984)

Figure 2.3.6 Velocity and turbulence characteristics above ripple crest and trough (Sato et al., 1984)
Kajiura and Brevik tested their analytical results against the experimental work done by Jonsson (1966), see also Jonsson and Carlsen (1976), and obtained reasonable agreement. However, three shortcomings in Kajiura's and Brevik's theory must be mentioned: (i) they did not take into account that the eddy viscosity is a function of time as well as of the distance from the wall, (ii) the thickness of the wave boundary layer was also taken as a time-independent quantity, and finally (iii) the variation of the bed-shear stress was assumed to be sinusoidal.

Jonsson and Carlsen (1976) used the momentum equation integrated over the boundary layer thickness and the assumption of a logarithmic velocity profile to determine the wave friction factor for a rough wall. Their model does, however, not describe the time variation of the shear stress and the phase lag of the maximum shear stress relative to the maximum velocity in the outer layer.

Fredsoe (1984) made further developments along the line of Jonsson and Carlsen, still using the depth-integrated momentum equation and a logarithmic velocity profile, assuming that the flow field in each half period starts from rest. Fredsoe (1984) used the momentum equation to calculate the detailed variation of the boundary layer thickness and the shear stress during the wave period.

The most recent developments of mathematical models for the wave boundary layer have mainly been concentrated on the application of higher order turbulence models to describe the details of the flow. Bakker (1974) used a mixing length model to describe the wave boundary layer and the suspended sediment concentrations under waves. A two-equation model was used by Hagetum and Eidsvik (1986).

2.3.5 Bed shear stress and bed friction

1. Definition

Waves exert friction forces at the bed during propagation. The bed shear stress, which is important for wave damping and sediment entrainment, is related to the friction coefficient by:

\[ \tau_{b,w,t} = \frac{1}{2} \rho f_w \bar{U}_{\delta}^2 \]  \hspace{1cm} (2.3.13)

in which:
- \( \tau_{b,w,t} \) = instantaneous wave-related bed-shear stress (N/m²)
- \( f_w \) = friction coefficient (-)
- \( \bar{U}_{\delta} \) = instantaneous fluid velocity just outside boundary layer (m/s)
- \( \rho \) = fluid density (kg/m³)

The friction factor \( f_w \) is assumed to be constant over the wave cycle and is determined from the peak values as: \( f_w = 2 \hat{\tau}_{b,w}/(\rho \hat{U}_{\delta}^2) \).

The time-averaged (over half a wave cycle) bed shear stress is:

\[ \tau_{b,w} = \frac{1}{4} \rho f_w \hat{U}_{\delta}^2 \]  \hspace{1cm} (2.3.14)
2. Laminar flow

In the laminar range \((\tilde{U}_\delta \hat{A}_\delta/v < 10^4\), see Fig. 2.3.2) the friction coefficient can be determined analytically from Eq. (2.3.12) applying \(\tau = \rho v \frac{dU}{dz}\). This yields (Jonsson, 1966):

\[
f_w = 2 \left( \frac{\tilde{U}_\delta \hat{A}_\delta}{v} \right)^{-0.5}
\]  

(2.3.15)

3. Smooth turbulent flow

In the hydraulic smooth regime \((10^4 < \tilde{U}_\delta \hat{A}_\delta/v < 10^6\) and \(\hat{A}_\delta/k_s > 10^4\), zie Fig. 2.3.2) Jonsson (1966, 1980) obtained from his model:

\[
0.25 f_w^{0.5} + 2 \log(0.25 f_w^{-0.5}) = -1.55 + \log(\tilde{U}_\delta \hat{A}_\delta/v)
\]  

(2.3.16)

Equation (2.3.16) can be approximated by

\[
f_w = 0.09 \left( \frac{\tilde{U}_\delta \hat{A}_\delta}{v} \right)^{-0.2}
\]  

(2.3.17)

Kajiura (1968) obtained from his model:

\[
\frac{f_w^{-0.5}}{8.1} + \log(f_w^{-0.5}) = -0.135 + \log(\tilde{U}_\delta \hat{A}_\delta/v)^{0.5}
\]  

(2.3.18)

4. Rough turbulent flow

In the rough turbulent regime \((\tilde{U}_\delta \hat{A}_\delta/v > 10^5\) and \(\hat{A}_\delta/k_s < 100\), see Fig. 2.3.2) Jonsson (1966) proposed:

\[
0.25 f_w^{-0.5} + \log(0.25 f_w^{-0.5}) = -0.08 + \log(\hat{A}_\delta/k_s)
\]  

(2.3.19)

with \(f_{w,max} = 0.3\) for \(\hat{A}_\delta/k_s \leq 1.57\).

Equation (2.3.19) can also be expressed as (Swart, 1976):

\[
f_w = \exp\left[ -6 + 5.2 \left( \frac{\hat{A}_\delta}{k_s} \right)^{-0.19} \right]
\]  

(2.3.20)

with \(f_{w,max} = 0.3\) for \(\hat{A}_\delta/k_s \leq 1.57\).

Equations (2.3.15), (2.3.16), (2.3.19) are shown in Fig. 2.3.7. Equation (2.3.20) is shown in Fig. 2.3.8.

2.16
Figure 2.3.7  Wave friction coefficient for hydraulic smooth and rough conditions (Jonsson, 1966)

Figure 2.3.8  Wave friction coefficient for hydraulic rough conditions (Jonsson, 1966)
Little is known about the friction coefficient in the transition regime. It is proposed to use Eq. (2.3.20) replacing $k_s$ by $k_s' + 3.3$ $v/u_{*,w}$ with $u_{*,w} = (\tau_{b,w}/\rho)^{0.5}$, similar to Eq. (2.2.24).

2.3.6 Breaking waves

1. Limiting steepness

Waves break when the wave steepness ($H/L$) exceeds a critical value. A criterion for a horizontal bottom has been given by Miche (1944):

$$(H/L)_{br} = 0.142 \tanh(kh) \tag{2.3.21}$$

yielding a maximum value of 0.14 for deep water.

Miche (1951) also derived an expression for the limiting (deep water) wave steepness on a sloping bottom assuming full reflection ($\beta = \text{bottom slope angle}$):

$$(H_o/L_o)_{br} = \left[\frac{2\beta}{\pi}\right]^{0.5} \frac{\sin^2\beta}{\pi} \tag{2.3.22}$$

2. Limiting wave height on a horizontal bottom

The limiting wave height corresponds to the maximum wave height at the crest just before breaking. Miche (1944) computed the limiting wave height on a horizontal bed from a Stokes wave solution by assuming that breaking occurs when the fluid velocity at the crest equals the wave propagation velocity, yielding for shallow water ($kh << 1$):

$$\gamma = \frac{H_{br}}{h} = 0.88 \tag{2.3.23}$$

Since Stokes waves are symmetrical, Equation (2.3.23) represents a spilling breaking wave. For a solitary shallow-water wave the theoretical limiting value is:

$$\gamma = \frac{H_{br}}{h} = 0.78 \tag{2.3.24}$$

Based on the analysis of flume tests with regular and irregular waves over a horizontal bottom, it was found that (Nelson, 1983):

$$\gamma = \frac{H_{br}}{h} \leq 0.55 \tag{2.3.25}$$

3. Limiting wave height on a sloping bed

Nelson (1983) also found that bottom slopes as small as 0.01 (1:100) affect the wave mechanics sufficiently to increase the $H_{br}/h$-parameter for individual waves. The wave motion on a plane sloping bed is mainly determined by the bottom slope ($\tan \beta$), the incident wave height ($H$) just before breaking, the wave period ($T$) and the gravity acceleration ($g$) giving two dimensionless parameters: $\tan \beta$ and $H(gT^2)$. The latter parameter usually is expressed as $H/L_o$ with $L_o = (gT^2)/2\pi$. 

2.18
Waves reflect rather than break when the bed slope is rather large (full reflection against a vertical wall). When the bottom slope $\beta$ decreases beneath a certain critical value, the waves become unstable and break.

A common breaking criterion is:

$$\xi = \frac{\tan \beta}{(H/L_0)^{0.5}} < 2.3$$  \hspace{1cm} (2.3.26)

Generally, three types of breaking waves are distinguished:

- surging breaker in case of flat incident waves and a large bed slope; the amount of breaking is small and occurs close to the shoreline, reflection is large ($2 < \xi < 2.3$),
- plunging breaker when the wave steepness is larger or the bed steepness is smaller; a curling jet is generated plunging in the wave trough ahead accompanied by large turbulence production and a forward marching bore after breaking ($0.4 < \xi < 2$),
- spilling breaking in case of relatively steep waves and/or a relatively flat bed; the water surface near the crest spills as the wave crest sharpens with the decrease in water depths; breaking occurs at a greater distance offshore where the wave crest is still symmetric and is accompanied by the generation of a foamy roller at the wave front while the wave trough ahead is not visibly disturbed ($\xi < 0.4$).

Plunging breakers show strong mixing properties. During breaking a jet is generated which strikes upon the forward slope of the wave generating a series of eddies that contribute to the rapid decay of the wave. Each eddy moves forward and downward up to the bottom yielding a strong mixing process. The forward velocity of the eddies is smaller than the wave (bore) velocity and therefore the eddies move toward the back of the wave where they expand under escape of the entrained air bubbles which rise to the surface (Miller, 1976).

This process eventually transforms into a bore that moves up to the shore. New waves may be generated and go through the breaking process again, but closer to the shore.

Spilling breakers have less effective mixing properties because of the relatively small scale eddies, which are confined to the near-water surface region and do not extend much below the trough level.

Waves heights of individual breaking waves in field conditions ($\gamma = H_{br}/h$) are given by Kana (1979):

- Spilling breakers : $\gamma = 0.55 - 0.65$
- Transitional breakers : $\gamma = 0.65 - 0.75$
- Plunging breakers : $\gamma = 0.75 - 0.90$

Within each range the $\gamma$-values were found to depend on the local bottom slope. A larger bottom slope yields a larger breaker height.

In flumes the following values have been observed:

- Spilling breakers : $\gamma = 0.65$ for slope 1:20
  $\gamma = 0.60$ for slope 1:40
  $\gamma = 0.55$ for slope 1:60
- Plunging breakers : $\gamma = 0.90$ for slope 1:20
  $\gamma = 0.80$ for slope 1:40
  $\gamma = 0.70$ for slope 1:60

2.19
Battjes (1974) related the $\gamma$-ratio to the $\xi$-parameter. Based on the analysis of field and flume data, the following values are given:

\[
\begin{align*}
\gamma &= 0.8 \quad \text{for} \quad \xi \leq 0.2 \\
\gamma &= 0.9 \quad \text{for} \quad \xi = 0.4 \\
\gamma &= 1.0 \quad \text{for} \quad \xi = 0.6 \\
\gamma &= 1.1 \quad \text{for} \quad 1 \leq \xi \leq 2.
\end{align*}
\]

Based on the analysis of laboratory experiments with regular waves and plane sloping bottoms, Weggel (1972) found:

\[
\gamma = \frac{1.56}{(1 + e^{-19.5 \tan \beta})} - \frac{43.8}{2\pi} \frac{1 - e^{-19 \tan \beta}}{H_{br}} \frac{H_{br}}{L_o}
\]

(2.3.27)

with $L_o = g T^2/2\pi$ = wave length at deep water.

Equation (2.3.27) yields $\gamma = 0.78$ for $\tan \beta = 0$.

Thornton - Guza (1982) and Sallenger - Howd (1989) state that in the surf zone $\gamma_s$ (or $\gamma_{rms}$) is approximately constant and independent of the offshore wave conditions because nearly all waves are breaking in the surf zone (wave energy saturation).

Thornton and Guza (1982) report field measurements on a sloping bottom 1:50. They observed a limiting breaker height (in terms of the rms wave height) of $\gamma_{rms} = 0.4$ ($\gamma_s = 0.55$) for spilling breakers. Sallenger and Howd (1989) report $\gamma_{rms} = 0.3$ ($\gamma_s = 0.45$) for a local bottom slope of $\tan \beta = 0.03$ and $\gamma_{rms} = 0.4$ ($\gamma_s = 0.55$) for $\tan \beta = 0.06$.

Hotta and Mizuguchi (1980) did detailed wave height measurements in the surf zone with a sloping bottom (varying from 1:100 to 1:50). Wave heights were measured simultaneously over a distance of about 100 m with space intervals of 2 m. The breakerline (maximum wave height) was about 100 m offshore. They found an almost constant ratio $\gamma_s = H_{br}/h = 0.7$ in the region where the water depth was continuously decreasing, indicating fully energy-saturated waves. The significant wave period $T_s$ was also constant in this zone.

Based on measurements of Kroon and Van Rijn (1992) performed in the inner surfzone (near Egmond, The Netherlands) with bed slopes in the range of 0.03 to 0.1 and water depths in the range of 0.4 to 1.4 m, the following results were found:

- no breakers: $\gamma_s \leq 0.4$
- spilling breakers: $\gamma_s = 0.4 - 0.6$
- plunging breakers: $\gamma_s = 0.6 - 0.9$
- surging breakers (swash zone): $\gamma_s = 0.9 - 1.2$

2.3.7 **Mass transport in non-breaking waves**

1. *Non-viscous oscillatory flow*

Stokes (1847) first pointed out that the fluid particles do not describe exactly closed orbital trajectories in case of small-amplitude surface waves propagating in a perfect non-viscous (irrotational) oscillatory flow. The particles have a second-order mean Lagrangian velocity (called "Stokes' drift") in the direction of wave propagation. It results from the fact that the
horizontal orbital velocity increases with height \(z\) above the bed. Consequently, a particle at the top of an orbit beneath a wave crest moves faster in the forward direction than it does in the backward direction at the bottom of the orbit beneath a wave trough. By definition the Lagrangian Stokes drift cannot be detected by taking measurements at a fixed point. The instantaneous horizontal Stokes drift \(U_i\) of a water particle with a mean position of \(x_i\) and \(z_i\) is \(U_i(x_i + \alpha, z_i + \beta)\) where \(\alpha\) and \(\beta\) are the coordinates of the particle position on the trajectory. An approximation of \(U_i\) is:

\[
U_i(x_i + \alpha, z_i + \beta) = U(x_i, z_i) + \alpha \frac{\partial U}{\partial x} + \beta \frac{\partial U}{\partial z} \tag{2.3.28}
\]

Applying linear wave theory, and averaging over the wave period, yields the time-averaged velocity (indicated by over bar). (Fig. 2.3.9):

\[
\overline{U}_i(z) = \frac{1}{8} \omega \ k \ H^2 \ \frac{\cosh[2k(z-h)]}{\sinh^2(kh)} \tag{2.3.29}
\]

in which:
- \(\overline{U}_i\) = Stokes net velocity (ratio of net horizontal displacement and wave period)
- \(\omega\) = \(2\pi/T\) = wave frequency
- \(k\) = \(2\pi/L\) = wave number
- \(z\) = vertical coordinate (positive downwards from mean water level)

At the bed \((z=h)\):

\[
\overline{U}_i = \frac{\omega \ k \ H^2}{8 \ sinh^2(kh)} \tag{2.3.30}
\]

At the surface \((z=0)\):

\[
\overline{U}_i = \frac{\omega \ k \ H^2 \ \cosh(2kh)}{8 \ sinh^2(kh)} \tag{2.3.31}
\]

For waves propagating in a horizontally unbounded domain the depth-integrated volume flux \(m^2/s\) is:

\[
M_s = \int_h^\circ \overline{U}_i(z) \ dz = \frac{\omega \ H^2 \ \sinh(2kh)}{16 \ sinh^2(kh)} = \frac{\omega \ H^2}{8} \ \coth(kh) = \frac{g \ H^2}{8 \ c} \tag{2.3.32}
\]

in which:
- \(c = (g/T)/(2\pi \ \coth(kh))\) = wave celerity

Equation (2.3.32) reduces to \(M_s = \omega \ H^2/8\) for deep water \((kh > 1)\).

For waves propagating in a horizontally bounded domain it is appropriate to impose a condition of zero volume flux at each location \((x)\), yielding (see Fig. 2.3.9):

\[
\overline{U}_i(z) = \frac{\omega \ k \ H^2}{8 \ sinh^2(kh)} \left[ \cosh[2k(z-h)] - \frac{\sinh(2kh)}{2kh} \right] \tag{2.3.33}
\]

Equation (2.3.33) can be seen as the sum of the forward Stokes drift and an uniform return flow. The generation of a positive volume flux near the surface and a negative flux near the bottom requires the presence of a horizontal pressure gradient (shear stresses are absent in
a non-viscous flow) caused by a "set-up" of the free surface towards the coast (similar to wind set-up).

The volume flux \((m^2/s)\) at a fixed location \((x)\) in an unbounded domain can also be determined in an Eulerian way, as follows:

\[
M_e = \frac{1}{T} \int_0^T \int_0^h u(t,z) \, dz
\]

in which:
\(U\) = instantaneous horizontal velocity at height \(z\)
\(\eta\) = water surface displacement from the mean surface level

Below the wave trough the time averaged value of the horizontal velocities is zero. However, in the region between the wave crest and the trough there is an asymmetry of the horizontal velocity indicating that more fluid moves in the wave direction under the crest than in the trough region.

For small-amplitude waves Eq. (2.3.34) yields:

\[
M_e = \frac{g H^2}{8 \, c}
\]

The Eulerian and the Lagrangian method yield the same depth-integrated volume transport. The vertical distribution of the time-averaged transport velocities, however, is different for both methods.

Dalrymple (1976) has presented information of the vertical distribution of the Eulerian velocities in the region between the wave crest and the trough, as shown in Fig. 2.3.10.

2. **Viscous and turbulent oscillatory flow**

Longuet-Higgins (1953) has shown that for real fluids with viscosity \(\nu\) there is a time-averaged net downward transfer of momentum into the boundary layer by viscous diffusion \((\nu \, \partial U/\partial z)\) causing a mean Eulerian flow \((\bar{U}_e)\) in addition to the Stokes drift \((\bar{U}_s)\). The mean Eulerian flow can be seen as the mean velocity of the orbit centers.

The total time-averaged transport velocity \((\bar{U}_m)\) is defined, as:

\[
\bar{U}_m = \bar{U}_e + \bar{U}_s = \bar{U}_e + \frac{\partial U}{\partial x} \int_0^T U \, dt + \frac{\partial U}{\partial z} \int_0^T V \, dt
\]

For laminar flow in the boundary layer Longuet-Higgins (see Russel and Osorio, 1957) derived:

\[
\bar{U}_m(z) = \frac{\omega k H^2}{16 \sinh^2(kh)} \left[ 5 - 8e^{-z/\delta} \cos(\pi z/\delta) - 3e^{-2z/\delta} \right]
\]

in which:
\(\delta = (2\nu/\omega)^{0.5}\) = thickness of laminar boundary layer
Equation (2.3.38), shown in Fig. 2.3.11, has a maximum value

$$\bar{U}_{m,max} = 1.376 \frac{\omega \ k \ H^2}{4 \ \sinh^2(\kh)} = 1.376 \frac{(\bar{U}_b)^2}{c} \quad (2.3.39)$$

For $z/\delta \to \infty$, Equation (2.3.38) yields:

$$\bar{U}_m = \frac{5}{16} \frac{\omega \ k \ H^2}{\sinh^2(\kh)} = \frac{5}{4} \frac{(\bar{U}_b)^2}{c} \quad (2.3.40)$$

in which:
- $\bar{U}_b$ = peak value of orbital velocity just outside boundary layer
- $c$ = wave celerity ($\omega/k$)

Assuming zero volume flux over the full water depth, Longuet-Higgins (1953) derived:

$$\bar{U}_m(z) = \bar{U}_s(z) + \bar{U}_e(z) = \frac{\omega \ k \ H^2}{8 \ \sinh^2(\kh)} F(z/h) \quad (2.3.41)$$

$$F(z/h) = \cosh[2k(z-h)] + \frac{3}{2} + \frac{kh}{2} \sinh[(2kh)(3 \frac{z^2}{h^2} - 4 \frac{z}{h} + 1)] +$$

$$+ \frac{3}{2} \left[ \frac{\sinh(2kh)}{2kh} + \frac{3}{2} \left[ \frac{z^2}{h^2} - 1 \right] \right] \quad (2.3.42)$$

The function $F(z/h)$ is shown in Fig. 2.3.12 for some values of $kh$ (see also Fig. 2.3.9).

Equation (2.3.41) can be seen as the sum of the forward Stokes drift (Eq. (2.3.29)) and a parabolic velocity distribution. This latter distribution gives a forward flow at the bottom and a backward flow at mid-depth.

Equation (2.3.41) is valid for $H < 2\delta$, yielding a wave height range that is of little practical interest. Based on a comparison with experimental results (Russel and Osorio, 1957), fairly good predictions are obtained for $0.7 < kh < 1.5$.

At the bottom ($z=h$) Equation (2.3.41) yields (see also Fig. 2.3.11):

$$\bar{U}_s = \frac{1}{8} \frac{\omega \ k \ H^2}{\sinh^2(\kh)} = \frac{1}{2} \frac{(\bar{U}_b)^2}{c} \quad (2.3.43)$$

$$\bar{U}_e = \frac{3}{16} \frac{\omega \ k \ H^2}{\sinh^2(\kh)} = \frac{3}{4} \frac{(\bar{U}_b)^2}{c} \quad (2.3.44)$$

$$\bar{U}_m = \frac{5}{16} \frac{\omega \ k \ H^2}{\sinh^2(\kh)} = \frac{5}{4} \frac{(\bar{U}_b)^2}{c} \quad (2.3.45)$$
Figure 2.3.9  Net time-averaged transport velocities in non-breaking waves (Stokes 1847, Longuet-Higgins 1953, Craik 1982)

Figure 2.3.10  Net time-averaged transport velocities in non-breaking waves (Dalrymple, 1976)
For oscillatory flow in an unbounded domain the Eulerian time-averaged transport velocity (due to viscosity effects) can be described by (Craik. 1982):

\[
\bar{U}_e(z) = \frac{3}{16} \frac{\omega k H^2}{\sinh^2(kh)} + \frac{1}{2} \omega k^2 H^2 (h-z) \coth(kh)
\]  

Equation (2.3.46) is shown in Fig. 2.3.9 for \( kh = 1 \).

The depth-integrated volume flux is:

\[
\mathbf{M} = \int_{h}^{\infty} (\bar{U}_s + \bar{U}_e) dz = \frac{g H^2}{8 c} + \frac{3}{16} \frac{\omega k h H^2}{\sinh^2(kh)} + \frac{1}{4} \omega k^2 h^2 H^2 \coth(kh)
\]

Longuet-Higgins also showed that Eq. (2.3.45) can be used to describe the time-averaged transport velocity just outside the boundary layer in case of turbulent smooth oscillatory flow. This was confirmed by Johns (1970). For turbulent conditions the velocity distribution inside the boundary layer is different from Eq. (2.3.38) because the turbulent (eddy) viscosity differs from the molecular viscosity. Information of the time-averaged transport velocities in turbulent rough conditions is rather scarce. Computed results of Brøker Hedegaard (1985) are shown in Fig. 2.3.13.

Finally, some measuring results are presented. Figure 2.3.14 shows measured Eulerian net velocities for oscillatory flow over a smooth bottom in a laboratory channel (Borghei, 1982). The measurements were made in fixed points using a Laser Doppler velocity meter. Comparison with the net velocities of Longuet-Higgsens shows reasonable results.

Figure 2.3.15 presents measured residual Eulerian velocities for sinusoidal oscillatory flow over a symmetrical ripple, showing a pair of circulation cells. These patterns seem to be strongly influenced by the formation of lee vortices.

Bijker et al (1974) confirmed the presence of drift velocities as proposed by Longuet-Higgins. However, the measured velocities were found to be considerably smaller than the theoretical values. Bijker et al concluded that the near-bed net velocities on a sloping bottom (in the range of 1:10 to 1:40) are determined by the local depth rather than the slope angle. Their results also show a considerable reduction of the near-bed net velocity when ripples are present on the bottom.

![Net time-averaged velocities in wave boundary layer of non-breaking waves](image)

**Figure 2.3.11** Net time-averaged velocities in wave boundary layer of non-breaking waves (Longuet-Higgins, 1953)
Figure 2.3.12 Vertical distribution of net velocities in non-breaking waves (Longuet-Higgins, 1953)

Figure 2.3.13 Eulerian net velocities in non-breaking waves (B. Hedegaard, 1985)

Figure 2.3.14 Measured net velocities in non-breaking waves (Borghei, 1982)
Figure 2.3.15 Measured net (Eulerian) velocities over a rippled bed in non-breaking sinusoidal waves (Sato et al., 1984)

Horizontal component, \( \bar{u} \).

Figure 2.3.16 Net velocities in breaking waves (Nadaoka et al., 1982)

Figure 2.3.17 Net velocities in breaking waves
2.3.8 Mass transport by breaking waves

Breaking waves generate a net current in longshore direction (longshore current) and in offshore direction (undertow).

Herein, the attention is focussed on the undertow. The measurements of Nadaoka et al (1982) and Stive (1980, 1983) using a Laser Doppler velocity meter in flume conditions clearly show the generation of offshore-directed velocities inside the breaker zone, see Fig. 2.3.16. The measured velocities of Nadaoka et al represent time-averaged values of 70 wave cycles. The measured net velocities above the wave trough are based on the assumption of zero-velocity during the period that the measuring location was above the water surface. They also analyzed the time variations of the velocities showing a strong asymmetric motion with relatively large velocities of short duration in onshore direction and relatively small velocities of long duration in offshore direction.

Above the trough level there is a net volume flux in onshore direction. According to a first order approximation, the volume flux above the trough level can be estimated as:

\[ M = \frac{g H^2}{8 c} \]  
(2.3.48)

Applying \( c = (gh)^{0.5} \) in shallow water, it follows that:

\[ M = \frac{1}{8} g^{0.5} h^{-0.5} H^2 \]  
(2.3.49)

Assuming no net flow over the depth, the time-averaged return flow, which is also called the undertow, below the trough level is given by (see Fig. 2.3.17):

\[ \bar{U}_{m,off} = \frac{1}{8} g^{0.5} h^{-0.5} h_t^{-1} H^2 \]  
(2.3.50)

Taking \( h_t = 0.8h \), it follows that:

\[ \bar{U}_{m,off} = 0.15 g^{0.5} h^{-1.5} H^2 \]  
(2.3.51)

2.4 Combined current and waves

2.4.1 Introduction

When waves propagate into shallow waters near coasts they may encounter relatively strong currents which affect the wave characteristics, the current velocities and the bed-shear stresses.

Herein, the main attention is focussed on the latter two phenomena. Modification of the wave height by the current effect is not studied.

2.28
2.4.2 Wave characteristics

Especially, opposing currents have a significant influence by steepening the waves even to the point of breaking (Brevik and Aas, 1980). A following current enlarges the wave trough and thereby the wave length and reduces the wave height (Brevik and Aas, 1980).

Linear wave theory can be applied with respect to a coordinate system moving with the current velocity (Jonsson et al., 1970; Jonsson, 1978). In that case the wave length remains constant and the wave period is altered: larger for a following current ($\bar{u} > 0$) and smaller for an opposing current ($\bar{u} < 0$).

A general expression for the wave period ($T_r$) relative to the moving coordinate system is:

$$T_r = \frac{T}{1 - (\bar{u} T \cos \phi)/L'}$$

(2.4.1)

in which:
- $\bar{u}$ = depth averaged current velocity
- $\phi$ = angle between current and wave direction ($\phi = 0$ for following, and $\phi = 180^\circ$ for opposing)
- $L'$ = wave length in presence of waves from $[L'/T - \bar{u} \cos \phi]^2 = \left(\frac{gL'/2\pi}{\tanh(2\pi h/L')}\right)$
- $c$ = wave propagation velocity
- $T$ = absolute propagation velocity

2.4.3 Current velocities and bed-shear stresses

1. Experimental results

Laboratory experiments show a distinct influence of the waves on the current velocity profile (Bakker and Van Doorn, 1978; Kemp and Simons, 1982, 1983; Nieuwjaar and Van der Kaaij, 1987; Nap and Van Kampen, 1988; Van der Stel and Visser, 1985; Visser, 1986).

Kemp and Simons (1982, 1983) measured mean velocities in regular waves with following and opposing currents over a smooth bed and over a rough bed (5 mm high triangular strips spaced at 18 mm). The experiments were performed in a wave flume (length = 14.5 m, width = 0.46 m, height = 0.70 m) with wave generation by a bottom-hinged paddle. Considerable attention was given to the establishment of an optimum in- and outflow of the current. The still water depth was 0.2 m and the wave period was 1 sec. The relative wave heights ($H/h$) were in the range of 0.1 to 0.25. Figure 2.4.1 presents the results of Kemp and Simons. Some results were only reported qualitatively.

The following phenomena can be observed:

- following current:
  - smooth bed: increased velocities near the bed, reduced velocities near the water surface, larger effect for higher waves
  - rough bed: reduced velocities near the bed and water surface, increased velocities at intermediate depths
opposing current:

smooth and rough bed  •  reduced velocities near the bed  
                      •  increased velocities near the surface  
                      •  larger effect for higher waves

Kemp and Simons also determined the current-related bed-shear stress ($\tau_{b,c}$) and the effective roughness ($k_s$) by plotting the velocities measured in the lower layer on a semi-logarithmic scale. The current-related bed-shear stress for a combination of waves and current was a factor 2 to 3 larger than for a current alone. The effective roughness for a current with opposing waves was 0.18 m, which was as large as the water depth(!), while that for a current alone was 0.025 m. The bed roughness in combined current and waves should be interpreted as an apparent bed roughness ($k_s$) because it largely reflects flow resistance due to pressure forces generated by the strong vortices moving forward and backward over the roughness elements. Therefore, this $k_s$-value cannot be related to the geometrical properties of the roughness elements.

Nieuwjaar-Van der Kaaij (1987) and Nap-Van Kampen (1988) measured time-averaged velocities using an electromagnetic velocity meter in irregular waves over a (natural) rippled bed with following and opposing currents.

The experiments were performed in a wave flume (length = 45 m, width = 0.8 m and depth = 1 m). The height and length of the sand ripples were resp. 0.02 m and 0.15 m. The wave heights were in the range of $H_s/h = 0.15-0.35$. The peak period of the waves was about $T_p = 2.5$ s. The fluid velocities in each point were averaged over time (= 256 s) and over space (= 0.6 m). 

Space-averaging was performed by moving the measuring carriage on top of the flume slowly forward and backward during the measuring period.

The results of Nieuwjaar-Van der Kaaij are shown in Figure 2.4.2. The following phenomena can be observed.

following currents:

• reduced velocities near the bed ($z/h < 0.2$)
• reduced velocities near the water surface ($z/h > 0.6$)
• increased velocities at intermediate depths
• most pronounced effect in case of a weak current and high waves

opposing current:

• reduced velocities near the bed ($z/h < 0.4$)
• increased velocities near the water surface ($z/h > 0.5$)
• most pronounced effect in case of a weak current and high waves.

Visser (1986) measured time-averaged velocity profiles in a wave-current basin with an angle of 90° between the current direction and the wave direction (regular waves). A smooth bed and a gravel bed ($d_{90} = 0.005$ m) was used. The current velocities were measured with an immersible Laser Doppler velocity meter. The velocities were averaged over 100 s. Relative wave heights were in the range of 0.25 to 0.5. The wave period was 1 s.
Figure 2.4.1 Influence of waves on current velocity profile; waves following and opposing the current (Kemp and Simons, 1982, 1983)
Figure 2.4.2 Influence of waves on current velocity profile; waves following and opposing the current (Nieuwjaar and Van der Kaaij, 1987)

- $\bar{u} \approx 0.1 \text{ m/s following}$
- $\bar{u} \approx 0.4 \text{ m/s following}$
- $\bar{u} \approx -0.1 \text{ m/s opposing}$
- $\bar{u} \approx -0.4 \text{ m/s opposing}$

- $H_s \approx 0.75 \text{ m}$
- $H_s \approx 0.10 \text{ m}$
- $H_s \approx 0.12 \text{ m}$
- $H_s \approx 0.15 \text{ m}$

- $\circ$ current and waves
- $\times$ current alone

$H_s =$ significant wave height  
$U_m =$ depth-mean velocity
The mean current velocity was in the range of 0.1 to 0.2 m/s. Some results are presented in Figure 2.4.3, showing reduced velocities in the near-bed region and increased velocities in the near water surface region. Havinga (1992) measured time-averaged velocity profiles over a rippled sediment bed in a wave-current basin. Relative wave heights were in the range of 0.2 to 0.35. The mean current velocity was in the range of 0.1 to 0.3 m/s. The angle between the wave and current direction was 60°, 90° and 120° respectively. The largest reduction of the near-bed velocities was observed for an angle of 90°.

Summarizing all results, it can be concluded that the current velocities in the near-bed region are reduced by the wave-induced vortices in the wave boundary layer. This effect is most pronounced in case of a relatively weak current and relatively high waves. In case of an opposing current the reduction of the near-bed velocities is somewhat larger than in case of a following current. When the wave direction is perpendicular to the current direction, the reduction of the near-bed velocities is largest. The current velocities in the upper layers are increased in case of an opposing current and in case the waves are perpendicular to the current. When the waves are in the same direction as the current (following), the velocities near the surface are reduced.

2. Mathematical models

Most theoretical models have some common features which can be summarized as follows:

- the wave motion is a potential flow over the entire water depth except in the thin wave boundary layer,
- the wave potential flow does not interact with the turbulence of the steady flow,
- the steady flow outside the wave boundary layer can be treated independently from the wave motion.

Inside the wave boundary layer turbulence generated by the wave and the current motion will affect the velocity profile of the mean current. Due to the non-linear interaction of the mean current and the wave boundary layer all theoretical models predict an increase of the flow resistance for the mean current. The effect can be described as an apparent wave roughness (κw), which is larger than the physical bed roughness (κb). The presence of the waves will therefore cause a larger bed-shear stress for a given flow rate or a reduced flow rate for a given bed shear stress. The models for combined wave and current boundary layers can be divided in two main groups: models based on a combination of time-constant eddy viscosity profiles and models based on higher-order turbulence closures.

One of the first models was the engineering method of Bijker (1967). It is based on the mixing length concept. Bijker considered a certain point (z' - c zo) above the bed at which a line emanating from the bottom (z=0) is tangential to the current velocity profile. The wave-induced orbital motion in the boundary layer is assumed to have a logarithmic profile and thus a straight line can be drawn emanating from the bed to be also tangential to the wave velocity profile. Therefore, once the velocity at the level z' - c zo is described, the velocity gradient at this level is automatically known and the shear stress can be determined from the mixing length theory.

The model of Lundgren (1972) includes a description of the conditions in the wave boundary layer. The eddy viscosity distribution inside the boundary layer was estimated from measured values averaged over a wave period (see Fig. 2.4.4).

The mean current velocity profile is assumed to have the normal logarithmic shape above the wave boundary layer, but the increased eddy viscosity in the wave boundary layer will give a higher flow resistance than corresponding to the natural bed roughness alone.
Figure 2.4.3 Influence of waves on current velocity profile; waves perpendicular to current (Visser, 1986)
The characteristics of the wave boundary layer are determined from the wave parameters alone, without taking the current into account. The model is therefore only valid in situations with a relatively weak current.

The model for combined wave and current motion by Smith (1977) is based on a linear eddy viscosity distribution in the wave boundary layer (see Fig. 2.4.4). This eddy viscosity distribution is continuous at $z = \delta_w$, but it is hardly realistic that the eddy viscosity over the entire flow depth receives a contribution from the wave-induced eddy viscosity. It must be expected that the contribution from the wave-generated turbulence gradually decays with the distance from the boundary layer.

Grant and Madsen (1979) presented a model which in some respects is similar to that of Smith (1977). They also apply a time constant eddy viscosity. The model has been improved to cover an arbitrary angle between the waves and the current. The eddy viscosity varies linearly with the distance from the bed. Inside the wave boundary layer the slope is determined by the largest shear velocity from the waves and the current (see Fig. 2.4.4). Outside the wave boundary layer, the eddy viscosity is determined by the mean shear stress. The eddy viscosity is discontinuous at $z = \delta_w$. The mean current is given as two logarithmic velocity profiles intersecting at $z = \delta_w$.

Some modifications to the model of Grant and Madsen (1979) have been introduced by Christoffersen and Jonsson (1985). They apply a constant eddy viscosity in the wave boundary layer. The eddy viscosity distribution is given in Fig. 2.4.4.

The model of Fredsøe (1984) does not apply the eddy viscosity directly in the description of the phenomena. The basic assumption is that the time scale for production and decay of turbulent kinetic energy is small compared to the wave period. Based on this, the formation of the wave boundary layer can be treated independently for each half wave period, and the velocity profile in the wave boundary layer can be assumed to be logarithmic, corresponding to instantaneous equilibrium in the turbulence energy. The development of the wave boundary layer during each half wave period is described by the momentum equation integrated over the wave boundary layer thickness. The current velocity profile outside the wave boundary layer is used as a boundary condition in the momentum equation. This outer profile is described by a logarithmic velocity profile, determined through the apparent wave roughness ($k_w$) analogous to other models, e.g. Grant and Madsen (1979). The eddy viscosity concept is not used directly in this model, but the (time-variant) eddy viscosity can be estimated from the bed shear stress and boundary layer thickness calculated by the model.

The model of Bakker and Van Doorn (1978) applies a mixing length turbulence model; the eddy viscosity is a function of the local flow conditions. The eddy viscosity varies with time and with the distance from the bed. It is therefore interesting, that a comparison made by Bakker and Van Doorn between their theory and that of Lundgren shows results of the same order of magnitude, though Lundgren assumes a time constant eddy viscosity.

Davies et al. (1988) have presented a model based on a one equation turbulence model (k-equation). A two-equations model has been presented by Iliagatum and Eidsvik (1986). Similar models were presented by Myrhaug et al (1990) and Huyhn Thanh et al (1991). The flow resistance predicted by the models of Lundgren (1972), Bakker and Van Doorn (1978), Grant and Madsen (1979) and Fredsøe (1984) was compared (by Davies et al), where it was possible to make the calculations based on the information given in the papers.
Figure 2.4.4  Eddy viscosity distributions

Figure 2.4.5  Comparison of apparent roughness according to various models
Figure 2.4.6  Apparent roughness for following waves ($\phi = 0^\circ$) and waves perpendicular ($\phi = 90^\circ$) to current according to Fredsøe (1984)

Figure 2.4.7  Vertical distribution of current velocity and wave velocity profile
The results are shown in Fig. 2.4.5 depicting the increase in the apparent roughness \( k_a \) acting on the mean current outside the wave boundary layer versus the ratio between the maximum wave orbital velocity \( \bar{U}_d \) and the mean bed shear velocity from the current \( u_{*c} \).

 Soulsby et al (1991) presented an intercomparison of various numerical models. They found a broad similarity between theories in many cases.

The model of Fredsøe was found to give reasonable results. This is also concluded by Visser (1986) comparing his experimental results (waves perpendicular to currents) with the results of the Fredsøe model.

Finally, Figure 2.4.6 is presented showing basic results of the Fredsøe-method for \( \phi = 0^\circ \) (waves following the current) and \( \phi = 90^\circ \) (waves perpendicular to the current).

3. **Engineering methods**

A. **Bijker (1966, 1986)**

Bijker estimated the resulting bed-shear stress of combined wave-current conditions by considering the near-bed flow structure and by assuming the validity of the logarithmic distribution.

**current**

\[
u = \frac{u_{*c}}{\kappa} \ln \left( \frac{z}{z_o} \right)
\]

(2.4.2)

at level \( \eta = e z_o \), it follows that (see Fig. 2.4.7):

\[
u_\eta = \frac{u_{*c}}{\kappa} \text{ or } \tau_{b,c} = \rho \kappa^2 (u_{c,\eta})^2
\]

(2.4.3)

**waves**

The instantenous bed-shear stress is defined:

\[
\tau_{b,cw}(t) = \frac{1}{2} \rho f_w (U_{w,\delta})^2 = \frac{1}{2} \rho f_w \left[ \bar{U}_d \sin(\omega t) \right]^2
\]

(2.4.4)

Assuming an instantaneous logarithmic velocity distribution near the bed, it can also be defined (at level \( \eta = e z_o \)), that:

\[
\tau_{b,w}(t) = \rho \kappa^2 (U_{w,\eta})^2
\]

(2.4.5)

Define: \( U_{w,\eta} = p \bar{U}_d \sin(\omega t) \)

From Eqs. (2.4.4), (2.4.5) it follows that:

\[
p^2 = f_w/(2\kappa^2)
\]

(2.4.6)
Figure 2.4.8 Vector addition of instantaneous wave and current velocity at level $\eta = e_{z_0}$

Figure 2.4.9 $\beta$-coefficient as a function of $\zeta \hat{U}_f \tilde{u}$ according to Yoo (1989)
Assume that there is an angle $\phi$ between the wave and current direction (see Fig. 2.4.8). The instantaneous resulting velocity at level $\eta = e z_o$ can be expressed as:

$$(V_{cw,\eta})^2 = (u_{c,\eta})^2 + (U_{w,\eta})^2 + 2 U_{w,\eta} u_{c,\eta} \cos \phi \quad (2.4.7)$$

The instantaneous bed-shear stress is:

$$\tau_{b,cw}(t) = \rho \kappa^2 (V_{cw,\eta})^2 \quad (2.4.8)$$

Substitution of $U_{w,\eta} = p \hat{U}_\delta \sin(\omega t)$ in Eq. (2.4.7) yields:

$$(V_{cw,\eta})^2 = (u_{c,\eta})^2 + (p \hat{U}_\delta \sin(\omega t))^2 + 2p u_{c,\eta} \hat{U}_\delta \sin(\omega t) \sin(\phi) \quad (2.4.9)$$

The value of the time-averaged bed-shear stress vector can be found by averaging over the wave period of Eq. (2.4.9), yielding:

$$\overline{(V_{cw,\eta})^2} = (u_{c,\eta})^2 + 1/2 \rho (\hat{U}_\delta)^2 \quad (2.4.10)$$

$$\rho \kappa^2 \overline{(V_{cw,\eta})^2} - \rho \kappa^2 (u_{c,\eta})^2 + 1/4 \rho f_w (\hat{U}_\delta)^2 \quad (2.4.11)$$

or,

$$|\tau_{b,cw}| = \tau_{b,c} ; \quad |\tau_{b,w}| \quad (2.4.12)$$

in which:

- $\tau_{b,cw}$ = time-averaged value of absolute bed-shear stress for combined wave-current flow
- $\tau_{b,c}$ = time-averaged current-related bed-shear stress = $(1/8) \rho f_c \bar{u}^2$
- $\tau_{b,w}$ = time-averaged value of absolute wave-related bed-shear stress = $(1/4) \rho f_w \hat{U}_\delta^2$
- $\bar{u}$ = depth-averaged current velocity
- $f_c = 8g/C = 0.24 \log^2(12h/k_s) = \text{current friction factor}$
- $C = 18 \log(12h/k_s) = \text{Chézy-coefficient}$
- $k_s = \text{effective bed roughness of Nikunadse}$
- $f_w = \exp[-6 + 5.2(A_x/k_s)^{0.19}] = \text{wave friction factor (f_w \leq 0.3)}$
- $\hat{U}_\delta = \text{peak value of near-bed orbital velocity (= \hat{U}_\delta T/2\pi)}$
- $A_x = \text{peak value of near-bed orbital excursion}$
- $T = \text{wave period}$
- $\delta = \text{thickness of wave boundary layer}$

Equation (2.4.12) expresses the time-averaged value of the absolute bed-shear stress vector and is found to be independent of the angle $\phi$ between the wave and current direction.
This parameter is most important for the sediment transport process because it governs the agitation (stirring) of the bed particles.

The most important parameter for resisting the mean current (bed shear stress experienced by the current) is the time-averaged value of the component of the bed-shear stress parallel to the current direction ($\tau_{b,cw,s}$). Its effect on the current is reflected by the slope (outside wave boundary layer) of the time-averaged current velocity profile, as shown in Fig. 2.4.10. The $\tau_{b,cw,s}$ parameter cannot be obtained by analytical integration. Bijker presented an approximation function based on numerical integration.

$$\tau_{b,cw,s} = \beta \, \tau_c$$  \hspace{1cm} (2.4.13a)

in which:

$$\tau_{b,cw,s} = \text{time-averaged bed-shear stress parallel to current-direction.}$$

$$\beta = 0.75 + 0.45 \left( \zeta \, \hat{U}_d \hat{U}_f \right)^{1.13} \quad \text{for } \phi < 70^\circ$$

$$\zeta = (f_w \, C^2 / 2g)^{1/2} = 2(f_\zeta f_\zeta)^{1/2}$$

For $\phi = 90^\circ$ it can be derived that:

$$\tau_{b,cw,s} = (2\pi)^{-1} \rho \left( f_c \, f_w \right)^{0.5} \bar{u} \, \hat{U}_d$$  \hspace{1cm} (2.4.13b)

Yoo (1989) presented a graphical form of the $\beta$-coefficient (Eq. 2.4.13) for $\phi = 0^\circ$, $45^\circ$, $90^\circ$, as shown in Fig. 2.4.9.

The major defect of the Bijker-method is the neglect of the current velocity reduction in the near-bed region due to wave-current interaction. O'Connor and Yoo (1988) introduced a velocity reduction factor $\alpha$ to account for this effect. Figure 2.4.9 shows $\beta$-values for $\alpha = 0.38, 0.58$ and 0.78.

Tolman (1992) studied the effect of the current on the wave-related friction and found little effect in the practical range of interest.

Roelvink (1990) modified the Bijker-approach and represented the wave-current interaction effect by calibration using experimental data.

He proposed:

$$\tau_{b,cw,s} = \frac{0.2}{\pi} \rho \left( 1 + \cos^2 \phi \right) (f_w \, f_c)^{0.5} \bar{u} \, \hat{U}_d$$  \hspace{1cm} (2.4.14)

B. Coffey-Nielsen (1986):

Coffey and Nielsen (1986) proposed an engineering model based on a logarithmic velocity profile. The wave effect on the steady current is represented as a shift of the velocity profile yielding an apparent roughness $k_s$ (Fig. 2.4.10). The ratio $k_s / k_i$ was obtained from experimental results and related to the ratio of the wave-related and current-related bed-shear velocities ($u_{w,s} / u_{w,c}$).

A disadvantage of this model is that iterative computations are required.
Van Rijn proposed a straightforward method for the rough flow regime, which is based on the application of logarithmic velocity profiles.

The velocity profile for a current in the presence of waves is described as a two-layer system (see Fig. 2.4.10):

- logarithmic velocity profile affected by bed-form roughness \( (k_a) \) inside the near-bed mixing layer \( (\delta) \), which is assumed to be equal to three times the thickness of the wave boundary layer,
- logarithmic velocity profile affected by apparent roughness \( (k_s) \) outside the near-bed mixing layer \( (\delta) \).

![Graph showing current modified by waves and current alone with height z on the y-axis and relative velocity \( v_z/\sqrt{v} \) on the x-axis.]

**Figure 2.4.10 Influence of waves on current velocity profile**

The expressions for rough turbulent flow are:

\[
\begin{align*}
z \geq \delta & : \quad v_{t,z} = \frac{\bar{v}_R \ln(30z/k_a)}{-1 + \ln(30h/k_a)} \\
\delta < z & : \quad v_{t,z} = \frac{v_{t,\delta} \ln(30z/k_s)}{\ln(30\delta/k_s)} \\
\text{with} & : \quad v_{t,\delta} = \frac{\bar{v}_R \ln(30\delta/k_s)}{-1 + \ln(30h/k_s)}
\end{align*}
\]

(2.4.15) (2.4.16) (2.4.17)

in which:

- \( v_{t,z} \) = modified current velocity at height \( z \) above bed
- \( \bar{v}_R \) = magnitude of depth-averaged velocity vector = \( [\bar{u}^2 + \bar{v}^2]^{0.5} \)
- \( \delta \) = thickness of wave-related near-bed mixing layer \( (= 3 \delta_w) \), \( (\delta > 0.033 \, k_a) \)
- \( \delta_w \) = thickness of wave boundary layer, Eq. (2.3.8)
- \( v_{t,\delta} \) = current velocity at \( z = \delta \)
- \( k_s \) = apparent bed roughness
- \( k_a \) = apparent bed roughness

2.42
The overall resistance (in current direction) experienced by the current due to friction and vortices-related pressure effects as modified by the wave motion, follows from Eq. (2.4.15) yielding:

\[ v_{*,cw,s} = \frac{\kappa \bar{v}_R}{-1 + \ln(30h/k_{\delta})} \]  \hspace{1cm} (2.4.18)

or

\[ \tau_{b,cw,s} = \frac{\rho \kappa^2 (\bar{v}_R)^2}{[-1 + \ln(30h/k_{\delta})]^2} \]  \hspace{1cm} (2.4.19)

Defining \( \tau_{b,cw,s} = \frac{1}{8} \rho f_{cw}(\bar{v}_R)^2 \), it follows that:

\[ f_{cw} = \frac{8 \kappa^2}{[-1 + \ln(30h/k_{\delta})]^2} = 0.24 \left[ \log(12h/k_{\delta}) \right]^2 \]

in which:

\( \tau_{b,cw,s} = \) time-averaged shear stress experienced by the current in combined wave-current conditions

The current-related shear stress (\( v_{*,rc} \) or \( \tau_{b,rc} \)) acting on the bed is reduced due to the reduced current-velocities in the near-bed region due to the wave-current interaction.

From Equations (2.4.16), (2.4.17), it follows that:

\[ v_{*,rc} = \frac{\kappa \ln(30\delta/k_{\delta})}{[-1 + \ln(30h/k_{\delta})] \ln(30\delta/k_{\delta})} \bar{v}_R \]  \hspace{1cm} (2.4.20)

or

\[ \tau_{b,rc} = \frac{\rho \kappa^2 [\ln(30\delta/k_{\delta})]^2}{[-1 + \ln(30h/k_{\delta})]^2 [\ln(30\delta/k_{\delta})]^2} \bar{v}_R^2 \]  \hspace{1cm} (2.4.21)

The bed-shear stress in a current alone (no waves) is:

\[ \tau_{b,c} = \frac{\rho \kappa^2 (\bar{v}_R)^2}{[-1 + \ln(30h/k_{\delta})]^2} \]  \hspace{1cm} (2.4.22)

From Eqs. (2.4.21), (2.4.22) it follows that:

\[ \tau_{b,rc} = \alpha_r \tau_{b,c} \]  \hspace{1cm} (2.4.23)
with
\[ \alpha_r = \left[ \frac{\ln(30\delta/k_s)}{\ln(30\delta/k_a)} \right]^2 \left[ \frac{-1 + \ln(30h/k_a)}{1 + \ln(30h/k_a)} \right] \]  
(2.4.24)

= bed-shear stress reduction factor (≥ 1)

The \( \alpha_r \)-coefficient is considerably smaller than one in case of relatively high waves combined with a weak current.

The time-averaged value of the absolute bed-shear stress (Eq. 2.4.12) acting on the bed material particles, now becomes:

\[ |\tau_{b,cw}| = \alpha_r \tau_{b,c} + |\tau_{b,w}| \]  
(2.4.25)

with \( \tau_{b,c} \) and \( \tau_{b,w} \) according to Eq. (2.4.12) and \( \alpha_r \) according to Eq. (2.4.24).

The time-averaged shear-stress experienced by the current in the presence of waves is given by Eq. (2.4.19). For application in mathematical models, it is proposed to use:

\[ \tau_{b,cw,x} = \frac{1}{8} \rho f_{cw} \bar{u} \bar{v}_R \]  
(2.4.26)

\[ \tau_{b,cw,y} = \frac{1}{8} \rho f_{cw} \bar{v} \bar{v}_R \]  
(2.4.27)

in which:
\( \bar{v}_R \) = depth-averaged velocity vector = \[ \frac{\bar{u}^2 + \bar{v}^2}{l} \]
\( f_{cw} \) = current-related friction factor = 0.24[log(12h/k_s)]^{-2}

The apparent roughness \( (k_s) \) was determined by fitting of logarithmic profiles to time-averaged velocity profiles (outside the wave boundary layer) measured in the presence of waves. The experimental data of Nieuwjaar-Van der Kaaij (1987), Nap-Van Kampen (1988) and Havinga (1992) representing a data set with \( \phi \)-values of 0°, 60°, 90°, 120° and 180° \( (\phi = \text{angle between wave and current direction}) \).

Basically, the ratio \( k_s/k_s \) should be a function of:
• the ratio of the peak orbital velocity and depth-averaged velocity \( \hat{U}_\delta/\bar{v}_R \),
• the ratio of the peak orbital excursion and the bed roughness, \( \hat{A}_\delta/k_s \),
• the angle \( \phi \) between the current and wave direction.

Thus,
\[ \frac{k_s}{k_s} = F \left( \frac{\hat{U}_\delta}{\bar{v}_R}, \frac{\hat{A}_\delta}{k_s}, \phi \right) \]  
(2.4.28)

The \( \hat{U}_\delta/\bar{v}_R \) ratio represents the relative strength of the wave and current motion. For \( \hat{U}_\delta/\bar{v}_R \to 1 \) it follows that \( k_s/k_s \to 1 \). The \( \hat{A}_\delta/k_s \) ratio represents the influence of the wave period because at the same \( \hat{U}_\delta/\bar{v}_R \)-value a decrease of the \( \hat{A}_\delta \)-value means a smaller wave period (faster oscillations) which probably results in an increase of the apparent roughness.
The mathematical model results of Fredsøe (1984) show a rather weak influence of the $A_s/k_s$-ratio for rough flow in the practical range of interest (see Fig. 2.4.6). Therefore, this parameter was neglected.

Based on analysis of the experimental data of flumes with rippled beds (see Fig. 2.4.11), it was found that:

$$\frac{k_s}{k_r} = \exp(\gamma \frac{\hat{U}_s}{\sqrt{\nu}})$$

(2.4.29)

The $\gamma$-coefficient was found to be dependent on the angle between the current and the waves, as follows:

<table>
<thead>
<tr>
<th>Angle</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0^\circ$ (following waves)</td>
<td>$0.75 \pm 0.4$</td>
</tr>
<tr>
<td>$\phi = 60^\circ$</td>
<td>$1.1 \pm 0.4$</td>
</tr>
<tr>
<td>$\phi = 90^\circ$ (perpendicular waves)</td>
<td>$2.1 \pm 0.9$</td>
</tr>
<tr>
<td>$\phi = 120^\circ$</td>
<td>$1.7 \pm 0.4$</td>
</tr>
<tr>
<td>$\phi = 180^\circ$ (opposing waves)</td>
<td>$1.1 \pm 0.4$</td>
</tr>
</tbody>
</table>

(2.4.30)

A reasonable estimate of $\gamma$ can be obtained by:

$$\gamma = 0.8 + \phi - 0.3 \phi^2$$

(2.4.31)

in which:

$\phi$ = angle between current and wave direction

(in radians between 0 and $\pi$; $1/2 \pi = 90^\circ$, $\pi = 180^\circ$)

The experimental data show that the largest $k_s$-values and hence the largest $\tau_{bcw}$-values do occur for a wave-current angle of $\phi = 90^\circ$. Figure 2.4.12 shows $\tau_{bcw}$-values as a function of $\tau_{bc}$-values for $\phi = 90^\circ$ and $0^\circ$ according to the method of Van Rijn. Both parameters were made dimensionless with $\tau_{bc} + \hat{\tau}_{bw}$, where $\tau_{bc} =$ current-related bed-shear stress (no waves) and $\hat{\tau}_{bw} =$ peak wave-related bed-shear stress (no current). The largest $\tau_{bcw}$-values do occur for $\phi = 90^\circ$. For example, the $\tau_{bcw}$-value for $\hat{U}_s/\nu = 1$ and $\phi = 90^\circ$ is about 30% larger than that for $\phi = 0^\circ$.

This latter behaviour is opposite to the mathematical model results of Fredsøe (1984) for plane beds who found smaller $k_s$-values for $\phi = 90^\circ$ than for $\phi = 0^\circ$.

Figure 2.4.13 shows a comparison of computed $\tau_{bcw}$-values according to six different methods for $\phi = 0^\circ$, $A_s/z_s = 10^3$, $z_s/h = 10^4$. The results of the numerical models of Fredsøe (1984), Davies et al (1988), Myrhaug et al (1990) and Huynh Thanh et al (1991) were presented by Soulsby et al (1991). The Bijker-model yields the largest $\tau_{bcw}$-values because the velocity reduction in the near-bed region due to wave-current interaction has been neglected resulting in an overprediction of the $\tau_{bcw}$-value.

The method of Myrhaug et al yields relatively large values for $\tau_{bc}/(\tau_{bc} + \hat{\tau}_{bw}) > 0.5$. The engineering method of Van Rijn yields the smallest $\tau_{bcw}$-values.

Computed velocity profiles based on Eqs. (2.4.15), (2.4.16), (2.4.17), (2.4.29) and (2.4.31) are shown in Fig. (2.4.3) for a current in a wave field ($\phi = 90^\circ$) as measured by Van der Stel and Visser (1985). The computed current profiles are in good agreement with the measured values.
Figure 2.4.11 Ratio of apparent and physical roughness \((k_a/k_s)\)

Figure 2.4.12 Computed \(\tau_{bcw}\) values for \(\phi = 0^\circ\) and \(90^\circ\) according to method of Van Rijn
Figure 2.4.13 Comparison of computed $\tau_{b,aw}$ values for $\phi = 0^\circ$
REFERENCES


REFERENCES (continued)


REFERENCES (continued)


2.50
REFERENCES (continued)


REFERENCES (continued)


3. FLUID AND SEDIMENT PROPERTIES

3.1 Fluid properties

3.1.1 Introduction

All real fluids have certain measurable characteristics or properties such as density, viscosity, compressibility, capillarity, surface tension etc. Some properties are combinations of other properties. For example, kinematic viscosity involves dynamic viscosity and density. Herein, the two most basic properties being the density and the viscosity are given.

3.1.2 Fluid density

The density of fresh water varies with temperature, as follows:

\[
\begin{align*}
\rho &= 999.9 \text{ kg/m}^3 \quad T_e = 0^\circ\text{C} \\
\rho &= 1000 \text{ kg/m}^3 \quad T_e = 4^\circ\text{C} \\
\rho &= 999.5 \text{ kg/m}^3 \quad T_e = 12^\circ\text{C} \\
\rho &= 999.0 \text{ kg/m}^3 \quad T_e = 16^\circ\text{C} \\
\rho &= 998.3 \text{ kg/m}^3 \quad T_e = 20^\circ\text{C} \\
\rho &= 995.7 \text{ kg/m}^3 \quad T_e = 30^\circ\text{C} \\
\rho &= 992.3 \text{ kg/m}^3 \quad T_e = 40^\circ\text{C}
\end{align*}
\]

The density of sea water (\(= 1025 \text{ kg/m}^3\)) can be determined from the following expression:

\[
\rho = 1000 + 1.455 \text{ CL} - 0.0065 (T_e - 4 + 0.4 \text{ CL})^2
\]

(3.1.1)

in which:

- CL = Chlorinity (in %o)
- T_e = temperature (in °C)
- \(\rho\) = fluid density (in kg/m³)

The Chlorinity follows from:

\[
S = 0.03 + 1.805 \text{ CL}
\]

(3.1.2)

in which:

- S = Salinity = total quantity of dissolved salt in grammes per kilogramme of sea water (in %o, weight ratio).

3.1.3 Fluid viscosity

The kinematic viscosity coefficient \(\nu\) is defined as:

\[
\nu = \frac{\eta}{\rho}
\]

(3.1.3)

in which:

- \(\nu\) = kinematic viscosity coefficient (m²/s)
- \(\eta\) = dynamic viscosity coefficient (Ns/m²)
- \(\rho\) = fluid density (kg/m³)
The kinematic viscosity coefficient can be approximated by:

\[ \nu = \left[ 1.14 - 0.031(\text{Te}-15) + 0.00068(\text{Te}-15)^2 \right] 10^{-6} \]  

(3.1.4)

The dynamic viscosity coefficient is influenced by the sediment particles. For dilute suspensions \((c < 0.1)\) Einstein (1906) found:

\[ \eta_m = \eta(1 + 2.5\ c) \]  

(3.1.5)

in which:

- \(\eta_m\) = dynamic viscosity coefficient of fluid-sediment mixture
- \(\eta\) = dynamic viscosity coefficient of clear water
- \(c\) = volumetric sediment concentration

Based on experiments with volume concentrations in the range of \(c = 0.1\) to \(0.6\), Bagnold (1954) found:

\[ \eta_m = \eta(1 + \lambda)(1 + 0.5\ \lambda) \]  

(3.1.6)

in which:

- \(\lambda = [(0.74/c)^{1/3} - 1]^{-1}\) = dimensionless concentration parameter

Equation (3.1.6) is shown in Fig. 3.1.2.

An empirical relationship was given by Do Ik Lee (1969), which reads as:

\[ \eta_m = \eta(1 - c)^{\alpha} \]  

(3.1.7)

with \(\alpha = -(2.5 + 1.9\ c + 7.7\ c^2)\).

An excellent review related to the effect of the sediment particles on viscosity was presented by Savage (1984).

The kinematic viscosity coefficient in a fluid-sediment mixture is given by:

\[ \nu_m = \frac{\eta_m}{\rho_m} \]  

(3.1.8)

in which:

- \(\eta_m\) = dynamic viscosity coefficient in fluid-sediment mixture
- \(\rho_m\) = fluid-sediment mixture density = \(\rho(1 + (s-1)c)\)
- \(c\) = sediment concentration (volume)
- \(s = \rho_s/\rho\) = specific density

3.2
3.2 Sediment Properties

3.2.1 Introduction

Sediment is fragmental material, primarily formed by the physical and chemical desintegration of rocks from the earth's crust. Such particles range in size from large boulders to colloidal size fragments and vary in shape from rounded to angular. They also vary in specific gravity and mineral composition, the predominant materials being quartz mineral and clay minerals (kaolinite, illite, montmorillonite and chlorite). The latter have a sheet-like structure, which can easily change (flocculation) under the influence of electrostatic forces (cohesive forces) in a saline environment. Consequently, there is a fundamental difference in sedimentary behaviour between sand and clay materials.

Sediments can be classified according to their genetic origin:
- Lithogeneous sediments, which are detrital products of disintegration of pre-existing rocks
- Biogeneous sediments, which are remains of organisms mainly carbonate, opal and calcium phosphate
- Hydrogeneous sediments, which are precipitates from seawater or from interstitial water

Descriptive sediment classifications can also be used and are related to characteristics like color, texture, grain size, organic content, etc. For example, a mixture of sand and clay is classified as a sandy clay when the percentage of sand is between 25% and 50%. Similarly, clayey sands, gravelly sands, sandy gravels, clayey gravels, and gravelly clays are distinguished.

Sediment particles larger than 62 μm and smaller than 2000 μm (see Section 3.2.4) are usually referred to as sand particles.

Based on mineral and chemical composition, three types of sands can be distinguished:
- silicate sands;
- carbonate sands;
- gypsum sands.

Silicate sands mainly consist of quartz and feldspar minerals, which are extremely insoluble in water.
Carbonate sands consist of calcite and aragonite, which are two different crystalline forms of calcium-carbonate (CaCO₃), originating from shell and coral fragments (coral sands). The percentage of carbonate in a sample usually is larger than about 80%. Carbonate sands are much more soluble in fresh water than silicate sands. In sea water (especially in the tropics) which is already supersaturated with carbonate it is hardly soluble. Carbonate sands usually exhibit some degree of cementation: weakly-cemented or well-cemented, which means that the fragments cannot be manually broken.
Gypsum sands consist of crystal forms of gypsum (CaSO₄·2H₂O), which is a moderately soluble mineral that can survive only in arid regions.

The sediment properties herein presented are: density, porosity, shape, size, fall velocity and angle of repose.
Figure 3.1.1 Viscosity coefficient

Figure 3.1.2 Viscosity coefficient as a function of concentration according to Bagnold (1954)
3.2.2 Density and porosity

The density of quartz and clay minerals is approximately equal to \( \rho_s = 2650 \text{ kg/m}^3 \). The density of carbonate material may be somewhat smaller (\( \rho_s = 2500 \text{ to } 2650 \text{ kg/m}^3 \)). The specific gravity is defined as the ratio of the sediment density and the fluid density, \( s = \rho_s / \rho = 2.65 \).

The dry sediment density is the dry sediment weight per unit volume (= concentration) and is equal to:

\[
\rho_{\text{dry}} = (1-p)\rho_s \tag{3.2.1}
\]

in which:
\( \rho_{\text{dry}} \) = dry sediment density (kg/m\(^3\));
\( \rho_s \) = sediment density (kg/m\(^3\));
\( p \) = porosity factor.

The wet density or volume weight of deposited material (assuming total saturation) is the weight of water and sediment per unit volume (sometimes called the wet bulk density) and is equal to:

\[
\rho_{\text{wet}} = p \rho + (1-p)\rho_s \tag{3.2.2}
\]

The porosity of sediment material is often related to the deposition history of the sediment bed. Loose packing occurs when sediments settle from suspension in still water. Basically, four packing arrangements are possible for spherical particles. The most unstable arrangement is the cubic arrangement with the sphere centres forming a cube yielding a porosity of 48%. The Rhombohedral arrangement with the spheres in the hollows of each other yields the most stable packing and the smallest porosity of 26%. Random packing of spheres yields porosity ranges from 36% to 40%. Natural sediments with particles of various sizes have relatively small porosity values because the smaller particles can occupy the large void spaces. A poorly sorted (many sizes) coarse sand has a porosity of about 40%. A well sorted (almost uniform) fine sand has a porosity of about 45%. The porosity of coral sand (mixture of coral and shell fragments) has been found in the range from 0.5 to 0.65 (Van der Meulen, 1988).

Deposits consisting of clay, silt, sand and organic material are called mud deposits and can have a large porosity factor (upto 80%).

The dry sediment weight of mixtures can be estimated from empirical relationships. Based on the analysis of samples from the toplayers (recent deposits) of reservoirs, Lane and Koelzer (1953) found:

\[
\rho_{\text{dry}} = 817 (100 p_{\text{sand}} + 2)^{0.13} \tag{3.2.3}
\]

in which:
\( p_{\text{sand}} \) = fraction of sand particles (d > 50 \( \mu \text{m} \)).

Lara and Pemberton (1963) analyzed reservoir samples always submerged (A) and submerged 50% of the time (B). They found:
A: \[ \rho_{\text{dry}} = 1550 \rho_{\text{sand}} + 1120 \rho_{\text{silt}} + 420 \rho_{\text{clay}} \] (3.2.4)

B: \[ \rho_{\text{dry}} = 1550 \rho_{\text{sand}} + 1135 \rho_{\text{silt}} + 560 \rho_{\text{clay}} \] (3.2.5)

in which:
- \( \rho_{\text{dry}} \) = dry sediment density (kg/m\(^3\))
- \( \rho_{\text{sand}} \) = fraction of sand particles (d \( \geq \) 62.5 \( \mu \)m)
- \( \rho_{\text{silt}} \) = fraction of silt particles (4 \( < \) d \( < \) 62.5 \( \mu \)m)
- \( \rho_{\text{clay}} \) = fraction of clay particles (d \( \leq \) 4 \( \mu \)m)

Murthy and Banerjee (1976) analyzed top layer samples (sampling depth = 0.4 m) from Indian Reservoirs, which were about 50% of the time empty (exposed to the sun) and found:

\[ \rho_{\text{dry}} = 1506 \rho_{\text{sand}} + 866 \rho_{\text{silt}} + 561 \rho_{\text{clay}} \] (3.2.6)

in which:
- \( \rho_{\text{sand}} \) = fraction of sand particles (20 \( \mu \)m \( \leq \) d \( \leq \) 200 \( \mu \)m)
- \( \rho_{\text{silt}} \) = fraction of silt particles (2 \( \mu \)m \( < \) d \( < \) 20 \( \mu \)m)
- \( \rho_{\text{clay}} \) = fraction of clay particles (d \( \leq \) 2 \( \mu \)m)

Equations (3.2.3) to (3.2.6) express values representing the initial (0 to 1 year) dry weight of the deposits. The dry weight increases with time due to consolidation.

Based on analysis results of a large amount of soil samples, Allersma (1988) proposed:

\[ \rho_{\text{dry}} = 480 \alpha + (1300 - 280 \alpha) \rho_{\text{sand}}^{0.8} \] (3.2.7)

in which:
- \( \alpha \) = consolidation coefficient (\( \alpha \) in the range from 0 to 2.4 with a mean value of \( \alpha = 1.2 \)).

Equation (3.2.7) is shown in Fig. 3.2.1.

The presence of organic material such as Cellulose, Lignin, Pectin, Coal (with densities in the range of 1200 to 1500 kg/m\(^3\)) can give a substantial reduction of the density of the mixture.

The wet sediment weight (\( \rho_{\text{wet}} \)) or wet bulk density (assuming total saturation) can be expressed as:

\[ \rho_{\text{wet}} = \rho + \left( \frac{\rho_{\text{s}} - \rho}{\rho_{\text{s}}} \right) \rho_{\text{dry}} \] (3.2.8)

Other soil parameters are (see Fig. 3.2.2):

- sediment concentration (c or \( \rho_{\text{dry}} \)) = ratio of dry solid mass (\( M_s \)) and total volume (\( V_i \)) = \( M_s / V_i \)
- solids content (sc) = ratio of the mass of dry solids (\( M_s \)) and the total mass (\( M_t \)) = \( M_s / M_t \)
- water content (wc) = ratio of the water mass (\( M_w \)) and the solid mass (\( M_s \)) = \( M_w / M_s \)
void ratio (e) = ratio of the void volume (V_v) and the solid volume = V_v/V_s

saturation degree (sd) = ratio of water volume (V_w) and void volume (V_v) = V_w/V_v

Relations are:

porosity (p) = \frac{\rho_s - \rho_{dry}}{\rho_s}

solid content (sc) = \frac{(\rho_s/\rho) \rho_{dry}}{\rho_s + [(\rho_s/\rho) - 1] \rho_{dry}}

void ratio (e) = \frac{\rho}{1 - p}

water content (wc) = \frac{e \cdot sd \cdot \rho}{\rho_s}

3.2.3 Shape

Most of the sand particles on the face of the Earth are more or less rounded because their edges and corners are smoothed by abrasion as running water or wind moves the sand particles from their origin (source) to their final resting place. Roundness is a function of abrasion induced by transport and it increases slowly with distance. Thousands of kilometers of transport in a river are required to achieve even moderate rounding. Beaches where sand moves in and out with each wave are ideal places for rounding of sand particles if they stay there for any length of time.

![Figure 3.2.1 Sediment densities according to Allersma (1988)](image-url)
The shape of particles generally is represented by the Corey shape factor, defined as:

\[
SF = \frac{c}{(a \ b)^{0.5}}
\]  

(3.2.9)

in which:

- \(a\) = length along longest axis perpendicular to other two axes
- \(b\) = length along intermediate axis perpendicular to other two axes
- \(c\) = length along short axis perpendicular to other two axes.

The SF-factor for natural sand is approximately 0.7.

The shape factor is essentially a flatness ratio and does not take into account the distribution of the surface area and the volume of the particle. For example, a cube of a given length and a sphere of a diameter equal to the length of the cube have the same shape factor (SF = 1). To overcome this, another shape factor is also applied, defined as:

\[
SF_* = SF \frac{d_s}{d_u}
\]  

(3.2.10)

in which:

- \(SF_*\) = shape factor according to Eq. (3.2.10)
- \(d_s\) = diameter of a sphere having the same surface area as that of the particle
- \(d_u\) = diameter of a sphere having the same volume as that of the particle.

3.8
SF, approaching unity implies increasing sphericity of the particles (sphericity is ratio of surface area of a sphere and surface area of the particle at equal volume). A behavioural measure of shape is expressed by the rollability parameter (Winkelmolen, 1971). The rollability is a functional shape property measured by the time it takes for grains of equal size and density to travel the length of a cylinder revolving with its axis inclined at an angle of 2.5° to the horizontal.

Grain fabric is the attitude in space and degree of preferred orientation displayed by the grains during sedimentation. Shape fabric has a physical cause and is therefore an indicator of hydraulic conditions.

3.2.4 Size

Usually, sediments are referred to as gravel, sand, silt or clay. These terms refer to the size of the sediment particle. The following Table 3.1 presents the grain size scale of the American Geophysical Union. This scale is based on powers of 2 mm, which yields a linear logarithmic scale via the phi-parameter defined as \( \phi = -2 \log d \) (with d in mm).

Various methods are available to determine the particle size. Cobbles can be measured directly with a ruler. Gravel, sand and silt are analyzed by wet or dry sieving methods yielding sieve diameters. Clay materials are analyzed hydraulically by using settling methods (Van Rijn, 1986) yielding the particle fall velocity from which the standard fall diameter is computed. Clay materials can also be analyzed with various electronic techniques such as the Coulter counter and the Laser Diffraction technique (Van Rijn, 1986). Thus, the size of a sediment particle is closely related to the analysis method.

Typical "diameters" are:

- **sieve diameter** which is the diameter of a sphere equal to the length of the side of a square sieve opening through which the given particle will just pass,

- **nominal diameter** which is the diameter of a sphere that has the same volume as the particle,

- **standard fall diameter** which is the diameter of a sphere that has a specific gravity of 2.65 and has the same fall velocity as the particle in still, distilled water of 24°C.

A natural sample of sediment particles contains particles of a range of sizes. The size distribution of such a sample is the distribution of sediment material by percentages of weight, usually presented as a cumulative frequency distribution (see Fig. 3.2.3).

The frequency distribution is characterized by:

- **median particle size** \( d_{50} \) which is the size at which 50% by weight is finer,

- **mean particle size** \( d_m = \Sigma (p_i d_i)/100 \) with \( p_i \) = percentage by weight of each grain size fraction \( d_i \),

- **standard deviation** \( \sigma_d = \Sigma p_i(d_i d_m)^2/100 \) or \( \sigma_d = 0.5(d_{50}/d_{10} + d_{90}/d_{50}) \) which is a measure based on graphic values.

Often the phi-scale is used for size distribution representation:

\[
\phi = -2 \log(d) \tag{3.2.11}
\]

where \( d \) is the particle diameter in millimeters.
<table>
<thead>
<tr>
<th>Class Name</th>
<th>Millimeters</th>
<th>Micrometers</th>
<th>Phi Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boulders</td>
<td>&gt; 256</td>
<td></td>
<td>&lt; -8</td>
</tr>
<tr>
<td>Cobbles</td>
<td>256 - 64</td>
<td></td>
<td>-8 to -6</td>
</tr>
<tr>
<td>Gravel</td>
<td>64 - 2</td>
<td></td>
<td>-6 to -1</td>
</tr>
<tr>
<td>Very coarse sand</td>
<td>2.0 - 1.0</td>
<td>2000 - 1000</td>
<td>-1 to 0</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>1.0 - 0.50</td>
<td>1000 - 500</td>
<td>0 to +1</td>
</tr>
<tr>
<td>Medium sand</td>
<td>0.50 - 0.25</td>
<td>500 - 250</td>
<td>+1 to +2</td>
</tr>
<tr>
<td>Fine sand</td>
<td>0.25 - 0.125</td>
<td>250 - 125</td>
<td>+2 to +3</td>
</tr>
<tr>
<td>Very fine sand</td>
<td>0.125 - 0.062</td>
<td>125 - 62</td>
<td>+3 to +4</td>
</tr>
<tr>
<td>Coarse silt</td>
<td>0.062 - 0.031</td>
<td>62 - 31</td>
<td>+4 to +5</td>
</tr>
<tr>
<td>Medium silt</td>
<td>0.031 - 0.016</td>
<td>31 - 16</td>
<td>+5 to +6</td>
</tr>
<tr>
<td>Fine silt</td>
<td>0.016 - 0.008</td>
<td>16 - 8</td>
<td>+6 to +7</td>
</tr>
<tr>
<td>Very fine silt</td>
<td>0.008 - 0.004</td>
<td>8 - 4</td>
<td>+7 to +8</td>
</tr>
<tr>
<td>Coarse clay</td>
<td>0.004 - 0.0020</td>
<td>4 - 2</td>
<td>+8 to +9</td>
</tr>
<tr>
<td>Medium clay</td>
<td>0.0020 - 0.0010</td>
<td>2 - 1</td>
<td>+9 to +10</td>
</tr>
<tr>
<td>Fine clay</td>
<td>0.0010 - 0.0005</td>
<td>1 - 0.5</td>
<td>+10 to +11</td>
</tr>
<tr>
<td>Very fine clay</td>
<td>0.0005 - 0.00024</td>
<td>0.5 - 0.25</td>
<td>+11 to +12</td>
</tr>
<tr>
<td>Colloids</td>
<td>&lt; 0.0024</td>
<td>&lt; 0.24</td>
<td>&gt; +12</td>
</tr>
</tbody>
</table>

Table 3.1 Grain size scale American Geophysical Union

Characteristic values are the mean phi-value \( \phi_n \), the median value \( \phi_{50} \), the standard deviation (sorting) \( \sigma_\phi \), the skewness (asymmetry) and kurtosis (peakedness):

standard deviation : \[ \sigma_\phi = 0.5(\phi_{84} - \phi_{16}) \] (3.2.12)

skewness : \[ \alpha_\phi = \frac{\phi_m - \phi_{50}}{\sigma_\phi} \] (3.2.13)

kurtosis : \[ \beta_\phi = \frac{0.5(\phi_{95} - \phi_5) - \sigma_\phi}{\sigma_\phi} \] (3.2.14)

Ideally, a sample has a normal (Gaussian) distribution in terms of the \( \phi \)-scale (log-normal distribution), yielding:

\[ d_{10} = \sigma_s^{-13} d_{50} \] (3.2.15)

\[ d_{16} = \sigma_s^{-1} d_{50} \] (3.2.16)

\[ d_{84} = \sigma_s d_{50} \] (3.2.17)

\[ d_{90} = \sigma_s^{13} d_{50} \] (3.2.18)

with:
\[ \sigma_s = 0.5(d_{50}/d_{16} + d_{84}/d_{50}) \]

3.10
Figure 3.2.3 shows particle size curve for a log-normal distribution on a logarithmic-linear scale and on a logarithmic-probability scale.

Grain size can be used as an indicator of energy conditions. Fine grains usually are dominant in low-energy conditions near river banks, on tidal flats, in protected or sheltered basins. Coarse grains are found in high-energy conditions near breaker bars along the coasts and in the deeper channels of rivers and estuaries, where the finer grains cannot easily survive (strong currents). The main grain size decreases with distance from the source known as: "fining down the transport path", due to abrasion effects and deposition of fines in quiescent conditions. Near the source the size range usually is relatively wide (well sorted); while a narrow size range (poorly sorted) is found far away from the source. The coarsest particles are transported as bed load; the finest particles as suspended load.

The relationship between grain size, sorting and hydraulic conditions has been used to distinguish between different sedimentary environments like wave-dominated, current-dominated; eroding or depositing and paths between them.

3.2.5 Particle fall velocity

1. Sphere falling in a still fluid

Basically, the fall velocity is a behavioural property. The terminal fall velocity \( w_s \) of a sphere is the fall velocity when the fluid drag force \( = 1/2(C_D \rho \ w_s^3) \ 1/4(\pi \ d^4) \) on the particle is in equilibrium with the gravity force \( = 1/6(\rho_s - \rho)gd^3 \), giving:

\[
    w_s = \left[ \frac{4(s-1)gd}{3C_D} \right]^{0.5}
\]  

(3.2.19)

in which:
\( w_s \) = terminal fall velocity of a sphere in a still fluid
\( d \) = sphere diameter
\( s \) = specific gravity (= 2.65)
\( C_D \) = drag coefficient
\( g \) = acceleration of gravity

The drag coefficient \( C_D \) is a function of the Reynolds number \( Re = w_s d/v \) and shape factor (see Fig. 3.2.4). In the Stokes region (\( Re < 1 \)) the drag coefficient is given by: \( C_D = 24/Re \), yielding:

\[
    w_s = \frac{(s-1)gd^2}{18v}
\]  

(3.2.20)

Outside the Stokes region there is no simple expression for the drag coefficient. The \( C_D \)-value decreases rapidly outside the Stokes region (\( Re < 1 \)) and becomes nearly constant for \( 10^3 < Re < 10^5 \), yielding \( w_s \) proportional to \( d^{0.5} \).

The effect of temperature on the fall velocity is taken into account by the kinematic viscosity coefficient \( v \). The largest effect occurs for the smallest sphere diameters.
Figure 3.2.3  Particle size curve for a log-normal distribution on logarithmic-linear scale and on logarithmic-probability scale

Figure 3.2.4  Drag coefficient as a function of Reynolds number for different shape factors (Albertson, 1953)
2. *Non spherical particles*

The expressions valid for a sphere cannot be applied for a natural sediment particle because of the differences in shape. The shape effect is largest for relatively large particles (> 300 \( \mu m \)) which deviate more from a sphere than a small particle. Experiments show differences in fall velocity of the order to 30% for SF in the range from 0.5 to 1.

The terminal fall velocity of non-spherical sediment particles can be determined from the following formulae:

\[
w_s = \frac{(s-1)gd^2}{18v} \quad 1 \leq d \leq 100 \ \mu m \quad (3.2.21)
\]

\[
w_s = \frac{10v}{d} \left[ 1 + \frac{0.01(s-1)gd^3}{v^2} \right]^{0.5} - 1 \quad \text{for} \quad 100 \leq d < 1000 \ \mu m \quad (3.2.22)
\]

\[
w_s = 1.1[(s-1)gd]^{0.5} \quad \text{for} \quad d \geq 1000 \ \mu m \quad (3.2.23)
\]

in which:
- \( d \) = sieve diameter
- \( s \) = specific gravity (= 2.65)
- \( v \) = kinematic viscosity coefficient

Figure 3.2.5 shows fall velocities according to the Stokes Equation (3.2.21). Figure 3.2.6 shows fall velocities as given by the U.S. Inter-Agency Committee on Water Resources (1957) and is largely based on experimental results. Equation (3.2.22) is also shown.

The fall velocity of *coral sand* (particles larger than 300 \( \mu m \)) may be considerably smaller than that of quartz sand (Van der Meulen, 1988). Figure 3.2.8 shows fall velocities for both materials at a temperature of 20\(^\circ\)C. The differences are mainly caused by differences in shape. Coral sand particles are more angular and have, therefore, a smaller fall velocity. The density of coral sand may also be somewhat smaller (= 2500 to 2650 kg/m\(^3\)).

3. *Effect of sediment concentration*

The fall velocity of a single particle is modified by the presence of other particles. A small cloud of particles in a clear fluid will have a fall velocity which is larger than that of a single particle. Experiments with uniform suspensions of sediment and fluid have shown that the fall velocity is strongly reduced with respect to that of a single particle, when the sediment concentration is large. This effect, known as hindered settling, is largely caused by the fluid return flow induced by the settling velocities. A state of fluidization may occur when the vertical upward fluid flow is so strong that the upward drag forces on the particles become equal to the downward gravity forces resulting in no net vertical movement of the particles. According to Richardson and Zaki (1954), the fall velocity in a fluid-sediment suspension can be determined as:

\[
w_{s,m} = (1-c)^\gamma w_s \quad (3.2.24)
\]

- \( w_{s,m} \) = particle fall velocity in a suspension
- \( w_s \) = particle fall velocity in a clear fluid
- \( c \) = volumetric sediment concentration (-)
- \( \gamma \) = coefficient (-)
**Figure 3.2.5** Fall velocities for particle sizes smaller than 100 μm according to Stokes

**Figure 3.2.6** Fall velocity for particles sizes larger than 100 μm according to US Inter-Agency Committee (1957)
Figure 3.2.7 Influence of sediment concentration on fall velocity (Re small)

Figure 3.2.8 Fall velocity of coral sand
The $\gamma$-coefficient varies from 4.6 to 2.3 for $Re = \frac{w_s d}{v}$ increasing from $10^4$ to $10^3$. For particles in the range of 50 to 500 $\mu$m under normal flow conditions the $\gamma$-coefficient is about $\gamma = 4$.

Figure 3.2.7 shows Equation (3.2.24) and the experimental results of Oliver (1961) and McNown-Lin (1952). As can be observed, Eq. (3.2.24) yields $w_{s,m}$-values which are 20% to 30% too large for small concentrations.

The formula of Oliver which reads as:

$$w_{s,m} = (1 - 2.15 c)(1 - 0.75 c^{0.33}) w_s$$  \hspace{1cm} (3.2.25)

yields good results over the full range of concentrations (see Fig. 3.2.7).

4. Influence of oscillatory flow

The fall velocity of a single sediment particle in flowing water is generally assumed to be equal to its terminal fall velocity ($v_s$) in still water. Various researchers have investigated the possible reduction of the terminal fall velocity in case of an oscillatory flow. A review and new information have been presented by Hwang (1985) and Nielsen (1979, 1984). These studies show that the major mechanism governing the fall velocity reduction in an oscillating flow is the drag non-linearity effect. According to Hwang, the fall velocity reduction can be expressed as:

$$\frac{v_s}{w_s} = F \left( \frac{w_s d}{v}, \frac{v_{f,\text{max}}}{w_s} \right)$$  \hspace{1cm} (3.2.26)

in which:

$v_s$ = effective fall velocity  
$v_{f,\text{max}}$ = peak value of fluid velocity  
$w_s$ = terminal fall velocity in still water  
d = particle diameter

Equation (3.2.26) is presented in graphical form by Hwang (1985). The ratio $v_s/w_s$ decreases for increasing values of $w_s d/v$ and $v_{f,\text{max}}/w_s$. The results of Hwang suggest a rather large reduction of the fall velocity, $(v_s/w_s = 0.5$ for $v_{f,\text{max}}/w_s = 10)$. The results of Hwang are somewhat suggestive, because the influence of the oscillation period is not shown. Information of the influence of the oscillation period can be obtained from the results of Ho (1964) given in terms of (see Hwang, 1985):

$$\frac{v_s}{w_s} = F \left( \frac{v}{\omega d^2}, \frac{\omega v_{f,\text{max}}}{g} \right)$$  \hspace{1cm} (3.2.27)

in which:

$\omega = 2\pi/T$ = angular oscillation frequency.

Taking a 100 $\mu$m sediment particle, an oscillation period of $T = 10$ s, and $v_{f,\text{max}} = 0.1$ m/s, it follows that $v_s/w_s = 1$.

Similar values were also found by Nielsen (1984), who concluded that the reduction of the terminal fall velocity is negligible in a pure oscillating motion ($T \geq 1$ s).
Turbulence is a special type of (random) oscillating motion dominated by high frequencies. It has been shown by Murray (1970) that the particle fall velocity can be considerably reduced by isotropic turbulence effects due to drag non-linearities. Another mechanism may be intensive eddy production close to the bed inducing vertically upward motions which may reduce the fall velocity until the eddies dissolve at higher levels. Jobson and Sayre (1970) reviewing all available information conclude that the turbulent motions may slightly increase the particle fall velocity.

Ludwick and Domurat (1982) have simulated the movements of 100 µm and 200 µm sediment particles in a turbulent velocity field and found that the settling of fine sand is not significantly reduced when the vertical turbulent fluid velocities have a symmetric distribution. The basic question is what type of vertical velocity distribution is present in natural conditions? Analysis of (turbulent) velocity measurements near the bottom where most of the sediment is transported, shows the presence of bursting processes characterized by lift-up of low-momentum fluid (bursts) and a down-rush of high-momentum fluid to the bed (sweeps). This indicates an asymmetric fluid motion in vertical direction with relatively high (short duration) downward velocities, which may result in a slight increase of the fall velocity. This is in agreement with the findings of Jobson and Sayre (1970).

3.2.6 Angle of (natural) repose

The angle of (natural) repose is a behavioural property of sand particles. Grains piled up on each other have an equilibrium slope which is called the angle of natural repose ($\phi_n$).

This parameter appears to be a function of size, shape and porosity. The angle increases with decreasing roundness. Values from the literature are in the range of $\phi_n = 30^\circ$ to $40^\circ$ for sand sizes from 0.001 to 0.01 m. Observations in nature on the avalanche lee slope of desert dunes and river bed dunes also show values in the range of $30^\circ$ to $40^\circ$.

The angle of repose ($\phi$) also referred to as the angle of internal friction is a characteristic angle related to the particle stability on a horizontal or sloping bed (see Figs. 4.1.1 and 4.1.9). The angle of repose ($\phi$) may differ from the angle of natural repose ($\phi_n$). Usually, the angle of repose is determined from initiation of motion experiments for horizontal and sloping beds (see section 4.1). The critical bed-shear stress for a particle on a sloping bed can be expressed as:

$$\tau_{b,cr} = \tau_{b,cr,0} \frac{\sin(\phi - \beta)}{\sin\phi}$$  \hspace{1cm} (3.2.28)

$\tau_{b,cr} = $ critical bed-shear stress on a sloping bottom

$\tau_{b,cr,0} = $ critical bed-shear stress on a horizontal bottom

$\phi = $ angle of repose

$\beta = $ angle of (longitudinal) bottom slope

For a given $\beta$ value the $\tau_{b,cr}$ and the $\tau_{b,cr,0}$-values can be determined from the measured variables. Equation (3.2.28) can then be used to determine the $\phi$-value.

Table 3.2 shows $\phi$-values reported in the literature.

The $\phi$-values of carbonate sands were determined from results of triaxial tests (direct shear testing).

The $\phi$-values of the silicate sands determined from Eq. (3.2.28) using results of initiation of motion experiments express the angle of repose of the particles of the top layer of the bed surface. This value should approximately be equal to the angle of natural repose ($\phi_n$). The values observed (see Table 3.2) are in the range of $40^\circ$ to $50^\circ$ and are much larger than the
observed angles of natural repose (30° to 40°). The reason for this is not fully clear; most probably it is related to the validity of Eq. (3.2.28). The degree of packing of the entire bed layer should not have much influence on the angle of repose of the top layer. Most probably the top layer will always have a loose packing.

<table>
<thead>
<tr>
<th>Source</th>
<th>Type of sediment</th>
<th>Size $d_0$ ($\mu$m)</th>
<th>Shape</th>
<th>Packing</th>
<th>Angle of repose $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>White (1940)</td>
<td>silicate sand</td>
<td>900</td>
<td></td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>Lysne (1969)</td>
<td>silicate sand</td>
<td>5600</td>
<td></td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>Lugue (1974)</td>
<td>silicate sand</td>
<td>1000 - 4000</td>
<td>rounded</td>
<td>loose</td>
<td>38 - 50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>900</td>
<td>rounded</td>
<td>dense</td>
<td>40 - 45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1800</td>
<td>rounded</td>
<td>loose</td>
<td>40 - 45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3300</td>
<td>rounded</td>
<td>loose</td>
<td>45 - 50</td>
</tr>
<tr>
<td>Van der Wal (1991)</td>
<td>silicate sand</td>
<td>210</td>
<td>rounded</td>
<td>loose</td>
<td>45 - 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1600</td>
<td>rounded</td>
<td>loose</td>
<td>55 - 65</td>
</tr>
<tr>
<td>Delft Hydr. (1984)</td>
<td>silicate stone</td>
<td>36000</td>
<td>angular</td>
<td>loose</td>
<td>41 - 47</td>
</tr>
<tr>
<td>Airey (1988)</td>
<td>carbonate sand</td>
<td>300 - 500</td>
<td>angular</td>
<td>loose-dense</td>
<td>38 - 50</td>
</tr>
<tr>
<td>Hull (1988)</td>
<td>carbonate sand</td>
<td>200 - 1000</td>
<td>angular</td>
<td>loose-dense</td>
<td>38 - 50</td>
</tr>
<tr>
<td></td>
<td>silicate sand</td>
<td>200 - 1000</td>
<td>rounded</td>
<td>loose</td>
<td>38 - 41</td>
</tr>
</tbody>
</table>

**Table 3.2 Angle of repose for silicate and carbonate sands**

For the design of *stable channels* the following conservative values are recommended (see Table 3.3).

<table>
<thead>
<tr>
<th>Size $d_0$ ($\mu$m)</th>
<th>Angle of repose ($\phi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rounded</td>
</tr>
<tr>
<td>$\leq 0.001$</td>
<td>30°</td>
</tr>
<tr>
<td>0.005</td>
<td>32°</td>
</tr>
<tr>
<td>0.01</td>
<td>35°</td>
</tr>
<tr>
<td>0.05</td>
<td>37°</td>
</tr>
<tr>
<td>$\geq 0.1$</td>
<td>40°</td>
</tr>
</tbody>
</table>

**Table 3.3 Angle of repose for stable channel design (silicate material)**
REFERENCES


Albertson, M.L., 1953. Effect of Shape on the Fall Velocity of Gravel Particles. Proc. 5th Hydr. Conf., Univ., of Iowa, Bull. no. 34, USA.


REFERENCES (continued)


4. INITIATION OF MOTION AND SUSPENSION

4.1 Initiation of motion in currents

4.1.1 Introduction

Particle movement will occur when the instantaneous fluid force on a particle is just larger than the instantaneous resisting force related to the submerged particle weight and the friction coefficient. Cohesive forces are important when the bed consists of appreciable amounts of clay and silt particles. The driving forces are strongly related to the local near-bed velocities. In turbulent flow conditions the velocities are fluctuating in space and time which make together with the randomness of both particle size, shape and position that initiation of motion is not merely a deterministic phenomenon but a stochastic process as well. Early work on the initiation of motion was done by Brahms (1753), who proposed a sixth power relationship between the flow velocity and the required weight of a stone to be stable and by Dubuat (1786) who introduced the concept of critical bed-shear stress.

4.1.2 Critical bed-shear stress

The fluid forces acting on a sediment particle resting on a horizontal bed (see Fig. 4.1.1) consist of skin friction forces and pressure forces. The skin friction force acts on the surface of the particles by viscous shear. The pressure force consisting of a drag \( F_D \) and a lift force \( F_L \) is generated by pressure differences along the surface of the particle. The forces acting on a sediment particle resting on a horizontal bed surface are depicted in Fig. 4.1.1.

The angle of repose \( \phi \) is defined as the angle between the line through the particle centre and the point of contact and the line through the particle centre normal to the bed surface. Particle movement will occur when the moments of the instantaneous fluid forces \( F_D \) and \( F_L \) with respect to the point of contact are just larger than the stabilizing moment of the submerged particle weight \( G \), yielding (see Fig. 4.1.1):

\[
F_D a_1 + F_L a_3 \geq G a_2 \tag{4.1.1}
\]

\[
F_D \left( (b_1 + b_2) \cos \phi \right) + F_L \left( b_3 \sin \phi \right) \geq G (b_2 \sin \phi) \tag{4.1.2}
\]

or

\[
\frac{(b_1 + b_2)}{(b_2 - b_3)} \frac{F_D}{F_L/G} \geq G \tan \phi \tag{4.1.3}
\]

For reasons of simplicity, the ratio of the lift force and the submerged particle weight is assumed to be relatively small (\( \ll 1 \)). Physically, this is not necessarily true. The lift force depends, however, on the same variables as the drag force and therefore the effect of the lift force is automatically taken into account by the empirical coefficients which will be introduced. This yields:

\[
F_D \geq \alpha_1 G \tan \phi \tag{4.1.4}
\]
Figure 4.1.1 Forces on a sediment particle (horizontal bed)

\[ a_1 = (b_1 + b_2) \cos \phi \]
\[ a_2 = b_2 \sin \phi \]
\[ a_3 = b_3 \sin \phi \]

\( \phi = \) angle of repose

Figure 4.1.2 Transport rates at low bed-shear stresses
The value $\alpha_1 = b_2/(b_1 + b_2)$ depends on the local Reynolds number. At a high Reynolds number the pressure force will be much larger than the (viscous) skin friction force and the resulting fluid force $F_D$ will act through the centre of the particle ($b_1 = 0$ and thus $\alpha_1 = 1$), (White, 1940). At a low Reynolds number the viscous friction force at the top of the particle will dominate yielding $b_1 > 0$ and thus $\alpha_1 < 1$.

Equation (4.1.4) was first derived by White (1940) based on the equilibrium of the horizontal forces. The parameter $G \tan \phi$ can be seen as a stabilizing friction force with $\mu = \tan \phi$ as the friction coefficient.

The effect of both the (viscous) skin friction force and the pressure force, usually, is expressed as:

$$F_D = \frac{1}{2} \rho C_D \left( \frac{1}{4} \pi d^2 \right) (u_i)^2$$  (4.1.5)

in which:

- $F_D =$ drag force
- $C_D =$ drag coefficient
- $d =$ particle diameter
- $\rho =$ fluid density
- $u_i =$ fluid velocity at particle centre

The drag coefficient ($C_D$) is known to be a function of the local Reynolds number ($u_i d/\nu$). The fluid velocity ($u_i$) at the centre of the particle can be expressed as:

$$u_i = \alpha_2 u.$$  (4.1.6)

The $\alpha_2$-coefficient depends on the local Reynolds number when the flow regime is hydraulically smooth; $u_* = (\tau_\nu/\rho)^{0.5}$ = bed-shear velocity.

The fluid force can now be expressed as:

$$F_D = \alpha_3 \rho d^2 u_*^2$$  (4.1.7)

The $\alpha_3$-coefficient ($= \alpha_2^2 \pi C_D/8$) is a coefficient depending on the local Reynolds number.

The submerged particle weight $G$ can be expressed as:

$$G = \alpha_4 (\rho_* - \rho) g d^3$$  (4.1.8)

The $\alpha_4$-coefficient ($= \pi/6$) depends on the shape of the particle.

Substitution of Eqs. (4.1.7) and (4.1.8) into Eq. (4.1.4) yields:

$$\frac{u_*^2}{(s-1) g d} \geq \alpha_5 \tan \phi$$  (4.1.9)

or

$$\theta \geq \theta_\alpha.$$  (4.1.10)
in which:
\[ \theta = \frac{u^2}{(s-1)g d} = \frac{\tau_b}{(\rho_s-\rho)g d} = \text{mobility (Shields) parameter} \]
\[ \theta_{cr} = \alpha_s \tan\phi = \frac{4}{3} \frac{\alpha_1 \tan\phi}{\alpha_2 C_D} = \text{critical Shields parameter} \]
\[ s = \frac{\rho_d}{\rho} = \text{relative density} \]

The \( \theta_{cr} \)-factor depends on the hydraulic conditions near the bed, the particle shape and the particle position relative to the other particles. The hydraulic conditions near the bed can be expressed by the Reynolds number \( \text{Re}_* = \frac{u_d}{\nu} \). Thus \( \theta_{cr} = f(\text{Re}_*) \).

Many experiments have been performed to determine the \( \theta_{cr} \) as a function of \( \text{Re}_* \). The experimental results of Shields (1936) related to a flat bed surface are most widely used to represent the critical conditions for initiation of motion. Herein, the median particle diameter \( (d_{50}) \) is used as the characteristic particle diameter in the case of non-uniform sediment material.

Figure 4.1.3 shows the Shields curve in terms of \( \theta \) and \( \text{Re}_* \). Shields (1936) measured transport rates at bed-shear stresses just larger than the critical bed-shear stress \( (\tau_{cr}) \). As his definition of the critical bed-shear stress for initiation of motion \( (\tau_{cr}) \) Shields used the value of the bed-shear stress at which the extrapolated transport rate was zero (Fig. 4.1.2). Details of the extrapolation method were not given by Shields (1936).

The experimental range of Shields roughly is:
\[ \theta_{cr} \geq 0.035 \quad \text{for} \quad \text{Re}_* \leq 5, \text{hydraulic smooth regime} \]
\[ 0.03 \leq \theta_{cr} \leq 0.04 \quad \text{for} \quad 5 \leq \text{Re}_* \leq 70, \text{transitional regime} \]
\[ 0.04 < \theta_{cr} \leq 0.06 \quad \text{for} \quad \text{Re}_* \geq 70, \text{hydraulic rough regime} \]

A Reynolds number smaller than about 5 means that the particle diameter is smaller than the thickness of the viscous sublayer \( (\delta_v \geq d) \). The \( \theta_{cr} \)-value for laminar flow \( (\text{Re}_* < 1) \) was found to be equal to 0.2 (White, 1940).

The value of \( \theta_{cr} \) from Eq. (4.1.10) can be estimated for \( \text{Re}_* \) in the range of 10 to 100. Assuming \( \alpha_1 = 0.5 \) to 1, \( \alpha_2 = 5 \) to 10, \( C_D \approx 0.5 \) and \( \phi = 30^\circ \), it follows that \( \theta_{cr} = 0.01 \) to 0.06, which is in reasonable agreement with the Shields curve (see also Fig. 4.1.3).

The Shields curve in terms of \( \theta \) and \( \text{Re}_* \) is not practical because the \( \tau_{b,cr} \)-value can only be obtained by iteration.

Bonnefille (1963) and Yalin (1972) showed that the Shields curve can be expressed in terms of the dimensionless mobility parameter \( \theta \) and the dimensionless particle diameter \( D \) (see Figure 4.1.4).

Applying these parameters, the Shields curve can be represented as:
\[ \theta_{cr} = 0.24 \quad D^{-1} \quad \text{for} \quad 1 < D \leq 4 \]
\[ \theta_{cr} = 0.14 \quad D^{-0.64} \quad \text{for} \quad 4 < D \leq 10 \]
\[ \theta_{cr} = 0.04 \quad D^{-0.1} \quad \text{for} \quad 10 < D \leq 20 \]
\[ \theta_{cr} = 0.013 \quad D^{0.29} \quad \text{for} \quad 20 < D \leq 150 \]
\[ \theta_{cr} = 0.055 \quad \text{for} \quad D > 150 \]
in which:

\[
\theta_{cr} = \frac{\tau_{b,cr}}{(\rho_s - \rho)gd_{so}} = \text{critical Shields parameter}
\]

\[
D_s = [(s-1)g/v^2]^{1/3}d_{so} = \text{particle parameter}
\]

\[
\tau_{b,cr} = \text{time-averaged critical bed-shear stress.}
\]

Figure 4.1.5 shows the critical bed-shear stress \((\tau_{b,cr})\) according to Shields as a function of the particle diameter \((d)\) for a temperature of \(T = 10^o\), 20\(^o\) and 30\(^o\)C, \((\rho_s = 2650\ \text{kg/m}^3, \rho = 1000\ \text{kg/m}^3)\). The influence of temperature is only significant for a particle diameter smaller than 600 \(\mu\)m. For a particle diameter larger than 7000 \(\mu\)m (7 mm) the \(\tau_{b,cr}\)-value is independent of temperature (viscosity).

Figure 4.1.3 *Initiation of motion for a current over a plane bed, \(\theta = f(Re_s)\) Shields (1936)*
Figure 4.1.4 Initiation of motion and suspension for a current over a plane bed, \( \theta = f(D_s) \), Van Rijn (1989)
Figure 4.1.5 Initiation of motion for a current over a plane bed, $\tau_{b,cr} = f(d)$ Shields (1936)

Figure 4.1.6 Initiation of motion and suspension for a current over a plane bed, $\theta = f(Re)$, Delft Hydraulics (1972)
1. Influence of criterion

The complexity of defining a critical bed-shear stress for initiation of motion is mainly caused by the stochastical character of the driving fluid forces and the stabilizing resisting forces and by the lack of an unambiguous definition of initiation of motion.

The most commonly used definitions (criterions) for the critical bed-shear stress are:
- the zero transport rate after extrapolation of measured transport rates (Shields, 1936), see Fig. 4.1.2,
- the number of particles displaced per unit area and time (Neill and Yalin, 1969; Graf and Pazis, 1977), see Fig. 4.1.7,
- the qualitative transport stage based on visual observation (Kramer, 1932; Delft Hydraulics, 1972), see Fig. 4.1.6.

The extrapolation method of Shields is depicted in Fig. 4.1.2. Paintal (1971), Delft Hydraulics (1972), Graf and Paris (1977) and others have found measurable quantities of transported particles at much smaller bed-shear stresses than the Shield-values. Figure 4.1.6 shows that the Shields curve represents permanent grain movement at all locations of the bed surface (seven modes of particle movement were distinguished by Delft Hydraulics, 1972). It is noted that the transport rates (q_b) are extremely sensitive to changes in the \( \theta \)-parameter at low shear stresses (just beyond the critical shear stresses). Paintal (1971) reported \( q_b = \theta^{15} \). Extrapolation will easily lead to large errors in \( \theta_{cr} \) using this method.

Grass (1970) introduced a stochastic approach to define initiation of motion. Particle motion will occur when the probability functions of both the driving bed-shear stresses and the resisting stresses overlap each other (see Fig. 4.1.8). A minimum overlap region must be present before initiation of motion is generated. Grass (1970) defined the minimum overlap region by means of a multiplication factor \( n \) for the standard deviation of the bed shear stress: \( \tau_{b} + n \sigma_{\tau_{b}} = \tau_{c} - n \sigma_{\tau_{c}} \), with \( \tau_{c} \) = space and time-averaged critical bed-shear stress (constant for each type of sediment material) and \( \sigma_{\tau_{c}} \) = standard deviation of critical bed-shear stress. Using \( \sigma_{\tau}/\tau = 0.4 \) and \( \sigma_{\tau_{c}}/\tau_{c} = 0.3 \), Grass found \( n = 0.625 \) for critical values corresponding to the Shields curve, which means that \( \tau_{cr,shields} = 0.65 \tau_{c} \). Lavalle and Mofjeld (1987) also question the existence of critical conditions for initiation of motion. Given adequate time, movement of particles somewhere will always be observed because turbulent flow basically is a stochastic process and there will always be a bed-shear stress large enough to move a particle.

Although a critical stage below which no single grain is moving, may not really exist, a critical stage is necessary for practical design purposes.

2. Influence of shape, gradation and size

Experiments with particles of different shapes show that the \( \theta_{cr} \)-parameter is not much affected by the shape of the particles when the nominal diameter (diameter that yields the same volume) is used as the characteristic parameter. Very flat particles have larger \( \theta_{cr} \)-values (factor 1.5 to 2).

Gradation has an effect when the size range is rather wide (\( d_{90}/d_{50} > 3 \)), because the larger particles will be more exposed, while the smaller particles are shielded by the larger particles. Egiazaroff (1965) proposed a method to determine the critical bed-shear stress of each fraction of the bed material, as follows:

\[
\tau_{b,cr,i} = \xi_{i} \tau_{b,cr}
\]  

(4.1.12)
\[ N = \text{number of particles moving per unit area (m}^2) \]

**Figure 4.1.7** Initiation of motion for a current over a plane bed, Graf and Paris, 1977

\[ \tau_b, \tau_c (\rho_s - \rho) \text{g}d_{50} \]

\[ u_\ast d_{50}/v \]

**Figure 4.1.8** Initiation of motion for a current over a plane bed according to stochastic approach (Grass, 1970)
in which:

\[ \tau_{b,cr,i} = \text{critical bed-shear stress of fraction } i \text{ with size } d_i \]
\[ \tau_{b,cr} = \text{critical bed-shear stress of the average diameter } d_m \]

\[ \xi_i = \left[ \frac{\log(19)}{\log(19 \frac{d_i}{d_m})} \right]^2 = \text{exposure coefficient} \tag{4.1.13} \]

Equation (4.1.13) yields: \( \xi_i = 5 \) for \( d_i/d_m = 0.2 \) and \( \xi_i = 0.4 \) for \( d_i/d_m = 5 \). Thus, particles smaller than \( d_m \) have a much larger \( \tau_{b,cr} \) because they are shielded between the large particles, whereas the particles larger than \( d_m \) have a smaller \( \tau_{b,cr} \) because they are more exposed.

Armouring will occur when the bed-shear stress is not large enough to move the largest particles of the bed material. When there is no supply of smaller particles from upstream, all smaller particles will eventually be eroded, and the coarser particles will form an armour layer preventing further scour. Based on experimental results, the median diameter (\( d_{50} \)) of the armour layer will approximately be equal to the \( d_{95} \) of the initial bed material. This phenomenon has been observed downstream of weirs.

Mantz (1977) studied the initiation of motion of fine cohesionless flaky sediments with particle sizes in the range of 10 to 100 \( \mu m \) and proposed the following critical bed-shear stress (see Fig. 4.1.4):

\[ \frac{\tau_{b,cr}}{(\rho_s - \rho)g d_{50}} = 0.1 \left( \frac{u_{*d_{50}}}{v} \right)^{-0.3} \quad \text{for} \quad 0.03 < \frac{u_{*d_{50}}}{v} < 1 \tag{4.1.14} \]

The data presented by Miller et al. (1977) also indicate that \( \theta_{cr} \) of Shields is too large for fine sediments (\( D_* < 10 \)). A better representation of the data presented by Miller et al. is given by (see Fig. 4.1.14):

\[ \theta_{cr} = 0.11 \ D_*^{-0.54} \quad \text{for} \quad D_* < 10 \tag{4.1.15} \]

Flume tests on loose (not cemented) calcareous sediments (coral sands) showed a similar behaviour as quartz sediments of the same size (Delft Hydraulics, 1972).

3. Influence of bed slope

a. Longitudinal slope \( \beta \)

A sediment particle resting on a longitudinal sloping bed, will be set in motion (see Fig. 4.1.9), when the sum of the critical fluid force (assuming \( \alpha_1 = 1 \), Eq. 4.1.4) and the gravity force component is just equal to the stabilizing force (= \( N \tan \phi \)), yielding:

\[ F_{D,cr} + G \sin \beta = G \cos \beta \tan \phi \tag{4.1.16} \]

or

\[ F_{D,cr} = G \cos \beta \tan \phi - G \sin \beta \tag{4.1.17} \]

The critical fluid force for a horizontal bed \( (F_{D,cr,0}) \) is given by Eq. (4.1.4), giving \( \alpha_1 = 1 \):

\[ F_{D,cr,0} = G \tan \phi \tag{4.1.18} \]
Combining Eqs. (4.1.17) and (4.1.18), the critical fluid force \( F_{D,cr} \) on a sloping bed can be related to the critical fluid force on a horizontal bed \( F_{D,cr,o} \), as follows:

\[
\frac{F_{D,cr}}{F_{D,cr,o}} = \frac{G \cos \beta \tan \phi - G \sin \beta}{G \tan \phi} = \frac{\sin(\phi - \beta)}{\sin \phi}
\]  

(4.1.19)

or

\[
F_{D,cr} = k_\beta F_{D,cr,o}
\]  

(4.1.20)

in which:

\[
k_\beta = \frac{\sin(\phi - \beta)}{\sin \phi} \quad \text{for a downslping flow} \quad (k_\beta < 1)
\]

\[
k_\beta = \frac{\sin(\phi + \beta)}{\sin \phi} \quad \text{for upsloping flow} \quad (k_\beta > 1)
\]

Equation (4.1.16), first presented by Schoklitsch (1914), can also be expressed in terms of the bed-shear stress, giving:

\[
\tau_{b,cr} = k_\beta \tau_{b,cr,o}
\]  

(4.1.21)

Whitehouse and Hardisty (1988) have shown that Eq. (4.1.19) and (4.1.21) yield realistic results compared with experimental data.

![Figure 4.1.9 Forces on a sediment particle (longitudinal bed slope)](image-url)

**Figure 4.1.9** Forces on a sediment particle (longitudinal bed slope)
Figure 4.1.10 Forces on a sediment particle (transverse bed slope)

b. Transverse slope $\gamma$

A sediment particle resting on a transverse sloping bed will be set in motion (see Fig. 4.1.10) when the resulting driving fluid force is equal to the stabilizing fluid force, giving:

$$F_R = F_S$$  \hspace{1cm} (4.1.22)

$$\left( F_{D,cr}^2 + G^2 \sin^2\gamma \right)^{0.5} = G \cos\gamma \tan\phi$$  \hspace{1cm} (4.1.23)

or

$$F_{D,cr}^2 = G^2 \cos^2\gamma \tan^2\phi - G^2 \sin^2\gamma$$  \hspace{1cm} (4.1.24)

Combining Eqs. (4.1.24) and (4.1.18), it follows:

$$\frac{F_{D,cr}^2}{F_{D,cr,o}^2} = \frac{G^2 \cos^2\gamma \tan^2\phi - G^2 \sin^2\gamma}{G^2 \tan^2\phi} = \cos^2\gamma \left[ 1 - \frac{\tan^2\gamma}{\tan^2\phi} \right]$$  \hspace{1cm} (4.1.25)

or

$$F_{D,cr} = k_\gamma F_{D,cr,o}$$  \hspace{1cm} (4.1.26)

or

$$\tau_{b,cr} = k_\gamma \tau_{b,cr,o}$$  \hspace{1cm} (4.1.27)

4.12
in which:

\[ k_y = \cos \gamma \left[ 1 - \left( \frac{\tan^2 \phi}{\tan^2 \gamma} \right)^{0.5} \right] \]

This factor was first presented by Leiner (1912).

For a combination of a longitudinal and a transverse bed slope, it follows that:

\[ \tau_{b,cr} = k_h k_y \tau_{b,cr,o} \]  \hspace{1cm} (4.1.28)

4. Influence of bed forms

As soon as the sediment transport process is established, ripples and dunes are formed on the bed. The critical bed-shear stress for initiation of motion on a bed form is different from that for a flat bed. The bed shear stress \( \tau_b \) over a bed consisting of bed forms is composed of a part \( \tau_{b,cr} \) related to (skin) friction over the bed surface and another part \( \tau_{b,cr}'' \) related to the non-uniform pressure distribution over the bed form crest and eddy region (see Chapter 6).

\[ \tau_b = \tau_b' + \tau_b'' \]  \hspace{1cm} (4.1.29)

A sediment particle resting on the surface of a bed form will be set in motion by the friction force \( \tau_{b,cr} \) or by the turbulent fluctuations in the eddy region downstream of the crest \( \tau_{b,cr}'' \). This means that the critical bed-shear stress \( \tau_{b,cr} \) is always larger when bed forms are present than when the bed is flat. The initiation of motion on a bed form has been observed to occur at the downstream end of the eddy region, where the flow reattaches to the bed. Despite the fact that the velocity and shear stress are small near the reattachment point, the longitudinal turbulence intensity is relatively large at that location.

5. Influence of cohesive material

When the bed consists of silty and muddy materials, cohesive forces between the sediment particles become important. These forces cause a distinct increase of the strength of the soil against erosion. Depending on the type of clay minerals, this effect may be more or less pronounced. Flume experiments show that an amount of 25% of clay in a sand bed caused a dramatic reduction (factor 30) of the sand concentrations generated by wave action (Van Rijn, 1985).

Biological activity at the bed may also influence the critical values for initiation of motion, especially in muddy and silty environments.

An important factor governing the erodibility of cohesive soils is the rate of consolidation. Fresh mud deposits have a very loose texture of mud flocs which already have a low density themselves. The wet bulk density of such a deposit may be within the range of 1050 to 1100 kg/m³ of which 95% or more consists of water. In this stage the cohesive forces in the deposit are still very low, and erosion can occur easily. If the deposits are not eroded again, its density gradually increases as interstitial water is pressed out of the fresh soil by the weight of the deposit itself. This process of consolidation initially goes relatively fast but gradually slows down. With the compaction of the soil the erosion resistance rapidly increases. It is well known that old compact clay soils are highly resistant against erosion.
A sand bed with small percentages of silts and clays (silty or clayey sand) already shows a distinctly increased resistance against erosion. Little is known of the erodibility of cohesive soils in a quantitative sense.

Some information can be obtained from flume tests performed at Delft Hydraulics (1989) on bed samples from the North Sea with particle sizes ($d_{50}$) in the range of 100 to 200 $\mu$m and mud-silt percentages in the range of 2 to 20%. The measured critical bed shear stress was related to the critical bed-shear stress according to Shields, yielding:

$$\tau_{b,cr} = (p_s)^{0.5} \tau_{b,cr,Shields}$$  \hspace{1cm} (4.1.30)

in which:

$p_s$ = percentage of fines (mud, silt) smaller than 50 $\mu$m (in %, minimum value = 1%)

4.1.3 Critical depth-averaged velocity

The earliest studies were related to critical velocities of stones (Brahms, 1753 and Sternberg, 1875). They studied the critical near-bed velocity and found that it was related to the particle diameter, as follows:

$$u_{b,cr} = \propto d^{0.5}$$  \hspace{1cm} (4.1.31)

The near-bed velocity is, however, not very well defined and therefore it is preferred to use the critical depth-averaged velocity ($\bar{u}_{cr}$) as the characteristic parameter.

One of the most well-known relationships between the critical depth-averaged velocity and the particle diameter (in graphical form) is given by Hjulstrøm (1935). The influence of the relative bed roughness (ratio of water depth and particle diameter, $h/d$) was, however, neglected by Hjulstrøm.

The critical depth-averaged velocity can be derived from the critical bed-shear stress using the Chézy-equation. Assuming hydraulic rough flow conditions ($u^2k_g/u > 70$), the critical depth-averaged flow velocity ($\bar{u}_{cr}$) for a plane bed can be expressed as:

$$\bar{u}_{cr} = 5.75 \cdot u_{*,cr} \log \left( \frac{12h}{k_g} \right)$$  \hspace{1cm} (4.1.32)

in which:

$\bar{u}$ = depth-averaged critical velocity

$u_{*,cr}$ = $\theta_{cr}^{0.5} \cdot (s-1) \cdot g \cdot d_{50}^{0.5}$ = critical bed-shear velocity (m/s)

$h$ = water depth (m)

$k_g$ = effective bed roughness of a flat bed (m)

$\alpha$ = coefficient ($\alpha = 1$ for stones $d_{50} \geq 0.1$ m and $\alpha = 3$ for sand and gravel material)

Equation (4.1.32) can be expressed as:

$$\bar{u}_{cr} = 5.75 \cdot (s-1) \cdot g \cdot d_{50}^{0.5} \cdot \theta_{cr}^{0.5} \cdot \log \left( \frac{12h}{k_g} \right)$$  \hspace{1cm} (4.1.33)

4.14
Using Eq. (4.1.33), \( k_s = 3d_{50} \) and \( d_{90} = 2d_{50} \) and the Shields curve, the critical depth-averaged velocity for sand particles in the range of 0.0001 to 0.002 m can be expressed as:

\[
\bar{u}_{cr} = 0.19 \left( d_{50} \right)^{0.1} \log \left( \frac{12h}{3d_{50}} \right) \quad \text{for} \ 0.0001 \leq d_{50} \leq 0.0005 \text{ m} \quad (4.1.34)
\]

\[
\bar{u}_{cr} = 8.50 \left( d_{50} \right)^{0.6} \log \left( \frac{12h}{3d_{50}} \right) \quad \text{for} \ 0.0005 < d_{50} \leq 0.002 \text{ m} \quad (4.1.35)
\]

in which:
- \( d_{50} \) = median particle diameter (m)
- \( d_{90} \) = 90% particle diameter (m)

Equations (4.1.34) and (4.1.35) are shown in Figure 4.1.11 for a plane bed.

Many expressions for the critical depth-averaged velocity of (coarse) gravel and stone material \( (d_{20} \geq 0.002 \text{ m}) \) can be found in the Literature. Herein, the expressions of Neill (1968) and Maynord (1978) are given:

Neill : \[ \bar{u}_{cr} = 1.4 \left[ (s-1)g \right] d_{50}^{0.5} \left[ \frac{h}{d_{50}} \right]^{1/6} \quad (4.1.36) \]

Maynord : \[ \bar{u}_{cr} = 1.3 \left[ (s-1)g \right] d_{50}^{0.5} \left[ \frac{h}{d_{50}} \right]^{1/5} \quad (4.1.37) \]

![Figure 4.1.11 Critical depth-averaged velocities for a plane bed](image_url)
4.1.4 Design of stable channels

1. Introduction

The design and maintenance of stable channels is extremely important for irrigation systems. The design methods can be divided into two categories:
- the (empirical) regime method,
- the tractive force method.

The regime method originating from irrigation channel design in India and Pakistan, is based on a set of empirical equations which specify the water depth \( h \), width \( b \), slope \( I \) and mean velocity \( ar{u} \) as a function of discharge \( Q \) and particle diameter \( d_{50} \), (Lacey, 1930; Lane, 1955; Blench, 1957). Thus:

\[
h, b, I, \bar{u} = f(Q, d_{50}) \tag{4.1.38}
\]

Transport of sediments is allowed as long as there is no net annual scour or deposition in each channel. In that case the channel is considered to be in regime (equilibrium). The regime method is most suitable for large-scale irrigation systems with a wide range of discharges in soils of silts and fine sands. The equations may only be used within their validity ranges (no extrapolation!).

The tractive force method is based on the principle of allowing no or negligible motion of bed material, which may be the natural bed material or dumped (protective) material like gravel, stones or riprap. The tractive force method is most suitable for small-scale irrigation systems with small discharges in soils of coarse material \( d_{50} \geq 500 \mu m \).

Herein, the tractive force method is presented. Information of the regime method is given by Lacey (1930), Lane (1955) and Blench (1957). A critical review is given by Stevens and Nordin (1987).

2. Tractive force method

As shown in section 4.1.2 (see Figs. 4.1.6 and 4.1.7), the \( \Theta \), values of the Shields curve represent conditions with frequent particle movements at all locations of the bed. Therefore, the Shields curve can not be recommended for the design of a stable channel. A better approach is the application of the experimental results of Pimental (1971) who measured bed-load transport rates close to initiation of motion (particle size \( d_{50} = 0.0025 \) m, 0.008 m and 0.022 m). The following dimensionless transport rates were measured:

\[
\phi = \begin{array}{ccc}
10^{-9} & \text{for } \Theta = 0.01 \\
10^{-8} & \text{for } \Theta = 0.02 \\
10^{-7} & \text{for } \Theta = 0.025 \\
10^{-6} & \text{for } \Theta = 0.03 \\
10^{-4} & \text{for } \Theta = 0.04
\end{array}
\]

in which:
\[
\phi = \frac{Q}{d_{50}^2 g (s-1) \rho} \quad \text{dimensionless transport rate (-)}
\]
\[
Q_b = \rho_b \quad \text{bed load transport rate (kg/sm)}
\]
\[
s = \frac{\rho_b}{\rho} \quad \text{specific density (-)}
\]
\[
d_{50} \quad \text{median particle size (m)}
\]
\[
\Theta = \frac{\tau_b}{(\rho_b - \rho) g d_{50}} \quad \text{dimensionless mobility parameter (-)}
\]
The experimental results of Paintal (1971) have been used by the author to compute the bed-load transport rate (in kg per m width per day) and the corresponding number of moving stones (per m width per day) for stone sizes of 0.05, 0.1 and 0.3 m at small \( \Theta \)-values of 0.02, 0.025 and 0.03 (all below the Shields curve), see Table 4.1. A value of \( \Theta = 0.02 \) appears to be a safe value yielding a number of moving stones much less than one per m width per day.

Applying Eq. (4.1.33) and assuming \( \alpha = 1, d_{so} = 2d_{50} \); the critical depth-averaged velocity for stones is:

\[
\Theta = 0.02: \quad \bar{u}_{cr} = 0.8 \left[ (s-1) g \ d_{so}^{-0.5} \ . \ \log \left( \frac{6h}{d_{50}} \right) \right]
\]

\[
\Theta = 0.03: \quad \bar{u}_{cr} = 1.0 \left[ (s-1) g \ d_{so}^{-0.5} \ . \ \log \left( \frac{6h}{d_{50}} \right) \right]
\]

Equations (4.1.39) and (4.1.40) are valid for uniform river flow. For regions with additional turbulence production (downstream of structures; weirs, bridge piers) a reduction factor \( \alpha_r \) must be taken into account, yielding:

\[
\bar{u}_{cr} = 0.8 \ \alpha_r \left[ (s-1) g \ d_{so}^{-0.5} \ . \ \log \left( \frac{6h}{d_{50}} \right) \right]
\]

in which:

- \( \alpha_r = 1.45/(1 + 3r) \) = reduction factor related to additional turbulence
- \( \sigma_u/u \) = relative turbulence intensity (\( r = 0.15 \) for uniform flow, \( r = 0.35 \) for region downstream of hydraulic jump)
- \( \sigma_u \) = standard deviation of time-averaged velocity
- \( u \) = time-averaged velocity

<table>
<thead>
<tr>
<th>stone diameter ( d_{50} ) (m)</th>
<th>( q_b ) = bed load transport in kg per m width per day</th>
<th>( N ) = number of stones per m width per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Theta = 0.02 )</td>
<td>( \Theta = 0.025 )</td>
<td>( \Theta = 0.03 )</td>
</tr>
<tr>
<td>0.05</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>( N = 0.6 )</td>
<td>( N = 6 )</td>
<td>( N = 60 )</td>
</tr>
<tr>
<td>0.10</td>
<td>0.3</td>
<td>3</td>
</tr>
<tr>
<td>( N = 0.2 )</td>
<td>( N = 2 )</td>
<td>( N = 20 )</td>
</tr>
<tr>
<td>0.30</td>
<td>1.5</td>
<td>15</td>
</tr>
<tr>
<td>( N = 0.04 )</td>
<td>( N = 0.4 )</td>
<td>( N = 4 )</td>
</tr>
</tbody>
</table>

*Table 4.1* Bed load transport close to initiation of motion according to Paintal (1971)
3. Schematization of cross-section

The most practical cross-section that can be constructed is a trapezoidal cross-section. Analysis of computed and measured bed-shear stresses along the cross-section of trapezoidal channels (with a ratio of the surface width and the depth larger than 5) has shown the following results (Olsen and Florey, 1952; Lane 1955):

\[
\hat{\tau}_{b,\text{middle}} = \rho g h I \tag{4.1.42}
\]

\[
\hat{\tau}_{b,\text{side}} = 0.75 \rho g h I = 0.75 \hat{\tau}_{\text{middle}} \text{ for side slope 1:2}
\]

\[
\hat{\tau}_{b,\text{side}} = 0.85 \rho g h I = 0.85 \hat{\tau}_{\text{middle}} \text{ for side slope 1:3} \tag{4.1.43}
\]

\[
\hat{\tau}_{b,\text{side}} = 0.90 \rho g h I = 0.90 \hat{\tau}_{\text{middle}} \text{ for side slope 1:4}
\]

\[
\hat{\tau}_{b,\text{side}} = 0.95 \rho g h I = 0.95 \hat{\tau}_{\text{middle}} \text{ for side slope 1:6}
\]

in which:
\[
\hat{\tau}_{b,\text{middle}} = \text{maximum bed–shear stress in middle part of channel}
\]
\[
\hat{\tau}_{b,\text{side}} = \text{maximum bed–shear stress in side part (with slope vertical to horizontal)}
\]

The bed material will be stable when the prevailing maximum bed-shear stress is smaller than the critical bed-shear stress. Thus,

\[
\text{Middle part is stable if } \hat{\tau}_{b,\text{middle}} \leq \tau_{b,\text{cr,o}} \tag{4.1.44}
\]

\[
\text{Side part is stable if } \hat{\tau}_{b,\text{side}} \leq k_\gamma \tau_{b,\text{cr,o}} \tag{4.1.45}
\]

in which:
\[
\tau_{b,\text{cr,o}} = \text{critical bed-shear stress for horizontal middle part (N/m}^2\text{)}
\]
\[
k_\gamma = \cos \gamma \left[1 - \tan^2 \gamma / \tan^2 \phi \right]^{0.5} = \text{reduction factor (-)}
\]
\[
\gamma = \text{angle of side slope (}^\circ\text{)}
\]
\[
\phi = \text{angle of repose (}^\circ\text{)}
\]
\[
I = \text{surface slope (-)}
\]
\[
h = \text{water depth (m)}
\]

The angle of repose \( \phi \) (see section 3.2.6) is given in Table 4.2).

The angle of repose \( \phi \) (see section 3.2.6) is given in Table 4.2).

The shape of an ideal stable channel with the same critical conditions along the cross-section can be determined by assuming (see Fig. 4.1.12):

\[
\rho g h I \Delta y = \tau_h \frac{\Delta y}{\cos \gamma} \text{ or } \tau_h = \rho g h I \cos \gamma \tag{4.1.46}
\]

and

\[
\hat{\tau}_b = \rho g h I \text{ at } y=0 \text{ (middle of channel)} \tag{4.1.47}
\]

Thus,

\[
\tau_b = \hat{\tau}_b \left( h/\bar{h} \right) \cos \gamma \tag{4.1.48}
\]

At critical conditions, it follows that:

\[
\hat{\tau}_b = \tau_{b,\text{cr}} \text{ and } \tau_b = k_\gamma \tau_{b,\text{cr}} \text{ or } \tau_b/\hat{\tau}_b = k_\gamma, \text{ yielding}
\]

\[
h = \bar{h} \left[1 - \tan^2 \gamma / \tan^2 \phi \right]^{0.5} \tag{4.1.49}
\]

4.18
Finally, a conusoidal shape of the cross-section is found:

\[ h = \hat{h} \cos[(\tan\phi/\hat{h})y] \]  \hspace{1cm} (4.1.50)

in which:
\( \hat{h} \) = water depth at \( y \) from the middle
\( \hat{h} \) = water depth at \( y = 0 \) (middle)

For \( \phi = 35^\circ \), the following parameters can be computed:
- surface width \( 2b = 4.5 \hat{h} \)
- area \( A = 2.9 (\hat{h})^2 \)
- perimeter \( P = 5 \hat{h} \)
- hydraulic radius \( R = 0.6 \hat{h} \)
- critical discharge \( Q = AC(RI)^{0.5} = 2.2(\hat{h})^{2.5}C \)

Recent information on this topic was presented by Diplas and Vigilar, 1992.

<table>
<thead>
<tr>
<th>Particle size ( d_{50} ) (m)</th>
<th>Angle of repose ( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rounded material</td>
</tr>
<tr>
<td>( \leq 0.001 )</td>
<td>30°</td>
</tr>
<tr>
<td>0.005</td>
<td>32°</td>
</tr>
<tr>
<td>0.01</td>
<td>35°</td>
</tr>
<tr>
<td>0.05</td>
<td>37°</td>
</tr>
<tr>
<td>( \geq 0.1 )</td>
<td>40°</td>
</tr>
</tbody>
</table>

*Table 4.2 Angle of repose \( \phi \)*

![Figure 4.1.12 Ideal stable channel shape](image)

**Figure 4.1.12 Ideal stable channel shape**

4.1.5 Examples and problems

1. A wide open channel with a plane sloping bed has a water depth \( h = 2 \) m, slope \( I = 0.5 \times 10^{-3} \), bed material characteristics \( d_{50} = 0.002 \) m, \( d_{90} = 0.005 \) m, \( \rho_s = 2650 \) kg/m\(^3\), water temperature \( T_w = 20^\circ C \), and water density \( \rho = 1000 \) kg/m\(^3\). Is there movement of bed material according to the Shields curve?
The critical bed-shear for \( d_{50} = 0.002 \text{ m} \) is (Fig. 4.1.5): \( \tau_{b,cr} = 1.3 \text{ N/m}^2 \).

The applied bed-shear stress is \( \tau_b = \rho ghI = 9.8 \text{ N/m}^2 \).

Thus, \( \tau_b > \tau_{b,cr} \) and movement of bed material.

2. A wide open channel with a plane sloping bed has a slope \( I = 10^{-5} \), bed material characteristics \( d_{50} = 0.0002 \text{ m}, d_{90} = 0.0003 \text{ m}, \rho_s = 2650 \text{ kg/m}^3 \), water temperature \( T_e = 20^\circ \text{C} \), kinematic viscosity \( \nu = 1 \times 10^{-6} \text{ m}^2/\text{s} \), water density \( \rho = 1000 \text{ kg/m}^3 \).

What is the maximum discharge per unit width without movement of bed material (according to Shields curve)?

The critical bed-shear stress for \( d_{50} = 0.0002 \text{ m} \) is (see Fig. 4.1.5): \( \tau_{b,cr} = 0.18 \text{ N/m}^2 \). The water depth \( h \) follows from \( \tau_{b,cr} = \rho ghI = 0.18 \), yielding \( h = 1.83 \text{ m} \).

The discharge per unit width is given by (see Eq. 2.2.19): \( q = \bar{u} h = C h^{1.5} I^{0.5} \).

Assuming flow in the hydraulic transition regime, the Chézy equation is (see Eq. 2.2.24): \( C = 18 \log \left( \frac{12h}{k_s + 3.3\nu/u_*} \right) \).

The effective bed roughness of a plane bed is (see Eq. 4.1.32): \( k_s = 3 d_{90} = 0.0009 \text{ m} \).

The bed-shear velocity is \( u_* = (\tau_b/\rho)^{0.5} = (0.18/1000)^{0.5} = 0.0134 \text{ m/s} \).

Thus, the Chézy-coefficient is \( C = 77.1 \text{ m}^{0.5}/\text{s}, (u_*k_s/\nu = 12) \).

The maximum permissible discharge is \( q = 0.6 \text{ m}^3/\text{s} \).

One of the most uncertain parameters in this computation is the estimation of the effective bed-roughness of a plane bed (see Eq. 4.1.32). Taking \( k_s = 1 d_{90} \), it follows that \( C = 82.8 \text{ m}^{0.5}/\text{s} \) and \( q = 0.65 \text{ m}^3/\text{s} \). Taking the largest value \( k_s = 3 d_{90} \), a safer (more conservative) value of the maximum discharge \( q = 0.6 \text{ m}^3/\text{s} \) is obtained.

3. A wide open channel has a water depth of \( h = 5 \text{ m} \). The bed is covered with sand dunes. The bed material characteristics are \( d_{50} = 0.0003 \text{ m}, d_{90} = 0.0005 \text{ m}, \rho_s = 2650 \text{ kg/m}^3 \). The overall Chézy-coefficient is \( C = 50 \text{ m}^{0.5}/\text{s} \). The water temperature is \( 20^\circ \text{C} \). The kinematic viscosity coefficient is \( \nu = 10^{-6} \text{ m}^2/\text{s} \). The fluid density is \( \rho = 1000 \text{ kg/m}^3 \).

What is the depth-averaged current velocity at initiation of motion at the upsloping part of the dunes and what is the overall bed-shear stress?

Using the Shields curve (Fig. 4.1.5) for \( d_{50} = 0.0003 \text{ m} \): \( \tau_{b,cr} = 0.2 \text{ N/m}^2 \).

The grain-related bed-shear stress (see Chapter 6) is \( \tau_b = \rho g(\bar{u}/C')^2 \).

The grain-related Chézy coefficient is \( C' = 18 \log[12h/(3 d_{90})] = 83 \text{ m}^{0.5}/\text{s} \).
At initiation of motion
\[ \tau'_{b} = \tau_{b,cr} \]
\[ \rho g \bar{u}_{cr} (C')^2 = 0.2 \]
\[ \bar{u}_{cr} = \left[ 0.2(C')^2 / (\rho g) \right]^{0.5} = 0.38 \text{ m/s} \]

The overall bed-shear stress is
\[ \tau_{b} = \rho g \bar{u}_{cr} (C')^2 = 0.57 \text{ N/m}^2 \]

4. A wide open channel with a plane sloping bed has a water depth of \( h = 1.7 \) m, a depth-averaged velocity \( \bar{u} = 2.5 \) m/s, bed material characteristics \( d_{50} = 2 \) d_{s0}, water temperature \( T_e = 20^\circ \text{C} \), kinematic viscosity \( v = 1 \times 10^{-6} \text{ m}^2/\text{s} \), fluid density \( \rho_f = 1000 \text{ kg/m}^3 \), sediment density \( \rho_s = 2650 \text{ kg/m}^3 \).
What is the minimum size of the bed material to obtain a stable bed (no movement)?

Assuming hydraulic rough flow, the bed material size can be obtained from Eq. (4.1.33)
\[ \bar{u}_{cr} = 5.75 [(s-1)g d_{50}]^{0.5} \theta_{cr}^{0.5} \log \left( \frac{12h}{k_s} \right) \]

The effective bed roughness is estimated to be
\[ k_s = 1 \text{ d}_{50} = 2 \text{ d}_{50} \]

The critical \( \theta \)-value is taken as (safe value)
\[ \theta_{cr} = 0.02 \]

using \( s = 2.65 \) and
\[ g = 9.81 \text{ m/s}^2 \]

it follows that
\[ \bar{u}_{cr} = 3.27 d_{50}^{0.5} \log \left( \frac{12h}{2d_{50}} \right) \]

Using \( \bar{u}_{cr} = 2.5 \) m/s and
\[ h = 1.7 \text{ m} \]

yields
\[ d_{50} = 0.2 \text{ m} \]

Check hydraulic regime of rough flow which requires
\[ u_* k_s / v > 70 \]
yielding
\[ \frac{u_* k_s}{v} = \frac{g^{0.5} \bar{u}}{C} \frac{k_s}{v} = 1.02 \times 10^5 \]

Taking a critical value of \( \theta_{cr} = 0.03 \) yields a much smaller size of \( d_{50} = 0.1 \text{ m} \).

5. A trapezoidal channel has a water depth \( h \) and a bottom width \( b \) and side slopes of 1 to 2 (\( \gamma = 27^\circ \)); the water surface slope is \( l = 3.5 \times 10^{-4} \). The discharge is \( Q = 160 \text{ m}^3/\text{s} \). The bed material consists of rounded stones with \( d_{50} = 0.04 \text{ m} \) and \( d_{90} = 0.08 \text{ m} \). The water temperature is \( 20^\circ \text{C} \); the kinematic viscosity is \( v = 1 \times 10^{-6} \text{ m}^2/\text{s} \); the densities are \( \rho_f = 1000 \text{ kg/m}^3 \) and \( \rho_s = 2650 \text{ kg/m}^3 \).
What is the water depth \( h \) and the bottom width \( b \) to obtain a stable bed?

Take a safe \( \theta \)-value
\[ \theta_{cr} = 0.03 \]

The critical bed-shear stress at the middle part is
\[ \tau_{b,cr,o} = 0.03 (\rho_s - \rho_f) g d_{50} = 20 \text{ N/m}^2 \]

The critical bed-shear stress at the side part is
\[ \tau_{b,cr} = k_\gamma \tau_{b,cr,o} = 20 k_\gamma \]
Using $\phi = 36^\circ$ (see Table 4.2) and side slope angle $\gamma = 27^\circ$, yields

$\tau_{b,cr} = 20.64 = 12.8 \text{ N/m}^2$

The middle part is stable, if

$\hat{\tau}_b = \rho gh l = \tau_{b,cr,0}$

This yields

$h = 20/(\rho gl) = 5.8 \text{ m}$

The side part is stable, if

$\hat{\tau}_b = 0.75 \rho gh l = 12.8$, see Eq. (4.1.43)

$h = 12.8/(0.75 \rho gl) = 5 \text{ m}$

The smallest water depth is decisive, thus

$h = 5 \text{ m}$

The cross-section area is

$A = bh + 2h^2 = 5b + 50$

The wetted perimeter is

$P = b + 2(2.23 h) = b + 22.4$

The hydraulic radius is

$R = (5b + 50)/(b + 22.4)$

The Chézy coefficient is

$C = 18 \log(12R/k_s)$

The effective bed roughness is

$k_s = d_{so} = 0.08 \text{ m}$

The discharge is

$Q = AC(R)^{0.5} = 160 \text{ m}^3/\text{s}$

A bottom width $b = 10 \text{ m}$ yields

$A = 100 \text{ m}^2$, $P = 32.4 \text{ m}$, $R = 3.1 \text{ m}$

$C = 48 \text{ m}^{0.5}/\text{s}$ and $Q = 158 \text{ m}^3/\text{s}$.

Thus, water depth is $h = 5 \text{ m}$ and the bottom width is $b = 10 \text{ m}$.

6. A wide open channel with a plane sloping bed has a bed surface slope of $I = 10^4$. The bed material characteristics are $d_{so} = 0.003 \text{ m}$ and $d_{so} = 0.006 \text{ m}$. Other data are: $\rho = 1000 \text{ kg/m}^3$, $\rho_s = 2650 \text{ kg/m}^3$, $\nu = 1 \times 10^4 \text{ m}^3/\text{s}$.

What is the maximum discharge (per unit width) without movement of bed material?

Solution: $q = 1.9 \text{ m}^3/\text{s}$ for $k_s = 3 d_{so}$

$q = 2.2 \text{ m}^3/\text{s}$ for $k_s = 1 d_{so}$

7. The bottom of a wide open channel is protected with uniform round stones (diametered). The stones have a mass of 30 kg and a sediment density of $\rho_s = 2800 \text{ kg/m}^3$. The water depth is $h = 4 \text{ m}$. Other data are: $\rho = 1000 \text{ kg/m}^3$, $\nu = 1 \times 10^4 \text{ m}^3/\text{s}$.

What is the critical depth-averaged velocity using $\theta_{cr} = 0.03$?

Solution: $\overline{u}_{cr} = 4.9 \text{ m/s}$ for $k_s = d$

8. A trapezoidal channel has a water depth (in middle) of $h = 2 \text{ m}$, bottom width $b = 15 \text{ m}$ and side slopes $1$ to $2$ ($\gamma = 27^\circ$). The bottom is covered with rounded stones $d_{so} = 0.05 \text{ m}$, $k_s = 0.05 \text{ m}$.

What is the maximum bottom slope and the maximum discharge to obtain a stable bed (use Shields)?

Solution: $I_{cr} = 2 \times 10^{-3}$

$Q_{cr} = 100 \text{ m}^3/\text{s}$
4.2 Initiation of motion in waves

4.2.1 Critical velocity

In oscillatory flow there is no generally accepted relationship for initiation of motion on a plane bed. Many equations have been proposed. Silvester and Mogridge (1970) present 13 different equations collected from the Literature. One of the more popular equations is that of Komar and Miller (1975):

\[
\frac{(\hat{U}_{\delta,cr})^2}{(s-1)gd_{50}} = 0.21 \left( \frac{2\hat{A}_{\delta,cr}}{d_{50}} \right)^{0.5} \quad \text{for} \quad d_{50} < 500 \mu m \tag{4.2.1}
\]

\[
\frac{(\hat{U}_{\delta,cr})^2}{(s-1)gd_{50}} = 1.45 \left( \frac{2\hat{A}_{\delta,cr}}{d_{50}} \right)^{0.25} \quad \text{for} \quad d_{50} \geq 500 \mu m \tag{4.2.2}
\]

in which:
\( \hat{U}_{\delta,cr} \) = critical peak value of orbital velocity near the bed
\( \hat{A}_{\delta,cr} \) = critical peak value of orbital excursion near the bed

Analysis of Eqs. (4.2.1) and (4.2.2) shows a weak increase of \( \hat{U}_{\delta,cr} \) for an increase of the wave period \( T \).

Various sets of previously published data: Bagnold (1946), Manohar (1955), Rance and Warren (1968), Silvester and Mogridge (1970), Dingler (1974), Bosman (1981) and Davies (1985), were analyzed by Van Rijn (1989) to describe the critical velocity \( \hat{U}_{\delta,cr} \) as a function of the particle size \( d_{50} \) and the wave period \( T \). Only experiments with sand particles \( (\rho_s = 2650 \text{ kg/m}^3) \) and wave periods in the range of 4 to 15 seconds were considered. The data represent a particle range from 90 to 3300 \( \mu m \).

Figure 4.2.1 shows the peak value of the critical near-bed orbital velocity as a function of the wave period and sediment size. The average inaccuracy of the curves is about 25%. Detailed analysis of the data shows a clear influence of the wave period. For most data the critical stage (in terms of the peak orbital velocity, \( \hat{U}_{\delta,cr} \)) increases with the wave period, which is consistent with Eqs. (4.2.1), (4.2.2). The experimental results of Silvester and Mogridge, however, show an opposite trend.

4.2.2 Critical bed-shear stress

The experimental data presented in section 4.2.1 can also be expressed in terms of the Shield parameter using the time-averaged bed-shear stress \( (\tau_{b,cr}) \), yielding (Van Rijn, 1989):

\[
\frac{\tau_{b,w,cr}}{(\rho_s - \rho)gd_{50}} = f(D_s) \tag{4.2.3}
\]

in which:
\( \tau_{b,w,cr} = 0.25 \rho f_w(\hat{U}_{\delta,cr})^2 \) = time-averaged (over half period) wave-related bed-shear stress, Eq. (2.3.14)

\( f_w \) = wave friction factor
\( \hat{U}_{\delta,cr} \) = peak value of critical near-bed orbital velocity according to linear wave theory, Eq. (2.3.2)
Figure 4.2.1  Initiation of motion for waves over a plane bed based on critical velocity

Figure 4.2.2  Initiation of motion for waves over a plane bed based on critical bed-shear stress
The time-averaged bed-shear stress and not the maximum bed-shear stress was used by Van Rijn (1989) because the Shields curve, originally proposed for unidirectional flow, is based on time-averaged parameters.

To determine the (time-averaged) wave-related bed-shear stress, the friction factor must be known, which means that the grain roughness \( k_s \) and the kinematic viscosity \( \nu \) must also be known. The \( k_s \)-value is assumed to be equal to \( \alpha \cdot d_{50} \) with \( \alpha \) in the range of 1 to 3 (Chapter 6). In some cases the \( d_{50} \) was not reported in the literature. To overcome this, a value equal to \( d_{50} = 1.5 \cdot d_{50} \) was assumed. When the water temperature was not reported, a value of 20° was assumed.

In the case of turbulent smooth conditions the wave friction factor was computed from Eq. (2.3.17). In the case of turbulent rough conditions Eq. (2.3.20) was applied. For the transitional regime (in analogy with unidirectional flow) the wave friction factor was determined from the equation for turbulent rough conditions (Eq. 2.3.20) representing the wave-related roughness by:

\[
k_s = \alpha \cdot d_{50} + \frac{3.3 \nu}{(\tau_{b,cr}/\rho)^{0.5}} \tag{4.2.4}
\]

The \( \alpha \)-coefficient was assumed to be in the range of 1 to 3 (see Chapter 6). To determine the critical bed-shear stress for the transitional regime, an iterative solution method was used because the \( k_s \)-parameter is a function of the critical bed-shear stress (Eq. 4.2.4). Figure 4.2.2 shows the dimensionless critical bed-shear stress as a function of the dimensionless \( D_c \)-parameter for the selected data. The Shields curve which is valid for unidirectional flow data only, is also shown. A vertical bar is used to express the influence of the wave period and the \( \alpha \)-value (1 to 3) for each particle size. The variation between the results of different investigators is mainly caused by the definition problem for initiation of motion. For example, Rance and Warren (1968) define the critical stage as the stage when one or two particles are dislodged and moved over a small distance, while Bosman (1981) defines the critical stage as that when about 10% of the surface particles is in motion resulting in a relatively large critical bed-shear stress (see Figure 4.2.2). Based on these results, it seems justified to conclude that the Shields curve can also be applied as a criterion for initiation of motion for oscillatory flow over a plane bed. The curve represents a critical stage at which only a minor part (say 1 to 10%) of the bed surface is moving.

Figure 4.2.2 is only valid for plane bed, which is a rare phenomenon in natural conditions. Usually, small-scale ripples are present and in that case the critical velocities for initiation of motion are considerably smaller due to the generation of vortex motions. For example, Carstens et al (1969) report a 50%-reduction of the peak velocity \( \bar{U}_{b,cr} \) for a rippled bed.

**4.2.3 Examples and problems**

1. The water depth in a coastal sea with a plane bed is \( h = 5 \) m. The wave period is \( T = 7 \) s. The bed material characteristics are \( d_{50} = 0.0002 \) m, \( d_{90} = 0.0003 \) m, \( \rho_s = 2650 \) kg/m³. The water temperature is \( T_e = 20^\circ \)C. Kinematic viscosity coefficient \( \nu = 1.10^{-6} \) m²/s, fluid density \( \rho = 1025 \) kg/m³. What is the wave height at initiation of motion?
**Method 1 using Fig. 4.2.1**

The wave length can be computed from

\[ L = \frac{gT^2}{2\pi} \tanh(2\pi h/L) \]

yielding

\[ L = 46 \text{ m} \]

Using Fig. 4.2.1 and \( d_{50} = 200 \mu \text{m} \), the critical peak orbital velocity can be obtained

\[ \hat{U}_{b,cr} = 0.23 \text{ m/s} \]

The critical wave height can be obtained from Eq. (2.3.2)

\[ H_{cr} = \frac{1}{\pi} \hat{U}_{b,cr} T \sinh(2\pi h/L) = 0.38 \text{ m} \]

**Method 2 using Shields curve Fig. 4.2.2**

The dimensionless particle size is

\[ D_* = \left[ \frac{(s - 1)g}{v^2} \right]^{1/5} d_{50} = 5 \]

The dimensionless Shields value is

\[ \theta_{cr} = 0.14 D_*^{-0.64} = 0.05 \]

The critical bed-shear stress is

\[ \tau_{b,cr} = 0.05 (\rho_s - \rho) gd_{50} = 0.16 \text{ N/m}^2 \]

The mean wave-induced bed-shear stress is given by (Eq. 2.3.14)

\[ \tau_{b,w} = \frac{D}{4} f_w (\hat{U}_b)^2 \]

At initiation of motion

\[ \tau_{b,w} = \tau_{b,cr} \text{ or } \frac{1025}{4} f_w (\hat{U}_{b,cr})^2 = 0.16 \]

Assuming a transitional hydraulic regime, the friction coefficient is (Eq. 2.3.20)

\[ f_w = \exp \left[ -6 + 5.2 \left( \frac{\hat{A}_h}{k_s + 3.3v/u_{*w}} \right)^{0.19} \right] \]

Taking \( \hat{A}_h = \hat{U}_b T/2\pi \), \( k_s = 3 d_{90} \)

and \( u_{*w} = (\tau_{b,w}/\rho)^{0.5} = (0.16/1025)^{0.5} \)

\[ = 0.0125 \text{ m/s, yields} \]

\[ f_w = \exp \left[ -6 + 1.4(\hat{U}_{b,cr})^{-0.19} \right] \]

Thus \( \tau_{b,w} = \tau_{b,cr} \) yields

\[ \frac{1025}{4} (\hat{U}_{b,cr})^2 \exp \left[ -6 + 1.4(\hat{U}_{b,cr})^{-0.19} \right] = 0.16 \]

\[ (\hat{U}_{b,cr})^2 \exp \left[ -6 + 1.4(\hat{U}_{b,cr})^{-0.19} \right] = 0.00062 \]

yielding

\[ \hat{U}_{b,cr} = 0.19 \text{ m/s} \]

The critical wave height is

\[ H_{cr} = \frac{1}{\pi} \hat{U}_{b,cr} T \sinh(2\pi h/L) = 0.31 \text{ m} \]
Method 3 using Eq. (4.2.1) of Komar

Critical peak orbital velocity at the bed

\[ (\hat{U}_{b,cr})^2 = 0.21 \left( 2\hat{A}_{b,cr}/d_{50} \right)^{0.5} (s-1) g d_{50} \]

using \( \hat{A}_{b,cr} = \hat{U}_{b,cr} T/2\pi \)

\[ (\hat{U}_{b,cr})^2 = 0.21 \left( \hat{U}_{b,cr} T/\pi \right)^{0.5} (s-1) g d_{50} \]

\[ (\hat{U}_{b,cr})^3 = 0.014 T (s-1)^2 g^2 d_{50} \]

\[ (\hat{U}_{b,cr})^3 = 0.00471 \]

yielding

\[ \hat{U}_{b,cr} = 0.17 \, \text{m/s} \]

\[ H_{cr} = 0.28 \, \text{m} \]

2. A coastal sea (plane bed) has a water depth \( h = 5 \, \text{m} \). The peak wave period is \( t_p = 7 \, \text{s} \). The significant wave height is \( H_s = 1 \, \text{m} \). Other data are: \( \rho = 1025 \, \text{kg/m}^3 \), \( \rho_s = 2650 \, \text{kg/m}^3 \), \( \nu = 1 \times 10^{-6} \, \text{m}^2/\text{s} \), \( d_{50} = 2 \, d_{s0} \), \( k_s = 1 \, d_{s0} \).

What is the \( d_{50} \) of the bed material which is just stable at the given conditions (use methods of Shields, Komar and Van Rijn, Fig. 4.2.1)?

solution: \( d_{50} = 0.0026 \, \text{m} \) (Shields)
\( d_{50} = 0.0038 \, \text{m} \) (Komar)
\( d_{50} = 0.0032 \, \text{m} \) (Van Rijn)

4.3 Initiation of motion for combined current and waves

4.3.1 Critical bed-shear stress

Information of initiation of motion for combined unidirectional and oscillatory flow is rather scarce. Some results were presented by Larsen et al (1981) for three continental shelf sites in the United States and in Australia, and by Hammond and Collins (1979) for flume conditions. The data of Larsen et al included measurements of current velocity and direction at 1 m above the sea bed, mean bottom pressures, photographs of the sea bed and bed material samples. The bed material at the USA-sites was characterized as sandy silt (\( d_{s0} = 35 \, \mu \text{m} \) and 42 \( \mu \text{m} \)). The bed material at the Australian site was characterized as fine sand (\( d_{s0} = 170 \, \mu \text{m} \)). The water depths at the sites were in the range of 75 to 90 m. Information of the local bed forms (ripples or plane bed) was not reported by Larsen et al.

The data were used by Van Rijn (1989) to determine the initiation of motion parameters according to the Shields method.

The resulting time-averaged total bed-shear stress \( (\tau_{b,cw}) \) is computed from Eq. (2.4.25), neglecting the wave-current interaction. Thus, \( \tau_{b,cw} = \tau_{b,c} + \tau_{b,w} \). The (time-averaged) wave-related bed-shear stress \( (\tau_{b,w}) \) is determined from the peak value of the bed-shear stress \( (\hat{\tau}_{b,w}) \) reported by Larsen et al (1981), applying \( \tau_{b,w} = \frac{1}{2} \hat{\tau}_{b,w} \). The current-related bed-shear stress is computed from the measured velocity \( (\bar{u}_i) \) at a height \( (z_i) \) of 1 m above the bed, assuming a logarithmic velocity distribution in the near-bed region, as follows:

\[ \tau_{b,c} = \rho \left( \frac{\kappa \bar{u}_i}{\ln(30 z_i/k_s)} \right)^2 \]  \hspace{1cm} (4.3.1)

in which:

\( k_s = \) effective roughness height (Eq. 4.2.4).
The sea bed was assumed to be flat. The water temperature was assumed to be 15°C. Applying the above given approach, the computed total time averaged bed shear stresses ($t_{\text{tot}}$) were somewhat (15%) larger than those reported by Larsen et al, because the wave-current interaction was neglected by Van Rijn (1989). The dimensionless parameters for initiation of motion are presented in Fig. 4.3.2, showing critical values that are smaller than the Shields values, especially for the two sandy silt sites ($d_{50} = 35 \mu m$ and 42 $\mu m$). The results are in better agreement with the curve proposed by Mantz (1977).

**Figure 4.3.1** Critical bed-shear velocities for combined current and waves over a plane bed (Hammond and Collins, 1979)

**Figure 4.3.2** Initiation of motion for combined currents and waves over a plane bed, $\theta = f(D_s)$
The data of Hammond and Collins (1979) were measured in a flume (length = 3.7 m, width = 0.3 m, depth = 0.3 m). In the flume, a perspex carriage with sediment material \((d_{50} = 142, 363, 771 \text{ and } 1134 \mu m)\) was installed, which could be set into an oscillating motion. Time-averaged velocities were measured at 0.02 m above the sediment bed by using a miniature propeller type current meter. The test results of the two finest sediments (142 and 163 \(\mu m\)) were used by Van Rijn (1989). The test results of the two coarsest materials (771 and 1134 \(\mu m\)) were not used because they may have been influenced by relatively large inertial forces acting on the grains produced by the horizontal acceleration of the sediment bed (on the carriage).

Figure 4.3.1 shows the critical combinations in terms of the wave-related and the current-related bed shear velocities computed from Eqs. (2.3.14), (4.2.4) and (4.3.1). As can be observed, larger critical current velocities \(u_{*c,cr}\) are generally achieved for small wave periods than for large periods at a given \(u_{*,w}\)-value. Probably, this is the result of a relatively large reduction of the near-bed current velocities of short-period waves for which the wave-induced vortex motions near the bed are relatively intense (see also section 2.4). Consequently, the imposed current velocity has to be larger to initiate motion for a short period wave than for a long period wave at the same wave height.

Figure 4.3.2 shows the experimental results in terms of the Shields parameters. The resulting critical bed-shear stress is computed from Eq. (2.4.25) as the sum of the wave-related and the current related bed shear stress \(\tau_{b,cw} = \tau_{b,c} + \tau_{b,w}\). The vertical bar indicates the influence of the wave period and the error in the estimated \(k_r\)-value (1 to 3 \(d_{50}\)). Generally, the longer period wave produce the smallest critical mobility parameters. As can be observed, the mean critical values are close to the Shields curve. The extreme values are about 40% smaller and larger than the mean values.

4.3.2 Examples and problems

1. The water depth in a coastal sea with a plane bed is \(h = 5\) m. The wave period is \(T = 7\) s. The depth-averaged current velocity is \(\bar{u} = 0.3\) m/s. The angle between the wave and current direction is 60°. The bed material characteristics are \(d_{50} = 0.0003\) m, \(d_{90} = 0.0005\) m, \(\rho_s = 2650\) kg/m³. The water temperature is \(T_e = 20^\circ C\), the kinematic viscosity coefficient is \(\nu = 1.1 \times 10^{-6}\) m²/s. The fluid density is \(\rho = 1025\) kg/m³.

What is the wave height at initiation of motion?

The wave length follows from

(Eq. 2.4.1),

\[ \left[ \left( \frac{L}{T} - \bar{u} \cos \phi \right) \right] = \left( \frac{gL}{2\pi} \right) \tanh(2\pi h/L') \]

yielding

\[ L' = 47 \text{ m} \]

The relative wave period is

(Eq. 2.4.1)

\[ T_r = \frac{T}{1-(\bar{u}/T \cos \phi)/L')} = 7.2 \text{ s} \]

The particle size parameter is

\[ D_r = 7.5 \]

The critical Shields parameter is

\[ \theta_{cr} = 0.037 \] (see Fig. 4.2.2)

The critical bed-shear stress is

\[ \tau_{b,cr} = 0.037 (\rho_s - \rho)gd_{50} = 0.18 \text{ N/m}^2 \]

The mean bed-shear stress exerted by combined wave and current

(Eq. 2.4.25) is

\[ \tau_{b,cw} = \tau_{b,c} + \tau_{b,w} \]
The current-related bed-shear stress (assuming transitional flow and $k_s - 3 \delta_m$)

\[
\tau_{b,c} = \frac{\rho g(\bar{u})^2}{18^2 \log^2\left(\frac{12h}{k_s + 3.3v/u_{*c}}\right)}
\]

\[
\tau_{b,c} = \frac{2.8}{\log^2\left(\frac{60}{0.0015 + 1.06 \times 10^{-4}(\tau_{b,c})^{0.5}}\right)}
\]

yielding

\[
\tau_{b,c} = 0.14 \text{ N/m}^2
\]

The wave-related bed shear stress
(Eq. 2.3.14)

\[
\tau_{b,w} = 0.25 \rho f_w (\bar{U}_b)^2 = 256.2 (\bar{U}_b)^2 \exp\left[-6 + 5.2\left(\frac{\hat{A}_b}{k_s + 3.3v/u_{*w}}\right)^{-0.19}\right]
\]

At initiation of motion

\[
\tau_{b,ew} = \tau_{b,cr}
\]

\[
\tau_{b,c} + \tau_{b,w} = 0.18
\]

\[
\tau_{b,w} = 0.18 - 0.14 = 0.04 \text{ N/m}^2
\]

Taking $u_{*w} = (\tau_{b,w}/\rho)^{0.5} = 0.0063$,

\[
\hat{A}_b = \frac{\bar{U}_b T}{2\pi} = 1.12 \bar{U}_b, k_s = 3 \delta_0.
\]

\[
v = 10^{-6} \text{ m/s}, \text{ yields}
\]

\[
256.2 (\bar{U}_{b,cr})^2 \exp\left[-6 + 1.6(\bar{U}_{b,cr})^{-0.19}\right] = 0.04
\]

\[
(\bar{U}_{b,cr})^2 \exp\left[-6 + 1.6(\bar{U}_{b,cr})^{-0.19}\right] = 1.6 \times 10^4
\]

\[
\bar{U}_{b,cr} = 0.066 \text{ m/s}
\]

The critical wave height is

\[
H_{cr} = \frac{1}{\pi} \bar{U}_{b,cr} T_r \sinh(2\pi h/L') = 0.11 \text{ m}
\]

2. A coastal sea (plane bed) has a water depth of $h = 5 \text{ m}$. The significant wave height is $H_s = 1 \text{ m}$. The peak wave period is $T_p = 7 \text{ s}$. The depth-averaged current velocity is $\bar{u} = 1 \text{ m/s}$. The angle between the wave and current direction is 90°. Other data are: $\rho = 1025 \text{ kg/m}^3$, $\rho_g = 2650 \text{ kg/m}^3$, $v = 1 \times 10^{-6} \text{ m}^2/\text{s}$, $d_{50} = 2 \delta_0$, $k_s = 3 \delta_0$.

What is the $d_{50}$ of the bed material which is just stable at the given conditions (use Shields)?

solution: $d_{50} = 0.012 \text{ m}$

4.30
4.4 Initiation of suspension in currents

4.4.1 Critical bed-shear stress

For increasing values of the bed-shear velocity, the particles will be moving along the bed by more or less regular jumps (saltations). When the value of the bed-shear velocity becomes comparable to that of the particle fall velocity, the sediment particles may go into suspension. Bagnold (1966) stated that a particle only remains in suspension when the turbulent eddies have dominant vertical velocity components which exceed the particle fall velocity \( w_s \). Assuming that the vertical velocity component \( \bar{w} \) of the eddies are of the same order of magnitude as the vertical turbulence intensity \( \bar{w} \), the critical value for initiation of suspension can be expressed as:

\[
\bar{w} = \left[ \frac{(w')^2}{s} \right]^{0.5} \geq w_s
\]  

(4.4.1)

Studies on turbulence phenomena in boundary layer flow provide detailed information of the vertical distribution of the turbulence intensity (Hinze, 1975). The vertical turbulence intensity \( \bar{w} \) has a maximum value of the same order as the bed-shear velocity \( u_* \), both for hydraulic smooth and rough flow conditions. Thus: \( \bar{w} = u_* \).

Using the above-mentioned values of the vertical turbulence intensity, the criterion for initiation of suspension becomes:

\[
\frac{u_{*,\text{crs}}}{w_s} - 1
\]

(4.4.2)

which can be expressed as:

\[
\theta_{\text{crs}} = \frac{(u_{*,\text{crs}})^2}{(s-1)gd_{50}} - \frac{(w_s)^2}{(s-1)gd_{50}}
\]  

(4.4.3)

in which:
- \( u_{*,\text{crs}} \) = critical bed-shear velocity for initiation of suspension (m/s)
- \( w_s \) = particle fall velocity in clear (still) water (m/s)
- \( d_{50} \) = particle diameter (m)
- \( g \) = acceleration of gravity (m/s²)
- \( \rho_s \) = density of sediment (kg/m³)
- \( \rho \) = density of fluid (kg/m³)

Another criterion for initiation of suspension was given by Engelund (1965). Based on a rather crude stability analysis, he derived:

\[
\frac{u_{*,\text{crs}}}{w_s} = 0.25
\]  

(4.4.4)

Equations (4.4.3) and (4.4.4) are shown in Figure 4.1.4.

An experimental investigation was carried out at Delft Hydraulics (Delft Hydraulics, 1982) to determine the critical flow conditions for initiation of suspension defined as the stage of flow at which the particles perform a jump length larger than about 100 particle diameters.
Visual observations during the (critical) flow conditions showed instantaneous upward particle motions (turbulent bursts) with jump heights in the range of 0 to 100 particle diameters at various locations of the bed. The experimental results, which are shown schematically in Figure 4.1.4 can be presented by:

\[
1 < D_\ast \leq 10: \quad \frac{u_{*,crs}}{w_g} = \frac{4}{D_\ast}, \quad \text{or} \quad \theta_{crs} = \frac{16}{(U_g)^2} \frac{(w_g)^2}{(s-1)g d_{s0}} \]

\[
D_\ast > 10: \quad \frac{u_{*,crs}}{w_g} = 0.4, \quad \text{or} \quad \theta_{crs} = 0.16 \frac{(w_g)^2}{(s-1)g d_{s0}}
\]  

(4.4.5)

Summarizing, it is suggested that the criterion of Bagnold may define an upper-limit at which a concentration profile starts to develop, while Equation (4.4.5) may define an intermediate stage at which locally turbulent bursts with sediment particles are lifted from the bed into suspension.

**4.4.2 Critical depth-averaged velocity**

Equation (4.4.5) was used to compute the critical depth-averaged velocity \(\bar{u}_{cr,\ast}\) for a plane bed. The grain roughness was computed as \(k_\ast = 3 \ d_{s0}\) with \(d_{s0} = 2 \ d_{s0}\). The critical depth-averaged velocity follows from (see Eq. (4.1.33)):

\[
\bar{u}_{cr,\ast} = 5.75 \ [(s-1)g \ d_{s0}]^{0.5} \ \theta_{cr,\ast}^{0.5} \ \log \left( \frac{12h}{k_\ast} \right)
\]

(4.4.6)

Equation (4.4.6) is shown in Fig. 4.1.11.

**4.4.3 Examples**

1. A wide open channel with a plane sloping bed has a water depth of \(h = 5 \ m\). The bed material characteristics are \(d_{x0} = 0.0003 \ m\), \(d_{x0} = 0.0005 \ m\), \(\rho_s = 2650 \ kg/m^3\). The fall velocity is \(u_\ast = 0.04 \ m/s\). The water temperature is \(T_e = 20^\circ C\), the kinematic viscosity coefficient \(v = 10^{-6} \ m^2/s\), the density \(\rho = 1000 \ kg/m^3\).

What is the depth-averaged current velocity at initiation of suspension?

According to Bagnold

\[
\tau_{b,cr,\ast} = 0.33 \ (\rho_s - \rho)g \ d_{s0} = 1.6 \ N/m^2
\]

\[
u_{*,cr,\ast} = 0.04 \ m/s
\]

According to Van Rijn

\[
D_\ast = d_{s0} \left[ \frac{(s - 1)g}{v^2} \right]^{1/3} = 7.6
\]

\[
\theta_{cr,\ast} = 0.092
\]

\[
\tau_{b,cr,\ast} = 0.092 (\rho_s - \rho)g \ d_{s0} = 0.45 \ N/m^2
\]

\[
u_{*,cr,\ast} = 0.021 \ m/s
\]
The depth-averaged current velocity at initiation of suspension follows from

\[ \tau_u = \rho g \left( \bar{u} / C \right)^2 = \tau_{u,v_{cr}} \]

\[ \bar{u}_{v_{cr}} = \left[ \frac{C^2 \tau_{b,v_{cr}}}{\rho g} \right]^{0.5} \]

Assuming transitional flow

\[ C = 18 \log \left( \frac{12h}{3d_{s0} + 3.3 v/u_*} \right) \]

Thus, Bagnold

\[ C = 82.4 \text{ m}^2/\text{s} \]
\[ \bar{u}_{v_{cr}} = 1.06 \text{ m/s} \]

Van Rijn

\[ C = 82.1 \text{ m}^2/\text{s} \]
\[ \bar{u}_{v_{cr}} = 0.55 \text{ m/s} \]
REFERENCES


REFERENCES (continued)


REFERENCES (continued)


5. **BED FORMS**

5.1 **Introduction**

Bed forms are relief features initiated by the fluid oscillations generated downstream of small local obstacles over a bottom consisting of movable (alluvial) sediment materials. Herein, the bed forms are considered from a hydraulic point of view and not from a sedimentary point of view. This means that the type, shape and the dimensions of the bed forms are presented; the detailed internal sedimentary processes are not considered. Information of these latter processes is presented by Allen (1968, 1982).

Bed forms under the following conditions are discussed:
- unidirectional currents
- non-steady currents (river and tidal flow)
- waves
- current and waves.

5.2 **Bed forms in unidirectional currents**

5.2.1 **Classification**

1. **Sand-bed rivers**

Many types of bed forms can be observed in nature. When the bed form crest is perpendicular (transverse) to the main flow direction, the bed forms are called transverse bed forms, such as ripples, dunes and anti-dunes. Ripples have a length scale smaller than the water depth (see Figure 5.2.7), whereas dunes have a length scale much larger than the water depth (see Figure 5.2.7). Figure 5.2.1 shows ripple and dune-type bed forms as observed in rivers (Simons and Richardson, 1966). The crest lines of the bed forms may be straight, sinuous, catenary, linguoid or lunate (see Figure 5.2.2).

Ripples and dunes travel downstream by erosion at the upstream face (stoss side) and deposition at the downstream face (lee side; see Figure 5.2.3). Antidunes travel upstream by lee side erosion and stoss side deposition (Figure 5.2.3).

Bed forms with their crest parallel to the flow are called longitudinal bed forms such as ribbons and ridges.

The morphological regimes for unidirectional currents over a sand bed \( (d_{50} \leq 2000 \, \mu m) \) can be classified into (see also Fig. 5.2.1):

- lower transport regime with flat bed, ribbons and ridges, ripples, dunes and bars,
- transitional regime with washed-out dunes and sand waves,
- upper transport regime with flat mobile bed and sand waves (anti-dunes).

Many attempts were made to explain the type of bed forms generated under given flow conditions by use of (kinematic) instability analysis (Kennedy, 1963; Engelund, 1970). The instability analysis indicates the conditions at which an initial perturbation of the bed will grow, but it does not give information of the equilibrium dimensions of the bed forms. The most reliable classification of bed form types is based on the analysis of bed forms as observed in flume and field conditions.

In the literature, roughly four groups of classification methods for sand beds are presented. Engelund (1967) used the Froude number as a classification parameter, while Liu (1957) described the type of bed forms in terms of a suspension parameter and a particle-related
Figure 5.2.1 Bed form types in rivers according to Simons and Richardson (1966)

Figure 5.2.2 Crest line patterns

Figure 5.2.3 Bed form migration in lower and upper regime
Reynolds number. Simons and Richardson (1966) used the stream power \((\tau_{bc} \bar{u})\) and the median fall diameter as basic parameters. Van Rijn (1984) and Van den Berg et al. (1989) used a dimensionless bed-shear stress parameter and a dimensionless particle parameter to classify the bed form types.

Herein, the classification diagrams for sand beds of Liu, Simons-Richardson, Van den Berg - Van Gelder and Van Rijn are presented (see Figs. 5.2.4 to 5.2.7).

Liu (1957) : BF type = \(F\left(\frac{u_{sc} u_{sc} d_{s0}}{w_s} \right)\)

Simons-Richardson (1966) : BF type = \(F(\tau_{bc}, \bar{u}, d_i)\)

Van den Berg - Van Gelder (1989) : BF type = \(F(\theta', D_*)\)

Van Rijn (1984, 1989) : BF type = \(F(T, D_*)\)

in which:

- \(u_{sc}\) = bed-shear velocity related to current
- \(\tau_{bc}\) = overall current-related bed-shear stress
- \(w_s\) = particle fall velocity of bed material
- \(d_{s0}, d_{90}\) = particle diameters
- \(v\) = kinematic viscosity coefficient
- \(\bar{u}\) = depth-averaged velocity
- \(s = \rho_s/\rho\) = relative density
- \(h\) = water depth
- \(\theta' = \frac{\tau_{bc}^*/[(\rho_s - \rho) g d_{s0}]}{\tau_{bc}}\) = particle mobility parameter
- \(T = \left(\frac{\tau_{bc}^* - \tau_{bc}}{\tau_{bc}}\right)\) = bed-shear stress parameter
- \(\tau_{bc}^* = \rho g (\bar{u}/C')^2\) = grain-related bed-shear stress
- \(C' = 18 \log(12h/3d_{90})\) = grain-related Chezy-coefficient
- \(D_* - d_{s0} \left[(s - 1)g/v^2\right]^{1/3}\) = particle diameter parameter

The diagrams of Liu (1957) and Simons-Richardson (1966) are mainly based on flume data. Van den Berg - Van Gelder (1989) and Van Rijn (1984, 1989) used flume and field data to develop their classification diagrams.

According to Van Rijn, the asymmetrical dune-type bed forms with a length scale much larger than the water depth \((\lambda >> h)\) are the dominant features for \(T \leq 15\). Mega-ripples and mini-ripples may be superimposed on the dunes for \(3 \leq T \leq 10\) and \(D_* \leq 10\). Mega-ripples are ripples with a length scale of the order of the water depth \((\lambda \approx h)\); mini-ripples have a length scale of the order of the near-bed turbulence length scale \((\lambda < h)\). Mini-ripples are the dominant features for \(T < 3\) and \(D_* < 10\).
The upper regime which is defined to occur for \( T \geq 25 \), is characterized by a dominating suspended load transport. The characteristic bed forms are nearly symmetrical sand waves with a length scale much larger than the water depth (\( \lambda > h \)), which occur in large rivers at relatively small Froude numbers, (\( Fr < 0.8 \)), whereas plane bed and sand waves (also called anti-dunes) are generated in small rivers at relatively high Froude numbers (\( Fr > 0.8 \)). The actual onset of the generation of anti-dunes cannot be predicted from Figure 5.2.7, because this is mainly governed by the interaction of generated free surface waves with the stream bed (at Froude numbers > 0.8). A small discontinuity in the bed will cause a stationary surface wave which will increase in amplitude for a Froude number approaching unity. Fully established anti-dunes have a length scale equal to the length scale of the surface waves (in phase).

The type of bed forms in the transition regime (15 < \( T < 25 \)) is somewhat obscure. It may range from that typical of the lower regime (disappearing dunes) to that typical of the upper regime (disappearing sand waves) depending mainly on the preceding flow conditions (rising or falling stage).

Summarizing the following classification is proposed by Van Rijn:

<table>
<thead>
<tr>
<th>Transport regime</th>
<th>Particle size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 1 \leq D_\bullet \leq 10 )</td>
</tr>
<tr>
<td>Lower</td>
<td>mini-ripples</td>
</tr>
<tr>
<td>( 0 \leq T \leq 3 )</td>
<td></td>
</tr>
<tr>
<td>( 3 &lt; T \leq 10 )</td>
<td>mega-ripples and dunes</td>
</tr>
<tr>
<td>( 10 &lt; T \leq 15 )</td>
<td>dunes</td>
</tr>
<tr>
<td>Transition</td>
<td>washed out dunes, sand waves</td>
</tr>
<tr>
<td>( 15 &lt; T &lt; 25 )</td>
<td></td>
</tr>
<tr>
<td>Upper</td>
<td>(symmetrical) sand waves</td>
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<tr>
<td>( T \geq 25, Fr &lt; 0.8 )</td>
<td></td>
</tr>
<tr>
<td>( T \geq 25, Fr \geq 0.8 )</td>
<td>plane bed and/or anti-dunes</td>
</tr>
</tbody>
</table>

**Table 5.1 Classification of bed forms according to Van Rijn**

2. **Gravel-bed rivers**

Classification diagrams for gravel-bed rivers are not available. The bed materials of gravel rivers usually have a broad range of grain sizes from sand particles to large boulders. As a result selective transport processes and armouring of the bed surface may occur locally. The most common regime is the lower transport regime; the transition regime (with plane bed) is a rare event. Very regular bed-features such as mega-ripples and dunes have been observed in laboratory flumes and small-scale channels (Tsubaki and Shinozaka, 1959). Various types of three-dimensional bar forms (referred to as alternate bars, crescentic bars, transverse bars, deltaic bars, see Fig. 5.2.8) have been observed in depositional regions of the river (Richards, 1982).
Figure 5.2.4  Bed form classification diagram of Liu (1957)

Figure 5.2.5  Bed form classification diagram of Simons-Richardson (1966)

Figure 5.2.6  Bed form classification diagram for currents Van den Berg and Van Gelder (1989)
Figure 5.2.7 Bed form classification diagram for currents. Van Rijn (1984, 1989)
5.2.2 Shape and dimensions of bed forms

1. **Flat bed, lower regime**

A flat immobile bed may be observed just before the onset of particle motion, while a flat mobile bed will be present just beyond the onset of motion. The bed surface before the onset of motion may also be covered with relict bed forms generated during stages with larger velocities.

2. **Ribbons and ridges, lower regime**

Small-scale ribbon and ridge type bed forms parallel to the main flow direction have been observed in laboratory flumes and small natural channels, especially in case of fine sediments ($d_{50} < 100 \mu m$) and are probably generated by secondary flow phenomena and near-bed turbulence effects (burst-sweep cycle) in the lower and transition flow regime. These bed forms are also called parting lineations because of the streamwise ridges and hollows with a vertical scale equal to about 10 grain diameters and are mostly found in fine sediments (say 50 to 250 $\mu m$). The width scale is of the order of 100 $v/u_\ast$.

3. **Ripples, lower regime**

When the velocities are somewhat larger (10-20%) than the critical velocity for initiation of motion and the median particle size is smaller than about 500 $\mu m$, small (mini) ripples are generated at the bed surface. Ripples that are developed during this stage remain small with a ripple length much smaller than the water depth ($\lambda_r << h$). Yalin (1985) report values in the range of:

$$\Delta_r = 50 \text{ to } 200 \ d_{50}$$

$$\lambda_r = 500 \text{ to } 1000 \ d_{50}$$

(5.2.1) (5.2.2)

The characteristics of mini ripples are commonly assumed to be related to the turbulence characteristics near the bed (burst-sweep cycle). Current ripples have an asymmetric profile with a relatively steep downstream face (lee side) and a relatively gentle upstream face (stoss...
side). As the velocities near the bed become larger, the ripples become more irregular in shape, height and spacing yielding strongly three-dimensional ripples (Fig. 5.2.9 A). In that case the variance of the ripple length and height becomes rather large. These ripples are called lunate ripples when the ripple front has a concave shape in the current direction (crest is moving slower than wing tips) and are called linguoid ripples when the ripple front has a convex shape (crest is moving faster than wing tips), see Fig. 5.2.2. The largest ripples may have a length up to the water depth and are commonly called mega-ripples. Figure 5.2.10 shows the relative height \( \frac{\Delta_r}{h} \) of mega-ripples observed in Pakistan irrigation channels (Mahmood et al, 1984) as a function of a dimensionless bed-shear stress parameter \( T \). The relative height of the mega-ripples varies in the range of 0.02 to 0.06. A clear influence of the \( T \)-parameter cannot be detected. The length of the mega-ripples varies in the range of 0.5 to 1 h.

Herein it is assumed that the ripples will completely disappear for \( T = 10 \) (see Fig. 5.2.7). Tentative expressions for the relative height and length of mega-ripples are given by:

\[
\frac{\Delta_r}{h} = 0.02 \left(1 - e^{-0.1T}\right) \left(10 - T\right) \quad \text{for} \quad 1 \leq D_* \leq 10 \quad \text{and} \quad 3 \leq T \leq 10
\]

(5.2.3)

\[
\lambda_r = 0.5 \ h
\]

(5.2.4)

Equation (5.2.3) is shown in Figure 5.2.10.

Analysis of bed material samples has shown that coarser particles are concentrated in the trough areas of the ripples. Samples of the trough area show larger median particle diameters and are less sorted than samples from the crest area (Harms, 1969).

*Figure 5.2.9* Bed forms in Pakistan Irrigation Channels, Mahmood et al (1984)
4. Dunes, lower regime

Another typical bed form type of the lower regime is the dune type bed form. Dunes have an asymmetrical (triangular) profile with a rather steep leeside and a gentle stoss side (Fig. 5.2.9C). A general feature of dune type bed forms is leeside flow separation resulting in strong eddy motions downstream of the dune crest.

Experiments were carried out at Delft Hydraulics to get information of the flow field along a dune (Delft Hydraulics, 1988). Artificial dunes with a height of 0.08 m and a length of 1.6 m were made in a flume (width = 1.5 m, length = 50 m). The leeside slope of the dunes was 26°. A layer of sand grains ($d_{50} = 1600 \mu m$, $d_{90} = 1900 \mu m$) was attached to the upperside of the dunes. The average water depth was 0.33 m. The overall mean velocity was 0.51 m/s. The surface slope was 9.5 $10^{-4}$. The overall Froude number was 0.29. A Laser-Doppler velocity meter was used to measure the instantaneous horizontal and vertical velocity.

Figure 5.2.11A shows mean horizontal ($u$) and mean vertical ($w$) velocities at various locations along the dune. A recirculation zone can be observed downstream of the dune crest. Reattachment and acceleration of the flow does occur on the stoss side (luff) side of the dune. The presence of the flow separation zone can also be observed at the water surface in the form of large boils rising up to the surface.

Figure 5.2.11B shows the turbulence intensities. Maximum values can be observed in the recirculation zones.

The length of the dunes is strongly related to the water depth with values in the range of 3 to 15 h. Extremely large dunes with heights of the order of 7 m and lengths of the order of 500 m have been observed in the Rio Parana river at water depths of about 25 m, velocities of about 2 m/s and bed material sizes of about 300 $\mu m$. 

Figure 5.2.10 Relative bed form height, Pakistan irrigation channels
The formation of dunes may be caused by large-scale oscillations as described by Yalin (1972). Due to the presence of large (low frequency) eddies, there will be regions at regular intervals with decreased and increased bed-shear stresses, resulting in the local deposition and erosion of sediment particles.

When the bed material is non-uniform, vertical sorting takes places, which means that the coarser particles accumulate in the dune trough region.

The water temperature was found to have a substantial influence on the dune dimensions (Colby and Scott, 1965). A drop in water temperature of 20°C (summer to winter) at the same discharge resulted in a decrease of the dune height and an increase of the dune length and hence a significant decrease of the friction factor. The sand transport rate increased by a factor 2 from summer to winter.

The most detailed theoretical analysis of dune migration was given by Fredsøe (1980, 1982) based on the continuity equation for the sediment

\[
\frac{\partial z_b}{\partial t} + \frac{1}{(1-p)} \frac{\partial q_t}{\partial x} = 0 \tag{5.2.5}
\]

in which:

- \( z_b \) = bed level
- \( q_t \) = sediment transport rate (volume)
- \( p \) = porosity factor
- \( t \) = time

The migrating velocity (a) of a dune is given by

\[
a = \frac{q_D}{(1-p)\Delta_d} \tag{5.2.6}
\]

in which:

- \( q_D \) = amount of sediment per unit width and time deposited at the dune front
- \( \Delta_d \) = dune height

The magnitude of \( q_D \) depends on the total sediment transport rate at the dune crest (\( q_{\text{crest}} \)) and the relative amount of suspended load (\( q_s \)) and bed load transport (\( q_b \)).

The bed load transport consists of grains which move in almost continuous contact with grains in the bed, and will be deposited at the front where flow separation occurs. The suspended sediment grains move above the bed without contact and will not settle immediately. A suspended grain will contribute to \( q_s \) and the dune migration if it by settling or diffusion moves into the separation zone before it is carried past the separation zone by the flow. Whether the grain is deposited at the front or not will thus depend on its distance from the bed when it passes the dune crest.

The total sediment transport is expressed as a function of the depth-averaged velocity and the depth: \( q_t = f(\bar{u}, h) \).
Figure 5.2.11a  Mean horizontal and vertical velocities along a dune (Delft Hydraulics, 1988)
Figure 5.2.11b  Turbulence intensities along a dune (Delft Hydraulics, 1988)
The local change of the bed level near the crest can be expressed, as follows:

\[
\begin{align*}
\text{Geometry} & : \quad \frac{\partial z_b}{\partial t} = -a \frac{\partial z_b}{\partial x} = - \frac{q_d}{(1-p) \Delta d} \frac{\partial z_b}{\partial x} \quad (5.2.7) \\
\text{Continuity} & : \quad \frac{\partial z_b}{\partial t} = - \frac{1}{(1-p)} \frac{\partial q_t}{\partial x} = - \frac{1}{(1-p)} \frac{\partial q_t}{\partial h} \frac{\partial h}{\partial x} \quad (5.2.8)
\end{align*}
\]

Since \( \partial q_t / \partial x = - \partial h / \partial x \), the dune height \( \Delta d \) can be determined from Eqs. (5.2.7), (5.2.8) yielding:

\[
\Delta d = - \frac{q_d}{\partial q_t} = - \frac{q_d}{\partial q_t + \frac{\partial q_t}{\partial u} \frac{\partial u}{\partial h} + \frac{\partial q_t}{\partial h} - \frac{\partial q_t}{\partial u} \frac{\partial u}{\partial h}} \quad (5.2.9)
\]

The sediment transport attains a maximum at the crest of the dune. Immediately after the crest the flow separates and the dune ends. At situations with dominant bed load transport the sediment transport is in local equilibrium with the hydraulic conditions, and the point of maximum transport coincides with the maximum in bed-shear stress \( \tau_b \). The bed-shear stress has a local maximum at a distance of about 16\( \Delta d \) downstream of the preceding crest. At dominant bed load transport the dune length \( \lambda_d \) is thus given by:

\[
\lambda_d = 16 \Delta d \quad (5.2.10)
\]

Fredsoe included the effect of gravity in his analysis. The gravity on the moving grains will tend to retard the bed-load transport moving up the dune, modifying the transport relation which gives a predicted dune length larger than Eq. (5.2.10). The effect of gravity is important at low transport intensities.

The suspended load transport cannot adjust immediately to changes in the hydraulic conditions, as it takes time for the suspended sand grains to settle out from the concentration profile. The maximum in the suspended load will therefore have a lag distance \( \lambda_s \) behind the maximum in the bed-shear stress. The dune length in case of both suspended and bed-load transport is expressed as:

\[
\lambda_s = 16 \Delta_d + \frac{q_s}{q_t} \lambda_s \quad (5.2.11)
\]

The dune dimensions according to Fredsoe (1982) are given in Fig. 5.2.12.

Based on Figure 5.2.12, the maximum dune length for particle sizes smaller than 1000 \( \mu \)m is approximately 3.7 h, which is rather small compared with measured values.
Based on the analysis of flume and field data, Van Rijn (1982, 1984) proposed the following relationship for the dune height and length:

\[
\frac{\Delta_d}{h} = 0.11 \left( \frac{d_{c0}}{h} \right)^{0.3} (1 - e^{-0.5T}) (25-T) \tag{5.2.12}
\]

\[
\lambda_d = 7.3 \ h \tag{5.2.13}
\]

in which: \( h \) = water depth from water surface to mean bed-level (at half the bed form height, see Fig. 6.2.2).
The dune characteristics according to Eqs. (5.2.12) and (5.2.13) are shown in Figure 5.2.13. The dune height increases weakly with particle diameter for a given water depth. The dunes are assumed to be washed out for \( T \geq 25 \) (see Figure 5.2.7). Equation (5.2.12) yields a maximum dune height in the range of 0.1 to 0.2 h for \( T = 5 \). Figure 5.2.10 shows measured and computed dune heights for some Pakistan Irrigation channels (Malmood et al., 1984). The generation of mini or mega-ripples on the stoss sides of the dunes is also a typical phenomenon of the lower flow regime. The influence of the water temperature (viscosity) on the dune dimensions was not found by Van Rijn.

Other relationships for dune dimensions were proposed by:

\[
\frac{\Delta_d}{h}, \frac{\lambda_d}{h} = F(\theta')\quad (5.2.14)
\]

\[
\frac{\Delta_d}{h} = \frac{(1-Fr^2)}{2n \alpha} \left(1 - \frac{\tau_{b,c}}{\tau_{b,c}}\right)\quad (5.2.15)
\]

\[
\frac{\Delta_d}{h} = \frac{1}{6} \left(1 - \frac{\tau_{b,c}}{\tau_{b,c}}\right)\quad (5.2.16)
\]

\[
\frac{\lambda_d}{h} = 6.3\quad (5.2.17)
\]

\[
\frac{\Delta_d}{d_{50}} = F(\theta', Fr)\quad (5.2.18)
\]

\[
\frac{\lambda_d}{d_{50}} = F(\theta', d_{50}, Fr, h)\quad (5.2.19)
\]

\[
\frac{\Delta_d}{h} = 0.086 h^{0.10}\quad (5.2.20)
\]

\[
\frac{\lambda_d}{h} = h^{0.6}\quad (5.2.21)
\]

\[
\frac{\Delta_d}{h}, \frac{\lambda_d}{h} = F(\theta'_c, d_{50})\quad (5.2.22)
\]

in which:

- \( \Delta_d \) = dune height
- \( \lambda_d \) = dune length
- \( \theta' \) = particle mobility parameter related to grains
- \( \tau_{b,c} \) = grain-related bed-shear stress for currents
- \( h \) = water depth
- \( \tau_{b,c} \) = critical bed-shear stress (Shields)
- \( Fr \) = Froude number
- \( d_{50} \) = median particle diameter
- \( n \) = power coefficient of velocity in sediment transport formula
- \( n = 3 \) to 6
- \( \alpha \) = dune shape coefficient (\( \alpha = 0.5 \) to 0.7)

The results of Tsubaki-Shinohara (1959), Ranga Raju-Soni (1976) and Fredsøe (1980, 1982) were presented in graphical form.
### Table

<table>
<thead>
<tr>
<th>Source</th>
<th>Flow Velocity $\bar{u}$ (m/s)</th>
<th>Flow Depth $h$ (m)</th>
<th>Particle Size $d_{50}$ ($\mu$m)</th>
<th>Temperature (°C)</th>
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<td>Guy et al</td>
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<td>0.16 - 0.22</td>
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<td>0.11 - 0.21</td>
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### Figure 5.2.13

Dune dimensions according to Van Rijn (1984)
Figure 5.2.14 Comparison of computed dune heights
Several dune height predictors were used by Van Rijn to compute the dune height for a sand bed with a median particle diameter of \( d_{50} = 600 \ \mu m \) \( (d_{50} = 1500 \ \mu m) \), mean water depths of \( h = 1 \) and 10 m and mean flow velocities of \( \dot{u} = 0.5, 1.0 \) and 1.5 m/s.

The results are shown in Figure 5.2.14. As can be observed, the methods of Ranga Raju-Soni (1976), Tsubaki-Shinohara (1959) and Fredsøe (1980) show an increasing trend, the latter two yielding extremely large values. The method of Yalin produces values which approach to \( \Delta_h = 1/6 \ h \). The equation of Van Rijn (1984) and Fredsøe (1982) predict a decreasing bed-form height for increasing velocities (washed-out dunes). For a small depth \( (h = 1 \ m) \) the methods of Van Rijn and Fredsøe (1982) show good agreement, while for a large depth \( (h = 10 \ m) \) the method of Fredsøe yields values which are about twice as large as according to the method of Van Rijn. The method of Ranga Raju-Soni yields remarkably small bed-form heights in the case of large flow depths, probably because only flume data were used for calibration.

5. Bars, lower regime

The largest bed forms in the lower regime are sand bars (such as alternate bars, side bars, braid bars and transverse bars), which usually are generated in areas with relatively large transverse flow components (bends, confluences, expansions). Alternate bars are features with their crests near alternate banks of the river, see Fig. 5.2.15. Braid bars actually are alluvial "islands" which separate the anabranches of braided streams. Numerous bars can be observed distributed over the cross sections. These bars have a marked streamwise elongation. Transverse bars are diagonal shoals of triangular-shaped plan along the bed. One side may be attached to the channel bank. These type of bars generally are generated in steep slope channels with a large width-depth ratio. The flow over transverse bars is sinuous (wavy) in plan. Side bars are bars connected to river banks in a meandering channel. There is no flow over the bar. The planform is roughly triangular. Special examples of side bars are point bars and scroll bars.

6. Washed-out dunes and sand waves, transition regime

It is a well-known phenomenon that the bed forms generated at low velocities are washed out at high velocities. It is not clear, however, whether the disappearance of the bed forms is accomplished by a decrease of the bed form height, by an increase of the bed form length or both. Flume experiments (Termes, 1986) with sediment material of about 450 \( \mu m \) show that the transition from the lower to the upper regime is effectuated by an increase of the bed form length and a simultaneous decrease of the bed form height, as shown in Figures 5.2.16 and 5.2.17. Ultimately, relatively long and smooth sand waves with a roughness equal to the grain roughness were generated (see Figure 5.2.17).

In the transition regime the sediment particles will be transported mainly in suspension \( (u_\ast/w_s \geq 5) \). This will have a strong effect on the bed form shape. The bed forms will become more symmetrical with relatively gentle lee-side slopes. Flow separation will occur less frequently and the effective bed roughness will approach to that of a plane bed. Based on Figure 5.2.7, the transition regime will occur for \( T \geq 15 \). Julien (1992) has studied bed form generation in large rivers. Large-scale bed forms with a relative height \( (\Delta/h) \) of 0.1 to 0.2 and a relative length \( (\lambda/h) \) of 5 to 15 were present in the Mississippi river at \( T \)-values upto 50 (Froude numbers smaller than 0.4). The shape of the bed forms was not reported. Probably, more or less symmetrical sand waves were generated because the effective bed roughness was rather small at high velocities (see Chapter 6).

According to the author, the bed forms in the transition regime which will most likely occur, are washed-out dunes and (symmetrical) sand waves. Both types may occur simultaneously.
Figure 5.2.15 Alternate bars in a river

$T = 9.3$

$T = 11.2$

$T = 15.3$

$T = 24.9$

---

Figure 5.2.16 Bed form development in the lower and transition regime

($T = 9.3$ to $24.9$; $d_{50} = 450 \mu m$; maximum bed form height = $0.15$ m; water depth = $0.3$ to $0.35$ m; Termes, 1986)
Figure 5.2.17 Bed form development in transition regime

Figure 5.2.18 Bed forms in transition and upper regime for non-uniform sediments (Chiew, 1991)
The dimensions of the washed-out dunes are described by Eqs. (5.2.12) and (5.2.13); these bed forms will fully disappear for \( T = 25 \). Hence \( \Delta_a = 0 \) for \( T = 25 \).

The dimensions of the (symmetrical) sand waves are described by:

\[
\frac{\Delta_{s,w}}{h} = 0.15 \left( 1 - e^{-\frac{\Delta(T-15)}{15}} \right) (1 - Fr^2) \quad \text{for } T \geq 15
\]  
(5.2.23)

\[
\lambda_{s,w} = 10 \, h \quad \text{for } T \geq 15
\]  
(5.2.24)

7. Plane bed and sand waves, upper regime

Two sub regimes can be distinguished in the upper transport regime:

- subcritical upper transport regime : \( T \geq 25 \) and \( Fr < 0.8 \)
- supercritical upper transport regime : \( T \geq 25 \) and \( Fr \geq 0.8 \)

In the subcritical upper regime the bed forms will be symmetrical sand waves with dimensions according to Eqs. (5.2.23) and (5.2.24).

In the supercritical upper regime the bed form types will be plane bed and/or anti-dunes. The latter type of bed forms are sand waves with a nearly symmetrical shape in phase with the water surface waves.

The anti-dunes do not exist as a continuous train of bed waves, but they gradually build up locally from a flat bed. Anti-dunes move upstream due to strong lee-side erosion and stoss-side deposition (Figure 5.2.3). Anti-dunes are bed forms with a length scale of about 10 times the water depth (\( \lambda = 10 \, h \)).

When the flow velocity further increases, finally a stage with chute and pools may be generated (Figure 5.2.1).

Winterwerp et al (1992) did experiments in a tilting flume with sand of 120 \( \mu \text{m} \) and observed violent sub and super-critical flow over asymmetrical sand dunes with lengths between 2 and 5 m and heights between 0.1 and 0.2 m. The length was related to the sand transport rate \( (\lambda = 2 \text{ to } 3 \, m \text{ for } \bar{c} = 0.1 \text{ to } 0.2 \text{ and } q_c = 5 \text{ to } 20 \, kg/m \text{m} \text{h}) \). Just upstream and downstream of the dune crest the flow was super-critical with velocities between 1 and 2 m/s resulting in heavy erosion. Just downstream of the trough a hydraulic jump was generated giving a transition to a region with sub-critical flow where heavy deposition was observed (velocities from 0.5 to 1 m/s, water depth = 0.1 m). Downstream of this region the flow was accelerating again towards the crest region ending in super-critical flow.

This regime can be observed near pipeline outlets of sand-water mixture flows during closure of channels.

Most data are confined to nearly uniform sediment materials. The sediment transport process and hence the bed form development in non-uniform materials are different from those in uniform materials. Selective sediment transport may occur in non-uniform materials at low velocities resulting in the formation of an armoured layer. Chiew (1991) studied the influence of the geometric standard deviation of the bed material \( (\sigma_g = d_{54}/d_{90}) \) on the type of bed forms in the transition and upper regime. In all experiments the median particle size was 600 \( \mu \text{m} \). The \( \sigma_g \)-value was varied from 1.2 to 5.5. Figure 5.2.18 shows the observed bed form types as a function of the Froude number and the geometric standard deviation. For uniform material the dunes were fully washed-out at \( Fr = 0.8 \) and anti-dunes were generated.
at Fr = 1. For non-uniform sediments dune-type bed forms were present upto Fr = 1 and anti-dunes did not form at Froude numbers upto 1.5 and \( \sigma_g > 2 \).

### 5.2.3 Examples and problems

1. A wide open channel has a mean water depth \( h = 3 \) m, a mean bed slope \( l = 1.5 \times 10^{-4} \), a mean velocity \( \bar{u} = 1 \text{ m/s} \), the bed material characteristics are \( d_{x0} = 350 \mu \text{m} \), \( d_{x0} = 1000 \mu \text{m} \), sediment density \( \rho_s = 2650 \text{ kg/m}^3 \), fluid density \( \rho = 1000 \text{ kg/m}^3 \), fluid temperature \( T_e = 20^\circ \text{C} \) (\( \nu = 1.01 \times 10^{-6} \text{ m}^2/\text{s} \)).

What type of bed forms are generated?
What are the dimensions of the bed forms?

#### Bed form type

Liu (1957) : \[ \tau_{b,c} = \rho gh l = 4.4 \text{ N/M}^2 \]
\[ u_{s,c} = (ghl)^{0.5} = 0.066 \text{ m/s} \]
\[ w_s = 0.048 \text{ m/s} \text{ (Fig. 3.2.6)} \]
\[ u_{s,c} / w_s = 1.4 \]
\[ u_{s,c} d_{50} / \nu = 23 \]
→ plane bed

Simons-Richardson (1966) : \[ \tau_{b,c} \bar{u} = 4.4 \text{ N/sm} \]
\[ d_i = 0.35 \text{ mm} \]
→ dunes

Van Rijn : \[ C' = 18 \log(12h/3d_{x0}) = 73.4 \text{ m}^{1/3}/\text{s} \]
\[ \tau_{b,c}' = \rho g(\bar{u}/C')^2 = 1.82 \text{ N/m}^2 \]
\[ \tau_{b,cr} = 0.21 \text{ N/m}^2 \text{ (Fig. 4.1.5)} \]
\[ T = (\tau_{b,c}' - \tau_{b,cr}) / \tau_{b,cr} = 7.7 \]
\[ D_\alpha = ((s-1)g/\nu^2)^{1/3} d_{50} = 8.8 \]
→ mega-ripples and dunes

#### Bedform dimensions

Gill (1971) : Taking \( n = 4 \), \( \alpha = 0.62 \)
\[ Fr = \bar{u} / gh^{0.5} = 0.184 \]
\[ \Delta_d = \left(1 - \frac{Fr^2}{5}\right) \left(1 - \frac{\tau_{b,cr}}{\tau_{b,c}}\right) h = 0.55 \text{ m} \]
Yalin (1972) : \[ \Delta_d = \frac{1}{6} \left( 1 - \frac{\tau_{bc}}{\tau_{bc}} \right) h = 0.48 \text{ m} \]

Allen (1968) : \[ \Delta_d = 0.086 \ h^{1.19} = 0.32 \text{ m} \]
\[ \lambda_d = h^{1.6} = 5.8 \text{ m} \]

Van Rijn : \[ \Delta_{mr} = 0.02 \ h (1 - e^{-0.1T}) (10 - T) = 0.074 \text{ m} \]
\[ \lambda_{mr} = 0.5 \ h = 1.5 \text{ m} \]
\[ \Delta_d = 0.11 \ h \left( \frac{d_{50}}{h} \right)^{0.3} (1 - e^{-0.5T}) (25 - T) = 0.37 \text{ m} \]
\[ \lambda_d = 7.3 \ h = 21.9 \text{ m} \]

Fredsøe (1982) : estimate \[ \Delta_d = 0.1 \ h = 0.3 \text{ m} \]
\[ h_{\text{crest}} = 3 - 0.15 = 2.85 \text{ m} \]
\[ C'_{\text{crest}} = 18 \ \log(12h_{\text{crest}}/3d_{50}) = 73 \text{ m}^{1/2}/s \]
\[ u'_{\text{crest}} = (3/2.85)\bar{u} = 1.05 \text{ m/s} \]
\[ u'_{*,\text{crest}} = (g^{0.5}/C') \bar{u}_{\text{crest}} = 0.045 \text{ m/s} \]
\[ \theta'_{\text{crest}} = (u'_{*,\text{crest}})^2/(s-1)g d_{50} = 0.36 \]
\[ \Delta_d = 0.13 \ h = 0.39 \text{ m (Fig. 5.2.12)} \]
\[ \Delta_d/\lambda_d = 0.052 \text{ (Fig. 5.2.12)} \]
\[ \lambda_d = 7.5 \text{ m} \]

2. A wide open channel width \( h = 3 \text{ m} \), \( \bar{u} = 1.5 \text{ m/s} \), \( I = 3 \cdot 10^{-4} \). Other data as given in example 1.

What are the type of bed forms and the dimensions of the bed forms according to the method of Van Rijn?

**Bed form type**

\[ C' = 73.4 \text{ m}^{1/2}/s \]
\[ \tau'_{b,\omega} = \rho g \bar{u}/C'h^2 = 4.1 \text{ N/m}^2 \]
\[ \tau'_{b,cr} = 0.21 \text{ N/m}^2 \]
\[ T = (\tau'_{b,\omega} - \tau'_{b,cr})/\tau_{b,cr} = 18.5 \]
\[ D_s = 8.8, \quad Fr = 0.28 \]

→ washed-out dunes and symmetrical sand waves in the transition regime
Bedform dimensions : \( \Delta_d = 0.11 \ h \left( \frac{d_{so}}{h} \right)^{0.3} (1-e^{-0.5T}) (25-T) = 0.14 \ m \)
\( \lambda_d = 7.3 \ h = 21.9 \ m \)
\( \Delta_{sw} = 0.15 \ h (1-Fr^2) (1-e^{-0.5(T-15)}) = 0.34 \ m \)
\( \lambda_{sw} = 10 \ h = 30 \ m \)

Both types may occur simultaneously.

3. A wide open channel with \( h = 3 \ m, \bar{u} = 2 \ m/s, I_b = 4.10^4; \) other data as given in example 1.

What are the type of bed forms and the bed form dimensions according to the method of Van Rijn?

Bed form type : symmetrical sand waves in the (sub-critical) upper transport regime.

Bedform dimensions : \( \Delta_{sw} = 0.39 \ m \)
\( \lambda_{sw} = 30 \ m \)

4. A wide open channel has a water depth of \( h = 2 \ m, \) mean current velocity \( \bar{u} = 0.6 \ m/s, \) the Chézy-coefficient is \( C = 63 \ m^{1/2}/s. \) The bed material characteristics are \( d_{so} = 150 \ \mu m, d_{xo} = 300 \ \mu m. \)

Other data are: \( \rho = 1000 \ \text{kg/m}^3, \rho_s = 2650 \ \text{kg/m}^3, \nu = 1 \ 10^{-6} \ m^2/s. \)

What type of bed forms can be expected according to the methods of Liu, Simons-Richardson and Van Rijn?

What are the dimensions of the ripples?

Solution:

Liu : dunes (close to ripples)
Simons-Richardson : ripples
Van Rijn : mini-ripples (close to dunes)
Ripple dimensions : \( \Delta_r = 0.0075 \ to \ 0.03 \ m \)
\( \lambda_r = 0.075 \ to \ 0.15 \ m. \)

5.3 Bed forms in non-steady currents

5.3.1 Non-steady river flow

In river flow the discharge and the water level stage vary as a function of time (rising and falling stages) depending on seasonal and climatological conditions. Small-scale bed forms like ripples respond rapidly to a new situation but large-scale bed forms like dunes have a less rapid response and there may be a considerable phase lag between the establishment of the new flow conditions at time \( t_1 \) and the establishment of the new dune dimensions at time \( t_2. \)

The first laboratory experiments with respect to the non-steady behaviour of bed forms were carried out by Simons and Richardson (1962).

Allen (1976) developed a stochastic model describing the creation and destruction of the dunes. After a dune has traveled a certain assigned distance it is destroyed and a new dune is created. At the moment of creation the dune dimensions correspond to the prevailing flow conditions assuming steady flow. During the life of the dunes the dune height can adjust to changes in the flow conditions, but the dune length remains constant.

Gee (1973) and Goodwin (1986) performed flume experiments to study the non-steady effects of dune transformation.

Fredsøe (1979), (1982) proposed analytical relationships for the initial change of the dune height which is based on flow and sediment transport parameters at the dune crest. The dune shape is essentially constant.

Wijbenga and Klasseen (1981) also performed flume experiments to study the changes of dune dimensions for unsteady flow conditions (sudden increase and decrease of discharge and depth). Fig. 5.3.1 shows changes in Chézy-coefficient, dune height and dune length for a sudden change in water depth. Comparison of the experimental results and the model results of Allen and Fredsøe did not give satisfactory agreement.

Tsujimoto and Nakagawa (1983) performed flume experiments and proposed semi-empirical relationships for the change in time of the dune height and the dune length. Various cases were studied: dune development from an initially flat bed under a constant discharge, dune transition under a suddenly decreasing discharge and dune transition under a gradually varying discharge.

Iseya (1984) performed flume experiments related to the development of dunes from an initially flat bed. The bed material size was 570 μm. Flume runs with varying and constant discharges were carried out. Figure 5.3.2 shows the variation of the dune characteristics for a gradual change of the discharge. During the fall of the discharge curve the dune length was still increasing, because the run was started from a flat bed. Thus, at the start of the experiment, the bed form dimensions were not in equilibrium with the prevailing flow conditions.

Fournier (1984) also performed flume experiments and proposed expressions for the dune height change and the transition time scale. According to Fournier, the dune length first adjusts itself in a relatively short period to the new flow conditions, while the dune height remains essentially constant. After this period ("coalescence" time) the growth in dune height becomes significant.

Levey et al (1980) measured bed level changes in a bend of the Congaree river during extreme and average discharges. The bed material was in the range of 250 to 500 μm. The height of the bed forms (with wave lengths from 5 to 30 m) were largest during maximum discharge. The adjustment period to return to a lower bed form height at an average discharge was about six days.

According to Van Rijn (1989), the transition period $T_d$, during which the dune dimensions change from those of stage 1 to those of stage 2, is related to the ratio of the change in cross-sectional area of the dune and the average bed-load transport in the transition period.
Figure 5.3.1 Variation of Chézy coefficient, dune height and dune length for a sudden change in water depth (Wijbenga and Klaassen, 1983)
upper: water depth changes from 0.2 to 0.4 m
lower: water depth changes from 0.4 to 0.2 m

5.26
Figure 5.3.2  Variation of dune characteristics for a gradual change of discharge (Iseya, 1984)

Thus,

\[ T_d = \frac{\alpha (1-p) (\Delta_2 \lambda_2 - \Delta_1 \lambda_1)}{1/2(q_{b,1} + q_{b,2})} \quad (5.3.1) \]

Assuming a first order adjustment process, the changes in dune height and dune length can be expressed as:

\[ \frac{\Delta_2 - \Delta_1}{\Delta_2 - \Delta_1} = e^{-\beta t / T_d} \quad (5.3.2) \]

\[ \frac{\lambda_2 - \lambda_1}{\lambda_2 - \lambda_1} = e^{-\gamma t / T_d} \quad (5.3.3) \]
in which:

\[ \Delta \mu, \lambda_t = \text{dune height, length at time t} \]
\[ \Delta_{\lambda}, \lambda_1 = \text{equilibrium dune height, length at stage 1} \]
\[ \Delta_{\lambda_2} = \text{equilibrium dune height, length at stage 2} \]
\[ q_{b,1}, q_{b,2} = \text{equilibrium bed load transport rates at stage 1, 2} \]
\[ p = \text{porosity of bed material} \]
\[ T_d = \text{dune transition period} \]
\[ \alpha, \beta, \gamma = \text{coefficients} \]

The experimental results (T25, T26) of Wijbenga-Klaassen (1981) were used to determine the \( \alpha, \beta \) and \( \gamma \) - coefficients. The tests were performed in a large scale flume (length = 100 m, width = 1.5 m, depth = 1 m) with bed material of 770 \( \mu \)m. The dune height and length were measured as a function of time after a sudden increase of the discharge. The water depth was changed from \( h = 0.2 \) m to \( h = 0.4 \) m.

Applying the measured values, Van Rijn (1989) found that \( \alpha = 4, \beta = 3 \) and \( \gamma = 1 \). A value of \( \beta = 1 \) was also found by Fournier (1984).

The above given expressions can be applied to obtain an order of magnitude estimate of the dune transition parameters in unsteady river flow. The equilibrium dune dimensions are given by Eqs. (5.2.12) and (5.2.13).

Finally, the detailed literature review of Wijbenga (1990) is mentioned. Based on his work, it can be concluded that:

- the time lag is larger for larger bed forms,
- the time lag of the bed form length is larger than that of the bed form height,
- the time lag depends on the variation of the discharge with time (shape of the hydrograph),
- the time lag is relatively small during increasing discharge (rising stage) and relatively large during decreasing discharge (falling stage),
- the time lag is most importantly related to variations in flow velocity; variations in grain size and water depth are less important,
- the time scale of bed form changes can be represented by the change of the bed form volume divided by the sediment transport rate.

### 5.3.2 Tidal flow


#### 1. Germany

A very detailed study of bed form behaviour in the tidal Elbe and Weser rivers in Germany was made by Nasner (1974, 1976, 1978, 1983). Echo soundings of the bed were carried out at four locations in the lower Weser river, at one location in the lower Elbe river (near Hamburg) and at one location in the outer Elbe river (near Scharhörn). The characteristic data are presented in Table 5.2.

Most of the bed forms in the Elbe and Weser river are asymmetrical and are herein classified as sand dunes (see also Fig. 5.3.3). The length of the sand dunes shows much less variation than the height of the sand dunes (see Table 5.2). The height in the lower Weser river and the lower Elbe river was found to be related to the fresh water discharge. Maximum sand dune heights (about 2.4 m) were observed at high river discharges (1200 \( \text{m}^3/\text{s} \)), see Fig. 5.3.4. The extreme height and length of the sand dunes were found to be equal to approximately twice the mean values. The observed net migration rates (m/day) were:

<table>
<thead>
<tr>
<th>Weser</th>
<th>0.1 - 0.3 m/day in ebb direction at low river discharge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0 - 1.5 m/day in ebb direction at high river discharge</td>
</tr>
</tbody>
</table>

5.28
Elbe (Hamburg) : 0.05 - 0.1 m/day in flood direction at low river discharge
          0.1 - 0.3 m/day in ebb direction at high river discharge

Elbe (Scharhörn) : 0.05 - 0.3 m/day in flood direction.

Analysis of velocity measurements above the crests and the troughs of the bed forms showed a depth-averaged velocity above the crest of about 1 m/s during all conditions (independent of the river discharge). Thus, the sand dunes are eroded at high discharges until the depth-averaged velocity above the crest is about 1 m/s. The sand dune heights increase at low discharge until the depth-averaged velocity above the crest is again about 1 m/s.

The effect of dredging on the regeneration of the bed forms was studied in the Elbe river near Hamburg (Nasner, 1983). The vertical irregularity of the bed after dredging was about 0.5 m. After about one week the original sand dune dimensions (height = 1.2 m, length = 25 m) were observed.

Stehr (1975) also observed asymmetrical sand dunes with a length of about 45 m, a height of about 3 m in the Elbe river. The water depths were about 15 m. Particle sizes were in the range of 300 to 500 μm.

<table>
<thead>
<tr>
<th>Location</th>
<th>Particle sizes</th>
<th>Peak flood velocity (m/s)</th>
<th>Peak ebb velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d_{50} (μm)</td>
<td>d_{90} (μm)</td>
<td>Low river discharge</td>
</tr>
<tr>
<td>Weser 1</td>
<td>480 1020</td>
<td>0.8-1.0</td>
<td>0.6-0.8</td>
</tr>
<tr>
<td>Weser 2</td>
<td>490 1080</td>
<td>0.8-1.0</td>
<td>0.6-0.8</td>
</tr>
<tr>
<td>Weser 3</td>
<td>350 500</td>
<td>0.7-0.9</td>
<td>0.5-0.7</td>
</tr>
<tr>
<td>Weser 4</td>
<td>450 760</td>
<td>0.7-0.9</td>
<td>0.5-0.7</td>
</tr>
<tr>
<td>Elbe (near Hamburg)</td>
<td>420 810</td>
<td>1.0-1.2</td>
<td>0.8-0.9</td>
</tr>
<tr>
<td>Elbe (near Scharhörn)</td>
<td>320 620</td>
<td>0.7 - 1.1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>Mean depth h (m)</th>
<th>Mean bed form height (m)</th>
<th>Mean bed form length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>minimum</td>
<td>maximum</td>
<td>minimum</td>
</tr>
<tr>
<td>Weser 1</td>
<td>11.1</td>
<td>1.1</td>
<td>2.6</td>
</tr>
<tr>
<td>Weser 2</td>
<td>11.1</td>
<td>1.2</td>
<td>2.8</td>
</tr>
<tr>
<td>Weser 3</td>
<td>10.9</td>
<td>0.7</td>
<td>2.0</td>
</tr>
<tr>
<td>Weser 4</td>
<td>11.6</td>
<td>1.3</td>
<td>2.3</td>
</tr>
<tr>
<td>Elbe (near Hamburg)</td>
<td>14.4</td>
<td>0.8</td>
<td>1.2</td>
</tr>
<tr>
<td>Elbe (near Scharhörn)</td>
<td>22</td>
<td>2.8</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Table 5.2 Bed form dimensions of Weser and Elbe river, Nasner (1974)
2. The Netherlands

Terwind (1970a) studied bed forms in tidal channels and in tidal fluvial channels (lower Rhine river) in The Netherlands. The bed material sizes were in the range of 200 to 500 μm. Mega-ripples, sand dunes and sand waves were observed in sandy areas; no bed forms were observed in areas with a mud content larger than 15%. The bed form dimensions varied in the range:

Tidal channels : $\Delta = 0.3$ to $1.5$ m, $\lambda = 7$ to $40$ m
Fluvial tidal channels : $\Delta = 0.3$ to $1$ m, $\lambda = 7$ to $20$ m
The bed form height showed the largest variability (0.5 m per day). The height of the bed forms in the tidal fluvial channels was related to the local water depth; the relative height was in the range $\Delta h / h = 0.1$ to 0.15. Bed forms with a height smaller than 1 m showed asymmetry changes with the turn of the tidal currents. Bed forms with a height larger than 1 m did not show this behaviour.

Bucx and Tobias (1986) have analyzed a series of echo soundings carried out over a length of 20 km in the eastern (landward) part of the Westerschelde estuary in the Netherlands. The total track length of the bed forms in the various ebb and flood channels was about 100 km. The water depths in that part of the estuary were in the range of 7 to 25 m. The tidal range was 4 to 5 m. The current velocities were in the range of 1 to 1.5 m/s. The ebb currents were slightly dominating in most channels. Vertical salinity stratification did not occur. The size of the bed material was in the range of 200 to 300 $\mu$m. Analysis of the echo sounding tracks shows the presence of mega-ripples in most channels. The heights of the mega-ripples were in the range of 0.2 to 1 m with a mean value of about 0.6 m. The lengths were mostly in the range of 1 to 20 m with a mean value of about 10 m. About 20% of the bed forms had a length larger than 20 m. Large asymmetrical sand dunes with a length of 100 m and a height of 2 m were present in some deep channels (depth $= 25$ m). Most (80%) of the mega-ripples were slightly asymmetrical in the direction of the dominating current. About 10% of the mega-ripples had a symmetrical shape and about 10% of the bed forms were more or less plane (steep slope areas, dredging areas, muddy areas).

Voogt et al (1991) observed asymmetrical mega-ripples with a height of about 0.5 m and a length of about 10 m in water depths of about 10 m, bed material sizes of about 250 $\mu$m and mean velocities up to 1.75 m/s. These asymmetrical mega-ripples change into symmetrical sand waves with a reduced height of about 0.4 m and an increased length of 30 m for increasing velocities up to 2.5 m/s. The tidal period was too short to generate a fully plane bed even at the highest velocity of 2.5 m/s.

3. USA

Salsman et al (1966) observed mega-ripples with a height of about 0.5 m and a length of about 20 m in a tidal bay (St. Andrews Bay, Florida) with depths of about 10 m. The median bed material size was about 135 $\mu$m. The tidal velocities were in the range of 0.2 to 0.4 m/s. Net migration velocities of 0.01 to 0.1 m/day were observed.

Summarizing, the bed forms most frequently found in tidal currents in estuaries are mega-ripples, asymmetrical sand dunes and symmetrical sand waves. Tidal currents may have an asymmetrical variation depending on the channel and shoal system. Mega-ripples have a height of the order of 0.5 to 1 m and a length of the order of the water depth (5 to 20 m). Generally, they are generated in the main flood and ebb channels with bed material sizes smaller than 300 $\mu$m. The lee slope of the mega-ripples is approximately equal to the angle of repose (avalanche angle). The shape is typically asymmetrical in the direction of the main current. Thus, reorientation takes place with the reversal of the tide. The sand dunes and sand waves have lengths of several times the water depths. These lengths, however, are smaller than those of the sand dunes in steady river flow because the tidal period is not large enough for the generation of equilibrium values. The largest bed forms do occur in areas where the river discharge is dominating the tidal discharge. Generally, the sand dunes and sand waves are found in the deeper wider tidal channels with bed material sizes larger than about 300 $\mu$m. Sand dunes are typically asymmetrical in the direction of the dominating current velocity. Sand waves are typically symmetrical with their crests approximately midway between the troughs. The lee side angles are much smaller than the angle of repose and seldom exceed 10°. This means that flow separation will not occur yielding small effective roughness values.
Sand waves are typically found in areas with symmetrical tidal currents generating an equal amount of sediments transported backward and forward over the crest on each phase of the tide.

The dimensions of the tidal bed forms in relation to water depths (h) are, as follows:

\[
\begin{align*}
\text{mega-ripples} & : \Delta = 0.05 \text{ to } 0.1 \text{ h and } \lambda = 0.5 \text{ to } 2 \text{ h} \\
\text{in shallow channels} & \quad (d_{s0} \leq 300 \ \mu\text{m})
\end{align*}
\]

\[
\begin{align*}
\text{sand dunes, sand waves} & : \Delta = 0.1 \text{ to } 0.2 \text{ h and } \lambda = 3 \text{ to } 6 \text{ h} \\
\text{in deeper channels} & \quad (d_{s0} = 300 \text{ to } 600 \ \mu\text{m})
\end{align*}
\]

### 5.3.3 Examples and problems

1. A wide open channel has a water depth \( h_1 = 3 \) m, mean current velocity \( \bar{u}_1 = 1 \) m/s. The bed material characteristics are \( d_{s0} = 600 \ \mu\text{m}, d_{90} = 1000 \ \mu\text{m}, p = \text{porosity} = 0.4 \). The bed is covered with dunes. The bed load transport is \( q_{b,1} = 0.1 \) kg/sm.

   A flood wave is passing, giving a new hydraulic situation with \( h_2 = 4.5 \) m, \( \bar{u}_2 = 1.5 \) m/s and \( q_{b,2} = 0.3 \) kg/sm. Other data are: \( \rho = 1000 \) kg/m\(^3\), \( \rho_s = 2650 \) kg/m\(^3\), \( v = 1 \times 10^{-6} \) m\(^2\)/s.

   What is the dune transition period \( T_d \)?
   What is the dune length after 25 hours during the new situation?

   \[
   \begin{align*}
   \text{Solution: } T_d &= 40 \text{ hours} \\
   \lambda_d &= 28 \text{ m after 25 hours.}
   \end{align*}
   \]

2. A tidal channel in an estuary has a depth of 5 m near the landward boundary and a depth of 15 m near the seaward boundary. The maximum current velocity is 1.2 m/s. The tidal range is 4 m. The bed material characteristics are \( d_{s0} = 350 \ \mu\text{m}, d_{90} = 600 \ \mu\text{m} \).

   What types of bed forms may exist and what are the bed form dimensions?

   \[
   \begin{align*}
   \text{Solution: Mega-ripples near landward boundary} & : \Delta_{nr} = 0.25 \text{ to } 0.5 \text{ m,} \\
   \lambda_{nr} &= 2.5 \text{ to } 10 \text{ m} \\
   \text{Sand dunes and waves near seaward boundary} & : \Delta_{sw} = 1.5 \text{ to } 3 \text{ m,} \\
   \lambda_{sw} &= 45 \text{ to } 90 \text{ m}
   \end{align*}
   \]

### 5.4 Bed forms in waves

#### 5.4.1 Classification

Two typical regimes can be observed in nature:
- lower regime with flat immobile bed, ripples and bars,
- upper regime with flat mobile bed (sheet flow).

A typical transition regime does not occur.

Figure 5.4.1 shows a classification diagram given by Allen (1982) which is based on 648 data sets.
Ripples occur where the near-bed peak orbital velocity is about 1.2 times the critical velocity for initiation of motion.

Ripples are washed out when the parameter \( \psi = \frac{\left( \bar{U}_b \right)^2}{(s - 1)g d_{50}} \) is larger than about 200 to 250 (Dingler-Inman, 1976; Horikawa et al, 1982). This regime is referred to as the sheet flow regime.

According to Wilson (1989), the sheet flow regime is present for

\[ 0 - \frac{\hat{e}_{b,w}}{(\rho_s - \rho)g d_{50}} \geq 0.8. \]

Thus,

- Ripple regime for \( \psi < 250 \) or \( 0 < 0.8 \)
- Sheet flow regime for \( \psi \geq 250 \) or \( 0 \geq 0.8 \)

Surf zone bars may be generated near the breaker line and typically reflect transport processes related to breaking waves.

### 5.4.2 Shape and dimensions of bed forms

#### 1. Ripples

Wave ripples are formed once the oscillatory motion is of sufficient strength to cause general movement of the surface particles. The height and length of the ripples grow until a stable ripple is obtained depending on the prevailing conditions. Wave-generated ripples show an almost symmetrical and rounded profile. Wave ripples occur in a wide range of environments: flooded overbank areas of rivers, flood plains, lake bottoms, intertidal flats, deep-sea bottom, shelf bottoms and nearshore sea bottoms. Wave ripples have been observed in depths up to 200 m. In deep water wave ripples are probably formed by internal density currents rather than by surface waves.

Bagnold (1946) defined two types of ripples: two-dimensional ripples related to rolling grains and three-dimensional ripples related to eddy motions. According to Bagnold, the rolling grain ripples are stable at velocities smaller than two times that of initiation of motion. Rolling grain ripples have a small height to length ratio. They are generated during oscillatory flow at low Reynolds number. This latter type of motion over an initially wavy bed results in the formation of steady recirculating cells. Close to the bed the flow is from the trough to the crest. The sediment particles will tend to be transported to the crests and the ripples tend to grow. As the ripples get steeper, the opposing effect of the gravity force component giving a downward motion will be more and more important. When fully developed, rolling grain ripples are generally two-dimensional, regular and have a sinusoidal shape. At larger velocities the flow is separated from the ripples and strong lee-side eddies are generated which can sweep the particles from the troughs to the crests and vice versa. Separation starts when the orbital diameter exceeds the ripple length. At high velocities the sediment particles are carried away from the bed (suspension). At increasing velocities the ripples will eventually be washed-out (sheet flow regime).

The time scale of ripple growth from flat bed to equilibrium values can vary from about 1 hour in case of relatively small waves (Davies, 1985) to about 1 minute in case of relatively large waves (Dingler, 1975). In deeper water where the wave action generally is too weak to move the sand particles, the bed may consist of relict ripples formed under earlier more intense wave action.
Figure 5.4.1 Bed form classification diagram for waves, Allen (1982)
A conceptual model for wave-formed sedimentary structures was given by Clifton (1976). For waves of uniform height and period propagating normal to a straight shoreline over a gentle sloping bottom, Clifton assumes that the bed configuration is related to \( \dot{U}_n, \Delta \dot{U}_o, T \) and \( d \), where \( \dot{U}_o \) = peak orbital velocity at bed, \( \Delta \dot{U}_o \) = orbital velocity asymmetry, \( T \) = period and \( d \) = particle diameter.

The \( \Delta \dot{U}_o \) parameter is a measure of the velocity asymmetry and it applies to fully oscillatory flow and to oscillatory flow superimposed by a current. Thus, \( \Delta \dot{U}_o = \dot{U}_{on} - \dot{U}_{off} + u_c \). The main reason for wave-induced asymmetry is the shoaling process. As a wave begins to shoal, the crest elevates and steepens and the trough shallows and becomes flatter. The volume of water carried forward (in wave direction) under the crest must be equal to the volume of water carried backward under the trough. Because the steepened crest passes a given point in shorter time than the broader trough, the velocity under the wave crest will be larger than that under the trough. Clifton used second-order wave theory to estimate \( \Delta \dot{U}_o \). The transition from symmetric to asymmetric ripples will occur for \( \Delta \dot{U}_o > 0.05 \text{ m/s} \) caused by asymmetric oscillatory motion with or without a superimposed longshore current.

Mobile asymmetric ripples generally migrate in the direction of their leeside slope. According to Clifton, three types of ripples can be distinguished depending on the variables \( d_{so}, \dot{A}_o = \text{peak orbital excursion}, \text{ and } \lambda_r = \text{ripple length}:

- Orbital ripples \( (2\dot{A}_o/d_{so} < 1000) \), which form under short period waves, the length depends directly on the length of the orbital diameter;

- Suborbital ripples \( (1000 < 2\dot{A}_o/d_{so} < 5000) \) which form under longer period waves, the length increases with increasing grain size but decreases with increasing orbital diameter;

- Anorbital ripples \( (2\dot{A}_o/d_{so} > 5000) \) which form under waves with very large orbital diameters, the length depends on grain size but is independent on orbital diameter.

Observations in coastal zones with medium to coarse-grained sand \((250-750 \mu \text{m})\) indicate that asymmetric bed forms develop in a consistent pattern, as shown in Figure 5.4.2 (Clifton, 1976). The most significant features as:

- Ripples become increasingly irregular for increasing energy conditions (longshore bar and upper shoreface);

- Irregular ripples may grade into cross-ripples with increasing wave heights and orbital asymmetry; cross-ripples consist of two sets of ripples both oriented oblique to the oscillatory flow, one set tends to be long-crested and the other set is composed of shorter ripples in the troughs of the longer ripples; cross-ripples are not detectably related to longshore currents or to waves approaching from different directions,

- Cross-ripples may grade into lunate mega-ripples for increasing orbital asymmetry; these mega-ripples have a length scale of about 1 m and are common in medium to coarse-grained sand \((250-750 \mu \text{m})\) in conditions of intense asymmetric orbital motion \( (\Delta \dot{U}_o > 0.25 \text{ m/s}) \) generated by long-period waves.

Shipp (1984) used the classification of Clifton (1976) to describe the bed form types observed in a single barred coastal system at Long Island, New York. The results of Shipp are valid for fair weather conditions; longshore currents were not present. The most interesting features are (see also Figure 5.4.2B).
A. SEQUENCE OF BED FORMS IN COASTAL ZONE (Clifton, 1976)

B. TYPE OF BED FORMS IN COASTAL ZONES DURING FAIR WEATHER CONDITIONS (Shipp, 1984)

Figure 5.4.2 Bed forms in the coastal zone according to Clifton (1976) and Shipp (1984)
Upper shore face: linear ripples, asymmetric ripples, flat bed (sheet flow)

Longshore trough: linear ripples ($\lambda_r = 0.7 \text{ m, } \Delta_r = 0.15 \text{ m}$)

Landward slope of bar: cross ripples, irregular ripples and linear ripples (from top to bottom)

Longshore bar crest: irregular and cross ripples for low-energy conditions lunate mega-ripples ($\lambda_r \approx 0.7 \text{ m, } \Delta_r \approx 0.15 \text{ m}$) for higher energy conditions

Seaward slope of bar: cross-ripples and linear ripples

Transitional zone: linear ripples of fine sand (200 $\mu$m); locally coarse-grain deposits (600 $\mu$m) forming linear mega-ripples ($\lambda_r \approx 0.7 \text{ m, } \Delta_r \approx 0.15 \text{ m}$)

Offshore: linear ripples of fine sand (150-200 $\mu$m)

Figure (5.4.3) shows ripple characteristics along the bed ($d_90 = 270 \mu$m), as measured by Sakakiyama et al (1985) in a large scale wave flume with regular waves. Case 3-2 shows a rippled bed in the offshore and onshore zone, while a flat bed can be observed near the breaker point (plunging breakers). In case 3-4 ripples can also be observed near the breaker point, probably because spilling breakers are present which cannot wash out the bed forms.

In the literature many equations are available to determine the dimensions of wave-generated bed forms (ripples). Important contributions were made by Inman (1957), Mogridge and Kamphuis (1972), Dingler (1975) and Nielsen (1981).

Based on the analysis of laboratory and field data, Nielsen concluded that:
• the size and shape of the ripples are influenced by the irregularity of the waves; for irregular waves the ripples are shorter and flatter than for regular waves,
• field data conform best with laboratory data when the field wave parameters are based on the significant wave height.

For laboratory conditions Nielsen proposed:

$$\frac{\Delta_r}{\lambda_o} = 0.275 - 0.022 \psi^{0.5} \quad (5.4.1)$$

$$\frac{\lambda_r}{\lambda_o} = 2.2 - 0.345 \psi^{0.34} \quad (5.4.2)$$

For field conditions Nielsen proposed:

$$\frac{\Delta_r}{\lambda_o} = 21 \psi^{-1.85} \quad \text{for } \psi > 10 \quad (5.4.3)$$

$$\frac{\lambda_r}{\lambda_o} = \exp \left( \frac{693 - 0.37 \ln^8 \psi}{1000 + 0.75 \ln^7 \psi} \right) \quad \text{for } \psi \geq 10 \quad (5.4.4)$$
in which:
\[ \Delta_r = \text{ripple height} \]
\[ \lambda_r = \text{ripple length} \]
\[ A_A = \text{peak value of orbital excursion near the bed} \]
\[ U_\delta = \text{peak value of orbital velocity near the bed} \]
\[ \psi = \text{mobility parameter} = \frac{(U_\delta)^2}{(s-1)gd_\phi} \]

.flat waves \[ H_\alpha/L_\alpha = 0.019 \]
.steep waves \[ H_\alpha/L_\alpha = 0.108 \]

**Figure 5.4.3** Ripple characteristics along the bed in a wave flume, Sakakibara et al (1985), \[ \eta = \text{ripple height}, \lambda = \text{ripple length}, d_\phi = \text{orbital diameter} \]

<table>
<thead>
<tr>
<th>Source</th>
<th>(D_{50}) ((\mu m))</th>
<th>(\hat{U}_\delta) (m/s)</th>
<th>(\Delta_r) (m)</th>
<th>(\lambda_r) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inman (1957)</td>
<td>250-460</td>
<td>0.2-0.4</td>
<td>0.05 -0.15</td>
<td>0.3 -0.9</td>
</tr>
<tr>
<td>Dingler (1975)</td>
<td>150-210</td>
<td>0.3-0.7</td>
<td>0.003-0.007</td>
<td>0.07-0.09</td>
</tr>
<tr>
<td>Boyd et al (1988)</td>
<td>100-120</td>
<td>0.1-0.5</td>
<td>-</td>
<td>0.06-0.24</td>
</tr>
</tbody>
</table>

**Table 5.3** Ripple data for field conditions

Van Rijn (1989) analyzed 140 sets of ripple data for regular waves and 50 sets of ripple data for irregular waves. The data were selected from laboratory and field experiments with sand beds consisting of particle sizes in the range of 100 to 500 \(\mu m\) under non-breaking wave conditions. The field data are those of Inman (1957) and Dingler (1975), see Table 5.3. Recent field observations of Boyd et al (1988) are also presented in Table 5.3. Long-crested
(2D) ripples were observed during persistent low-energy conditions; short-crested (3D) ripples were observed during high-energy conditions. Ripple orientation responded rapidly to changes in the wave propagation direction. Ripples were observed to migrate both in the direction of wave propagation (positive) and in the opposite direction (negative). Positive migration predominated.

Based on the analysis of Van Rijn (1989), the ripple height and length were found to be related to the peak value of the orbital excursion ($\Delta_\phi$) and a particle mobility parameter ($\psi$), as follows:

$$\frac{\Delta_r}{\Delta_\phi}, \frac{\Delta_r}{\lambda_r} = F(\psi) \quad (5.4.5)$$

in which:

$$\psi = (\hat{\nu}_o)^2/(s-1)g\ d_{50}$$

Figure 5.4.4 shows the ripple height for regular and irregular waves. Figure 5.4.5 shows the ripple steepness ($\Delta_r/\lambda_r$). The relationships of Nielsen (Eqs. 5.4.1 to 5.4.4) for regular and irregular waves are also shown.

Comparing the experimental results of regular and irregular waves, it appears that the ripple height and steepness are smaller for irregular waves than for regular waves when the $\psi$-parameter is larger than about 25. For these latter conditions the ripple generation is increasingly dominated by suspended sediment transport processes which have a more diffusive character resulting in a smoothing of the ripples. This was also reported by Nielsen (1981). The relationships of Nielsen for irregular waves seem to give a ripple height that is somewhat too small for $\psi$-values in the range of 20 to 60. Further, it is noted that the field data of Dingler (1975) are relatively small compared with the laboratory data for $\psi$-values in the range of 40 to 80. The reason for this is not clear. It may be caused by the limited accuracy of the ripple height measurements in field conditions, especially when the ripple heights are small. More field experiments are necessary to verify the results of Dingler (1975). Van Rijn (1989) proposed the following relationships for irregular waves:

$$\frac{\Delta_r}{\Delta_\phi} = 0.22 \quad \text{for} \quad \psi \leq 10$$

$$\frac{\Delta_r}{\Delta_\phi} = 2.8 \times 10^{-3}(250 - \psi)^5 \quad \text{for} \quad 10 < \psi < 250 \quad (5.4.6)$$

$$\frac{\Delta_r}{\Delta_\phi} = 0 \quad \text{for} \quad \psi \geq 250$$

$$\frac{\Delta_r}{\lambda_r} = 0.18 \quad \text{for} \quad \psi \leq 10$$

$$\frac{\Delta_r}{\lambda_r} = 2.10^{-7}(250 - \psi)^{2.5} \quad \text{for} \quad 10 < \psi < 250 \quad (5.4.7)$$

$$\frac{\Delta_r}{\lambda_r} = 0 \quad \text{for} \quad \psi \geq 250$$

The upper regime with sheet flow conditions is assumed to be present for $\psi \geq 250$. 

5.39
Figure 5.4.4 Ripple height in waves, Van Rijn (1989)
Figure 5.4.5 Ripple length in waves, Van Rijn (1989)
From Eqs. (5.4.6) and (5.4.7), it can be derived that:

\[ \lambda_r = 1.22 \, \lambda_o \quad \text{for} \quad \psi \leq 10 \]  

(5.4.8)

which is close to the value \( \lambda_r = 1.3 \, \lambda_o \) reported by Dingler and Inman (1976), Miller and Komar (1980) and Nielsen (1981).

The proposed expressions are valid for non-breaking wave conditions. In case of breaking wave conditions the mobility parameter (\( \psi \)) will, in general, be larger than 250 yielding sheet flow over a flat bed. In spilling breaking waves this may be realistic. However, in plunging breaking waves the interaction of the waves with the bed is so vigorously that a rather irregular bed surface may be generated.

2. **Sheet flow regime**

Based on Figure 5.4.4, the sheet flow regime with a plane mobile bed will occur for \( \psi = \frac{\bar{U}_\psi}{(s-1)g \, d_{50}} > 250 \). According to Wilson (1989), sheet flow conditions will be generated for \( \Theta = \frac{\tau_w}{(\alpha_r - \alpha_c)g \, d_{50}} > 0.8 \).

Generally, sheet flow conditions are assumed to be present in the surf zone where breaking waves are dominant. Kroon and Van Rijn (1989), however, did not observe a fully plane bed in the surf zone. The bed was always irregular with bumps and holes of the order of 0.02 m high and about 1 m long.

3. **Surf zone bars or longshore bars**

These type of bars have their orientation (crests) parallel to the coastline and are found in the surf zone near the breakerline (Figs. 5.4.2 and 5.4.3). The basic mechanism may be the generation of net onshore-directed velocities seaward of the breakerline and net offshore-directed velocities (undertow) in the surf zone. In case of high-energy coasts consisting of fine sediment material (200-300 \( \mu \)m) two or more parallel bars are generated, while no bars are generated in case of low-energy coasts of relatively coarse sediment (> 500 \( \mu \)m).

Longshore variations in bar shape and dimensions may be affected by the interaction of swell waves and edge waves yielding beach cusps and wing-type bars connected to the beach in case of high-energy reflective beaches.

5.4.3 Examples and problems

1. A coastal sea has a water depth of \( h = 5 \) m. Irregular waves with a peak period of \( T_p = 7 \) s are present. The bed material characteristics are \( d_{50} = 300 \, \mu \)m, \( d_{90} = 500 \, \mu \)m. Other data are: \( \rho = 1025 \, \text{kg/m}^3, \rho_s = 2650 \, \text{kg/m}^3, v = 1 \times 10^{-6} \, \text{m}^2/\text{s} \).

What is the significant wave height at initiation of sheet flow?

Initiation of sheet flow:

\[ \frac{(\bar{U}_\psi)^2}{(s-1)g \, d_{50}} = 250 \]

\( (\bar{U}_\psi)^2 = 250 \left( \frac{2650}{1025} - 1 \right) 9.81 \times 0.0003 = 1.17 \, \text{m}^2/\text{s}^2 \)

\( \bar{U}_\psi = 1.08 \, \text{m/s} \)
\[ \hat{U}_h = \frac{\pi H_s}{T_p \sinh(2\pi h/L_s)} = 1.08 \text{ m/s} \]

Wave length is \( L_s = 46 \text{ m} \) (\( h = 5 \text{ m} \), \( T_p = 7 \text{ s} \))
Thus, \( H_s = 1.8 \text{ m} \)

2. A coastal sea has a water depth of \( h = 5 \text{ m} \). The (irregular) wave characteristics are \( H_s = 1 \text{ m} \), \( T_p = 7 \text{ s} \). Ripples are present. Other data: see example 1.

What are the ripple dimensions according to the methods of Nielsen and Van Rijn?

<table>
<thead>
<tr>
<th>Nielsen</th>
<th>( \Delta_t = 0.0044 \text{ m} )</th>
<th>( \lambda_r = 0.088 \text{ m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van Rijn</td>
<td>( \Delta_t = 0.028 \text{ m} )</td>
<td>( \lambda_r = 0.36 \text{ m} )</td>
</tr>
</tbody>
</table>

5.5 Bed forms in currents and waves

5.5.1 Classification

Ripples are the dominating type of bed forms in the nearshore and surf zone. Two general types of currents exist: cross-shore return flow and longshore currents. Tidal currents are also important, especially in periods with calm weather. The cross-shore return flows refer to a general seaward flow or to a channelized narrow seaward rip current. General seaward flow is most significant close to the shore under breaking wave conditions and accounts for seaward-facing bed forms. In rip currents which are opposing the waves, seaward-facing megaripples have been commonly observed (Clifton, 1976).

Longshore currents refer to the current in the zone between the longshore bar and the shoreline. They feed the rip currents at breaks in the longshore bar. Usually, the wave propagation direction is almost normal to the current direction. Little is known of the type of bed forms which are generated under these conditions. Some information is given by Nielsen (1983), who reports the presence of wave-generated ripples with their crest lines parallel to the shore in the littoral drift zone, showing no influence of the (weak) longshore current of about 0.3 m/s. Shipp (1984) reports the generation of cross-ripples and irregular 3D-ripples in the surf zone (see Fig. 5.4.2). Complex ripple patterns (sometimes called interference ripples) are found in areas where wave and currents cross at a certain angle, rip currents with waves or cross-waves reflections with longshore currents.

Bed forms in the offshore zone are generated by tidal currents superimposed by waves. The currents may be following, opposing or oblique to the waves. Bed forms in tidal seas are related to the peak current velocities, water depth, sediment diameter and the availability of sediment.

The bed forms in combined current and wave conditions fall into four categories:

- symmetrical wave-induced ripples with their crest almost perpendicular to the wave direction in case of weak tidal current velocities (see Fig. 5.5.2).
- asymmetrical current-induced ripples and large symmetrical sand waves with their crest perpendicular to the tidal current direction in case of a strong tidal current and weak orbital velocities (see Fig. 5.5.3).
- wave-current ripples in a honeycomb pattern in case of equal strength of the current and peak orbital velocities (see Fig. 5.5.2).
longitudinal furrows, ribbons, ridges and banks with their crests and troughs almost parallel to the peak tidal current direction (see Figs. 5.5.4 and 5.5.5).

The ripple data of Nieuwjaar-Van der Kaaij (1987), Nap-Van Kampen (1988) and Havinga (1992) yield the following results (see Fig. 5.5.2):

- symmetrical wave-induced ripples (2D, 2½D) for \( u_{*,c}/u_{*,w} \leq 0.25 \)
- wave-current ripples (honeycomb pattern, 3D) for \( 0.25 < u_{*,c}/u_{*,w} < 2 \)
- asymmetrical current-induced ripples (3D) for \( u_{*,c}/u_{*,w} \geq 2 \)

According to Amos and Collins (1978), who analyzed field measurements in the intertidal zone of a sand flat, ripples can be classified as follows:

- wave-dominated ripples for \( \hat{U}_\delta/u_{*,c} > 10 \)
- current-dominated ripples for \( \hat{U}_\delta/u_{*,c} < 1 \)

in which: \( u_{*,c} \) = overall current-related bed-shear velocity.

The bed forms generated by combined currents and waves bear some features of both hydraulic effects. Where the wave component dominates, the bed forms are similar to fully developed wave-related bed forms. As the current component gains in strength, the bed forms become more asymmetrical and larger in height and length, especially in case of an opposing current. The influence of the waves is that the bed form crest will become more rounded.

Based on the available data, Van Rijn proposes a classification diagram for bed forms under combined current and wave conditions, which is shown in Fig. 5.5.1. The basic parameters are the current-related and the wave-related mobility parameters defined as follows:

\[
\begin{align*}
\theta'_c &= \frac{(u'_{*,c})^2}{(s-1)gd_{s0}} \\
\theta'_\delta &= \frac{(u'_{*,w})^2}{(s-1)gd_{s0}}
\end{align*}
\]  

(5.5.1)  
(5.5.2)

in which:

- \( u_{*,c} = (0.125 f'_{c})^{0.5} \bar{u} \) = current-related effective bed-shear velocity
- \( u'_{*,w} = (0.25 f'_{w})^{0.5} \hat{U}_\delta \) = wave-related effective bed-shear velocity
- \( f'_{c} = 0.24[ \log(12h/3d_{s0}) ]^{-2} \) = current-related friction factor
- \( f'_{w} = \exp\left[-6+5.2(\hat{A}_\delta/3d_{s0})^{-0.19}\right] \) = wave-related friction factor
- \( \bar{u} \) = depth-averaged velocity
- \( \hat{U}_\delta \) = peak orbital velocity at bed based on relative wave period
- \( \hat{A}_\delta \) = peak orbital excursion at bed based on relative wave period (\( \hat{U}_\delta = \omega \hat{A}_\delta \))
- \( h \) = water depth
- \( d_{s0}, d_{o0} \) = particle diameters
- \( s = \rho_p/\rho \) = relative density
Figure 5.5.1 Bed form classification for current and waves, Van Rijn
5.5.2 Shape and dimensions of bed forms

1. Wave-current ripples

The generation of ripples (length smaller than the water depth) in flumes with following and opposing waves with respect to the current was studied by Harms (1969), Tanaka and Shuto (1984), Nieuwjaar-Van der Kaaij (1987) and Nap-Van Kampen (1988). The wave-induced ripples are approximately symmetrical, but become almost immediately asymmetrical when a current is generated in the flume.

Generally, the ripples in a current opposing the waves are less asymmetrical than in a current following the waves. This is caused by the fact that there is a relatively strong (wave-induced) reduction of the near-bed current velocities in case of an opposing current. For $u_{*e}/u_{*w} > 1.5$ $\rightarrow$ 2 the asymmetry of the ripples is about equal to that for ripples in a current alone. A considerable increase (50%) of the ripple length was observed when a current was generated in the flume, especially in case of a current opposing the waves. The ripple height showed a minor increase (20%) when a current was generated.

The generation of ripples in case of current and waves under an angle was studied by Havinga (1992). The observed ripple patterns are shown in Fig. 5.5.2. The current-induced ripple height was about 0.02 to 0.03 h. The wave-induced ripple length (0.2 to 0.3 h) was about twice as large as the wave-induced ripple length. The ripple height and length were not noticeably affected by the wave-current angle ($60^\circ$, $90^\circ$ and $120^\circ$).

Wave-current ripples with lengths between 0.1 and 1 m have been observed on intertidal flats (Reineck and Wunderlich, 1968). These types of ripples have more rounded crests than current-ripples. The ripple steepness lies between those of wave ripples ($= 0.15$ to 0.2) and those of current ripples ($= 0.05$ to 0.15).

Current-induced mega-ripples are found to be the dominant features in the surf zone. They are also generated as secondary features on the back of sand waves in the offshore zone. Their shape is asymmetrical with a steep lee side slope and they respond rapidly to local currents and waves. They are generated easily, but they are also easily washed out at higher velocities.

2. Transverse sand waves

Field observations in shelf areas indicate that the presence of symmetrical and asymmetrical sand waves (Fig. 5.5.3) with their crest perpendicular to the main current direction (transverse) is related to the tidal current conditions. Stride (1982) states that the lower limit is a peak tidal current of 0.65 m/s for fine to medium sediments ($< 500 \mu m$). They are found in areas with available sediments. They could have grown up from a flat bed with only small-scale bed forms. Sand waves are restricted to tidal environments occurring in estuaries and in shelf seas.

At high current velocities the crests of the sand waves are eroded, similarly to that of dunes in the upper river flow regime. Sand waves can be symmetrical with an almost trochoidal shape or slightly asymmetrical with a gentle stoss side and a steeper lee side. Symmetrical sand waves are expected to be found under almost symmetrical tidal flow and are not expected to migrate substantially. The asymmetrical sand waves are associated with a net sediment transport rate in the direction of the largest velocity. The sand wave length is in the range of 3 to 20 times the water depth. The sand wave height is in the range of 0.1 to 0.3 times the water depth. Migration velocities are in the range of 0 to 5 m per year in (asymmetrical) tidal currents. Often mega-ripples are migrating over the sand waves. Wave-induced ripples will be present on the mega-ripples during rough weather conditions.
The transient behaviour of sand waves due tidal variations (neap-spring cycle) and storm events (high waves) is also of interest. The effect of a storm has been observed to cause a significant decrease of the sand wave height (30% reduction, Langhorne, 1982). The effects of tidal variations seem to be confined to the crest region of the sand waves (Langhorne, 1982), showing oscillating crest movements (over 2 m) in the neap-spring cycle.

Tobias (1989) analyzed echo sounding data collected in 10 areas near the approach channel (Eurogeul, North Sea) to the harbour of Rotterdam (period 1975-1985). The water depths (to mean bed level) were in the range of 20 to 35 m. The peak current velocities of the springtide were 0.8 m/s (flood) and 0.7 m/s (ebb), which implies an asymmetry of $\Delta \bar{u} = 0.1$ m/s in the flood direction. The monthly-mean significant wave height in the area was about $H_s = 1.4$ m during the winter period (November-February) and $H_s = 1.0$ m during the summer period (May-September). The monthly-maximum significant wave heights were resp. 4 m and 2.8 m. The bed material sizes were in the range of 250 to 500 $\mu$m. Analysis of the echo soundings shows the presence of transverse sand waves with mega-ripples migrating over the back of the sand waves (see Fig. 5.5.3). The basic data of the sand waves ($\lambda > h$) and mega-ripples ($\lambda < h$) are given in the following Tables 5.3 and 5.4.

<table>
<thead>
<tr>
<th>Location</th>
<th>Relative height $\Delta/h$</th>
<th>Relative length $\lambda/h$</th>
<th>Number of ripples on stoss-side</th>
<th>Number of ripples on lee-side</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.012</td>
<td>0.66</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>0.03</td>
<td>0.95</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0.014</td>
<td>0.37</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

*Table 5.3 Characteristics of mega-ripples super-imposed on sand waves in North Sea*

<table>
<thead>
<tr>
<th>Location</th>
<th>Relative height $\Delta/h$</th>
<th>Relative length $\lambda/h$</th>
<th>Asymmetry $\lambda_2/\lambda_1$</th>
<th>Relative crest width normal to current $\lambda_2/h$</th>
<th>Water depth $h$ (m)</th>
<th>Propagation velocity $c$ (m/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.122</td>
<td>9.4</td>
<td>0.76</td>
<td>82</td>
<td>24</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.260</td>
<td>12.6</td>
<td>0.42</td>
<td>104</td>
<td>20</td>
<td>2.5 m to NE</td>
</tr>
<tr>
<td>3</td>
<td>0.131</td>
<td>9.0</td>
<td>0.49</td>
<td>105</td>
<td>22</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>0.226</td>
<td>15.9</td>
<td>0.28</td>
<td>278</td>
<td>24</td>
<td>1 m to NE</td>
</tr>
<tr>
<td>5</td>
<td>0.103</td>
<td>10.0</td>
<td>0.76</td>
<td>58</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.153</td>
<td>12.4</td>
<td>0.37</td>
<td>65</td>
<td>28</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>0.148</td>
<td>15.6</td>
<td>0.36</td>
<td>52</td>
<td>24</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0.128</td>
<td>5.7</td>
<td>0.71</td>
<td>77</td>
<td>34</td>
<td>1 to SW</td>
</tr>
<tr>
<td>9</td>
<td>0.124</td>
<td>7.9</td>
<td>0.88</td>
<td>-</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>0.181</td>
<td>9.5</td>
<td>0.64</td>
<td>26</td>
<td>35</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 5.4 Characteristics of sand waves in North Sea*

As can be observed, the megaripples are an order of magnitude smaller than the sand waves.

Analysis of bed material samples shows the presence of smaller sizes in the troughs and larger sizes on the crests of the bed forms. Probably, the smaller particles are suspended by
relatively large velocities at the crest, after which the particles are deposited in the trough regions (smaller velocities).

Tobias (1989) computed the ratio \( \frac{U_{\delta,\text{max}}}{U_{\text{mean}}} \), with \( U_{\delta,\text{max}} \) = peak orbital velocity related to the maximum significant wave and \( U_{\text{mean}} \) = peak current velocity related to the mean tide and found a value smaller than unity for the summer period and a value larger than unity for the winter period. This means a current-dominated transport mechanism during the summer and a wave-dominated transport mechanism during the winter period.

Wright (1992) studied sand waves in the southern North Sea over a period of 20 years. Most of the sand waves appeared simply to oscillate back and forth over distances between zero and 25 metres per year (between zero and 25% of the average wave length). The heights of the sand waves changed by only between 1 and 3.5 metres during the 20 year study period (between 10 and 30% of the sand wave height).

3. Longitudinal furrows, ribbons, ridges and banks

Longitudinal furrows may develop in tidal seas overlying a rocky flat bottom covered with gravel, pebbles and cobbles and where there are strong currents (1 to 1.5 m/s) and a shortage of mobile sediments. Observations (Belderson et al., 1988) have shown that variable amounts of coarse sand and fine gravel are transported along the axes of the furrows. With decreasing velocities these furrows develop through a transition zone into sand ribbons (see Fig. 5.5.4). The furrows can have a length of the order of 1000 m, a width of the order of 10 m and a depth of the order of 1 m. The cross-sectional shape of the furrows may be somewhat asymmetrical. The plan form is slightly sinuous.

Small-scale ribbons (height = 0.1 m) are aligned (parallel) with the main flow direction and are believed to be generated by secondary currents superimposed on the main flow yielding a spiral type of fluid motion.

Sand ridges and banks (North Sea, East China Sea) are the largest sedimentary features in marine conditions with heights of the order of 10 m, widths of the order of 1 km and lengths of the order of 10 km. Their parallel spacing is of the order of the width (= 1 km). Generally, sand banks consist of medium to coarse sand (> 500 \( \mu \text{m} \)) and are large sources of sediment. Sand banks are quite stable features. Mega-ripples and sand waves may migrate over the banks in regions where the velocities are large enough to initiate particle motion. Closely related to the strength and direction of the currents, sediments are circulating round and over the bank. The crest axis of the bank deviates typically about 10° to 20° from the direction of the peak tidal current, which may be caused by Coriolis effects giving an anticlockwise rotation on the Northern hemisphere (Stride, 1982).

Houbolt (1968) studied the sand riges of the Well bank in the southern bight of the North Sea. The sand bed of these ridges consists of rather uniform material in the range of 200 to 300 \( \mu \text{m} \). Based on analysis of spatial grain size distributions and analysis of laminations of cores, a schematic pattern of the sand particle movement was presented by Houbolt (see Fig. 5.5.5). Sand seems to go round the ridge. The entire ridge seems to move slowly towards the north-east, perpendicular to its long axis and to the tidal currents.

Large tidal sand ridges have been observed in the East China Sea Shelf by Chang-Shu and Jia Song (1988). These ridges are mainly present in the submerged palaeovalley of the Changjiang river and the adjacent area. The sand ridges have a length in the range of 10 to 60 km, a width in the range of 2 to 5 km, a spacing of 8 to 14 km and a height of 5 to 20 m. They mainly consist of well-sorted fine sands with abundant debris of marine shells. The local water depths vary in the range of 50 to 100 m. The presence of (shallow) marine fossils suggests that the sand ridges probably are developed during the post-glacial sea level transgression (relict sand ridges) when the water depths were much shallower and the tidal
currents were stronger. As the sea level rose and the tidal currents became weaker, the sand ridges gradually ceased growing and became relict sand bodies. In the nearshore area they were later covered by fine-grained sediments of the late Holocene age (buried sand ridges). The present-day tidal peak currents (= 0.2 m/s) are almost parallel to the long axes of the ridges.

![Diagram of ripple formation](image)

**Figure 5.5.2** Ripple patterns in combined current and wave conditions

![Graph of transverse sand waves with mega-ripples](image)

**Figure 5.5.3** Transverse sand waves with mega-ripples in North Sea (Tobias, 1989)
Figure 5.5.4 Longitudinal furrows in North Sea (Belderson et al., 1988)

Figure 5.5.5 Schematic pattern of sand particle movement along sand ridge in North Sea (Houbolt, 1968)
5.5.3 Examples and problems

1. A coastal sea has a water depth of $h = 20$ m, the peak flood-current velocity is $u_{\text{max,flood}} = 0.6$ m/s; the peak ebb-current velocity is $u_{\text{max,ebb}} = 0.5$ m/s. Irregular waves perpendicular to the flood and ebb current directions are present. The significant wave height is $H_s = 1.5$ m; the peak period is $T_p = 8$ s. The bed material characteristics are $d_{50} = 300 \mu$m, $d_{90} = 600 \mu$m. Other data are: $\rho = 1025$ kg/m$^3$, $\rho_s = 2650$ kg/m$^3$, $\nu = 1.10^6$ m$^2$/s.

What type of bed forms are present?
What are the bed form dimensions?

The type of bed forms can be determined from Fig. 5.4.6. To apply this figure, the grain-related bed-shear velocities have to be computed ($u_{*,w}$ and $u_{*,s}$).

wave length : $h = 20$ m and $T_p = 8$ s, yields $L_s = 88$ m
peak orbital velocity : $\hat{U}_\delta = \frac{\pi H_s}{T_p \sinh(2\pi h/L_s)} = 0.306$ m
peak orbital excursion : $\hat{A}_\delta = \hat{U}_s(T/2\pi) = 0.39$
wave-related friction factor : $f_w' = \exp[-6 + 5.2(\hat{A}_s/3d_{90})^{-0.19}] = 0.0161$
current-related friction factor : $f_c' = 0.24[\log(12h/3d_{90})]^{-2} = 0.0091$
bed-shear velocities : $u_{*,w}' = (f_w'/4)^{0.5} \hat{U}_\delta = 0.00194$ m/s (waves)
$u_{*,s}' = (f_c'/8)^{0.5} \bar{u} = 0.0202$ m/s (flood)
$u_{*,s}' = (f_c'/8)^{0.5} \bar{u} = 0.0169$ m/s (ebb)
mobility parameters : $\theta_w' = 0.081$ (waves)
$\theta_c' = 0.087$ (flood)
$\theta_s' = 0.062$ (ebb)

Bed form types (Fig. 5.4.6) : 2D waves-ripples superimposed on 3D current ripples in honey comb pattern (see Fig. 5.4.7). Both types may be superimposed on large-scale sand waves, as observed by Tobias (1989).

Bed form dimensions

2D wave ripples : $\Psi = \frac{(\hat{U}_\delta)^2}{(s-1)g d_{50}} = 20$

Van Rijn : $\Delta_r = 0.07$ m , Eq. (5.4.6)
$\lambda_r = 0.44$ m , Eq. (5.4.7)

3D current-ripples : Tobias : $\Delta_r = 0.02h = 0.4$ m
$\lambda_r = 0.7h = 14$ m
sand waves : Tobias : \( \Delta_{sw} = 0.15h = 3 \text{ m} \)  
\( \lambda_{sw} = 10h = 200 \text{ m} \)

2. The water depth in a coastal sea is \( h = 5 \text{ m} \). The peak tidal ebb and flood velocities are 0.5 m/s. The significant wave height is \( H_s = 1 \text{ m} \). The peak wave period is \( T_p = 8 \text{ s} \). The bed material characteristics are \( d_{s0} = 250 \mu \text{m} \) and \( d_{\infty} = 500 \mu \text{m} \). Other data are: \( \rho = 1025 \text{ kg/m}^3 \), \( \rho_s = 2650 \text{ kg/m}^3 \), \( \nu = 1 \times 10^{-6} \text{ m}^2/\text{s} \)

Solution: Bed form type : 3D-wave ripples superimposed on 3D-current mega-ripples.

Bed form dimensions : \( \Delta_r = 0.016 \text{ m} \), \( \lambda_r = 0.3 \text{ m} \)  
\( \Delta_{mr} = 0.1 \text{ m} \), \( \lambda_{mr} = 3.5 \text{ m} \)
REFERENCES


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6. EFFECTIVE BED ROUGHNESS

6.1 Introduction

Nikuradse (1932) introduced the concept of an equivalent or effective sand roughness height \( (k_c) \) to simulate the roughness of arbitrary roughness elements of the bottom boundary. In case of a movable bed consisting of sediments the effective bed roughness \( (k_r) \) mainly consists of grain roughness \( (k_r') \) generated by skin friction forces and of form roughness \( (k_r'') \) generated by pressure forces acting on the bed forms. The effective bed roughness for a given bed material size is not constant but depends on the flow conditions. For example, Figure 6.1.1 shows \( k_r \)-values of the Mississippi river determined from measured surface slopes, water depths and discharges using the Chézy-equation. As can be observed, the \( k_r \)-value strongly decreases for increasing velocities, probably because the bed forms become more rounded or are washed out at high velocities.

6.2 Current-related bed roughness

6.2.1 Introduction

The fundamental problem of bed roughness prediction is that the bed characteristics (bed forms) and hence the bed roughness depend on the main flow variables (depth, velocity) and sediment transport rate (sediment size). These hydraulic variables are, however, in turn strongly dependent on the bed configuration and its roughness. Another problem is the almost continuous variation of the discharge during rising and falling stages. Under these conditions the bed form dimensions and hence the Chézy-coefficient are not constant but vary with the flow conditions. Basically, the Chézy-coefficient should be determined from the full momentum equation in stead of the Chézy-equation.

6.2.2 Available methods

Assuming an uniform channel with a non-movable plane rough bed, the bed-shear stress is proportional to the depth-averaged velocity, as follows: \( \tau_b \propto (u)^2 \). When bed forms are generated, the friction factor and hence the bed-shear stress will increase due to the presence of form roughness, as shown in Fig. 6.2.1.

Basically, two approaches can be found in the literature to estimate the bed roughness:

- methods based on bed-form and grain-related parameters such as bed-form length, height, steepness and bed-material size,
- methods based on integral parameters such as mean depth, mean-velocity and bed-material size.

The first method is more universal and can also be used to determine the roughness of a movable bed in non-steady conditions, provided that the bed-form characteristics are known.

6.2.3 Methods based on bed-form parameters

The bed-shear stress \( (\tau_b) \) in an alluvial channel can be divided into:

- grain-related bed-shear stress \( (\tau_b') \),
- form-related bed-shear stress \( (\tau_b'') \).

The distribution of the grain-related bed-shear stress (skin friction) along a bed form (dune) is shown in Figure 6.2.2. The grain-related stress is low in the flow separation zone and high near the crest of the bed form.
The form-related bed-shear stress is related to the fluid pressure distribution upstream and downstream of the crest. The flow is accelerating near the crest, requiring a decrease of the fluid pressure, whereas the flow is decelerating downstream of the crest requiring an increase of the fluid pressure. Integrated over the bed form length, there is a net horizontal pressure force \( F_p \) acting on the bed form. The skin-friction force \( F_s \) and the form-related pressure forces \( F_f \) acting on a bed form are shown schematically in Figure 6.2.3.

The total bed-shear stress \( \tau_b \) can be divided into \( \tau'_b \) and \( \tau''_b \), as follows:

Total bed-shear stress
\[
\tau_b = \frac{F_{sh} + F_f}{\lambda} - \frac{F_s}{\lambda} + \frac{F_f}{\lambda} = \tau'_b + \tau''_b \tag{6.2.1}
\]

Grain-related bed-shear stress
\[
\tau'_b = \frac{F_s}{\lambda} = \left( 1 - \frac{\alpha}{\Lambda} \right) \tau_s - \frac{1}{8} \rho \bar{f}' \bar{u}^2 \tag{6.2.2}
\]

Form-related bed-shear stress
\[
\tau''_b = \frac{F_f}{\lambda} = \frac{C_D}{2} \frac{1}{\rho} \int_0^\Lambda u^2 \, dz = \frac{1}{8} \rho \bar{f}'' \bar{u}^2 \tag{6.2.3}
\]

in which:
- \( F_s \) = skin-friction force
- \( F_f \) = pressure force
- \( \lambda \) = bed form length
- \( \Lambda \) = bed form height
- \( \tau_s \) = grain-related shear stress
- \( \alpha \) = coefficient (\( \sim 5 \))
- \( C_D \) = drag coefficient
- \( u \) = local flow velocity

![Figure 6.1.1 Effective bed-roughness of Mississippi river](image)
Figure 6.2.1 Bed-shear stress as a function of depth-averaged velocity

Figure 6.2.2 Fluid pressure and shear stress distribution along a dune, Langhorne (1978)

Figure 6.2.3 Skin-friction force and fluid pressure force on a dune
From Eqs. (6.2.1), (6.2.2) and (6.2.3) it follows that:

\[ f = f' + f'' \]  

(6.2.4)

For reasons of simplicity, Van Rijn (1984b, 1989) assumed that the effective roughness height of Nikuradse (\(k_s\)) also can be divided in a grain-related part (\(k_{s,c}'\)) and a form-related part (\(k_{s,c}''\)), as follows:

\[ k_{s,c} = k_{s,c}' + k_{s,c}'' \]  

(6.2.5)

in which: \(k_{s,c}'\) = current-related grain roughness height and \(k_{s,c}''\) = current-related form roughness height.

Assuming hydraulically rough flow, the current-related friction factor is:

\[ f_c = 8g C^{-2} = 0.24 \left( \log \left( \frac{12h}{k_{s,c}} \right) \right)^{-2} \]  

(6.2.6)

\[ C_c = 18 \log \left( \frac{12h}{k_{s,c}} \right) \]  

(6.2.7)

The friction factors for the smooth and the transition regime are presented in Chapter 2.

*Grain roughness*

Grain roughness is the roughness of individual moving or non-moving sediment particles as present in the toplayer of a natural plane movable or non-movable bed.

Experimental research shows that grain roughness in the lower regime is mainly related to the largest particles of the top layer of the bed (\(d_{90}\)). Van Rijn (1982) analyzed about 120 sets of flume and field data with and without a mobile bed to determine the grain roughness. The median particle sizes were in the range of 130 to 5000 \(\mu\)m. The \(k_{s,c}'\) values were computed from the Chézy-coefficients (Eq. 6.2.7), which were derived from the measured water depths, depth-averaged velocities and energy gradients. Based on this analysis, the grain roughness was found to be in the range of 1 to 10 \(d_{90}\) of the bed material, as shown in Figure 6.2.4. A clear influence of the mobility of the particles cannot be observed. Experiments with a mobile bed do not show a significantly larger grain roughness than those with an immobile bed. This effect may, however, be masked by the experimental scatter of the data.

Lyn (1991) studied the resistance of flat-bed sediment-laden and clear water flows in a flume. Bed materials in the range of 150 to 250 \(\mu\)m were used. The mobility parameter (\(\theta\)) was in the range of 0.5 to 0.7 (lower regime).

Using the experimental results of Lyn (1991), Van Rijn found:

\(k_{s,c}' = 2\) to 3 \(d_{90}\) for non-movable (rigid) plane bed,

\(k_{s,c}' = 3\) to 5 \(d_{90}\) for movable plane bed.

The \(k_{s,c}'\)-value of a movable plane bed (moving layer of grains) seems to be somewhat larger than that of a rigid plane bed.

Aguirre-Pe and Fuentes (1990) found \(k_{s,c}'\)-values in the range of 1 to 6 \(d_{94}\) for mountain rivers with beds consisting of gravel and cobbles (lower regime).
Similar values for sand and gravel beds in the lower regime were reported by:

Kamphuis (1974) for $540 \leq d_{90} \leq 4200$ $\mu$m: $k_{s,c}' = 2.5$ $d_{90}$ (6.2.8)

Gladki (1975) for $1000 \leq d_{90} \leq 20000$ $\mu$m: $k_{s,c}' = 2.3$ $d_{84}$ (6.2.9)

Hey (1979) for $1000 \leq d_{90} \leq 20000$ $\mu$m: $k_{s,c}' = 3.5$ $d_{84}$ (6.2.10)

Mahmood (1971) for $100 \leq d_{90} \leq 600$ $\mu$m: $k_{s,c}' = 5.1$ $d_{84}$ (6.2.11)

Wilson (1987, 1988) studied the effective roughness in the upper regime with high concentrations ($\theta > 1$) by performing experiments in a conduit with a square cross-section ($0.094 \times 0.094$ m$^2$). Sand particles ($d = 700$ $\mu$m) and nylon particles ($d = 3900$ $\mu$m) were tested.

Based on analysis of his experimental results, Wilson found: $k_{s}' = 5 \theta d_{90}$ with $\theta$ in the range of 1 to 7. Hydraulic rough flow conditions were assumed to be present by Wilson. This is not correct. The effect of the sediment particles on the viscosity in the near-bottom region where the concentrations are high, should be taken into account.

Einstein-Chien (1955) and Winterwerp et al (1990) studied the effect of high concentrations on the flow velocity profile (in the upper regime). Both research groups did experiments in a flume with a rigid plane bed coated by a layer of sand particles. Van Rijn used their data to compute the $k_s'$-value from $C = 18 \log \left( \frac{12 R_s}{k_s' + 3.3 v_m/u_*} \right)$. Side wall roughness was eliminated. The viscosity of the mixture in the near-bed region was estimated to be 10 times larger than that of clear water ($v_m = 10^{-5}$ m$^2$/s). The ratio of $k_s'$ and $d_{90}$ is plotted as a function of the mobility parameter $\theta$ in Fig. 6.2.5. The ratio $k_s'/d_{90}$ increases from 3 to about 20 for an increasing $\theta$-value.

A logical explanation for the increase in the $k_s'$-value in high-concentration flows is the interaction of the flow with the sediment particles in the near-bed region. The particle velocities are greatly reduced by collisions with the bed and with each other resulting in relatively large differences between the local fluid and particle velocity (slip). The associated fluid drag forces are the driving forces of the particle motions. Conversely, these drag forces are reducing the fluid velocities, which can be interpreted as a shear effect additional to the fluid shear.

Based on the available information (lower and upper regime), Van Rijn proposes to use:

$$k_{s,c}' = 3 \ d_{90} \quad \text{for} \ \theta < 1 \ (\text{lower regime}) \quad (6.2.12a)$$

$$k_{s,c}' = 3 \ \theta \ d_{90} \quad \text{for} \ \theta \geq 1 \ (\text{upper regime}) \quad (6.2.12b)$$

in which:

- $k_{s,c}' = $ grain-related effective roughness height of a plane bed
- $\theta = u_s^2/(s-1)g \ d_{90}$ = mobility parameter ($\theta = \theta'$ for plane bed)
- $u_* = g^{0.5} \bar{u}/C =$ bed-shear velocity ($u_* = u_*'$ for plane bed)
- $v_m = \eta_m/\rho_m =$ kinematic viscosity coefficient of fluid-sediment mixture ($\sim 10^{-5}$ m$^2$/s in near-bed region)
- $C = 18 \log \left( \frac{12 h}{k_{s,c}' + 3.3 \ v_m/u_*} \right) = 18 \log \left( \frac{12 h}{k_{s,c}' + v_m \ C/G} \right) =$ Chézy-coefficient
Equation (6.2.12b) can only be solved by interaction. First, the $k'_s$-value is estimated, then $C$, $u$, and $\theta$ are computed and $k'_{s}$ is computed from Eq. (6.2.12b). This procedure should be repeated until the computed $k'_s$ is equal to the estimated $k'_s$.

Equation (6.2.12b) shown in Fig. 6.2.5, is in good agreement with the equation proposed by Wilson (assuming $d_{s0} = 2d_{3p}$). Equation (6.2.12) predicts $k'_s$-values which are somewhat larger than the data points of Einstein-Chien (1955) and Winterwerp et al (1990), see Fig. 6.2.5. This conservative estimate is preferred because the data are related to experiments with rigid beds. A movable bed of loose material will have a somewhat larger roughness because a perfectly plane bed will not exist in natural conditions. Small irregularities will always be present resulting in a larger roughness. This was also shown by the experimental results of Lyn (1991).

The author advises to use a minimum value of $k'_s = 0.01$ m in case of flow in the upper regime with high concentrations over a plane bed.

**Form roughness**

The effective form roughness is related to the bed-form height $\Delta$, the bed-form steepness $(\Delta/\lambda)$ and the bed-form shape $(\gamma)$. The following functional relationship is assumed to be valid:

$$k''_{s,c} = F(\Delta, \frac{\Delta}{\lambda}, \gamma) \quad (6.2.13)$$

The most general case is that of a bed consisting of (mega)ripples superimposed on asymmetrical dunes and symmetrical sand waves (see Chapter 5). Herein it is proposed to determine the overall form roughness by summation of the individual values, as follows (see also Van Rijn, 1989):

$$k''_{s,c} = k''_{s,r} + k''_{s,d} + k''_{s,sw} \quad (6.2.14)$$

in which:

- $k''_{s,r} = $ overall current-related form roughness
- $k''_{s,d} = $ form-roughness related to ripples
- $k''_{s,sw} = $ form-roughness related to asymmetrical dunes
- $k''_{s,sw} = $ form-roughness related to symmetrical sand waves

**Ripples** are herein (see Chapter 5) defined as bed forms with a length smaller than the water depth. Based on the analysis of ripple data of Barton-Lin (1955), Ackers (1964), Vanoni-Brooks (1957), Mahmood et al (1984) from Pakistan irrigation channels, Nieuwjaar-Van der Kaaaj (1987), Nap-Van Kampen (1988) and Havinga (1992); Van Rijn proposed to use the following relationship (see Fig. 6.2.6):  

$$k''_{s,r} = 20 \gamma_r \Delta_r \left( \frac{\Delta_r}{\lambda_r} \right) \quad (6.2.15)$$

in which:

- $\Delta_r = $ ripple height
- $\lambda_r = $ ripple length
- $\gamma_r = $ ripple presence factor ($\gamma_r = 1$ for ripples alone, $\gamma_r = 0.7$ for ripples superimposed on dunes or sand waves).
**Figure 6.2.4** Effective grain roughness

**Figure 6.2.5** Effective grain roughness in high-concentration flows
The form roughness \(k_{s,d}^{/}/\rangle \) shows a strong increase for \(\Delta_r/\lambda_r > 0.1\), as presented in Figure 6.2.6.

Basically, the ripple presence factor is defined as \(\gamma_r = n \lambda_r/\lambda_d\), with \(\lambda_r\) = ripple length, \(\lambda_d\) = dune length (or sand wave length) and \(n\) = number of ripples present on one dune or sand wave.

When the bed is fully covered with ripples, the \(\gamma_r\)-parameter is \(\gamma_r = 1\). When the ripples are superimposed on dunes or on sand waves, the region near the crest and the trough of the dunes usually is free of ripples resulting in a \(\gamma_r\)-parameter of about 0.7.

**Dunes** are herein (see Chapter 5) defined as asymmetrical bed forms with a length of about 7 times the water depth. Based on the analysis of dune data, Van Rijn (1984b, 1989) proposed (Figure 6.2.7):

\[
k_{s,d}^{/} = 1.1 \gamma_d \Delta_d \left(1 - e^{-25 \Delta_d/\lambda_d}\right)
\]

(6.2.16)

in which:

\(\Delta_d\) = dune height
\(\lambda_d\) = dune length
\(\gamma_d\) = form factor \((\gamma_d = 0.7\) for field conditions\)

The form factor \((\gamma_d)\) expresses the influence of the dune form on the roughness height. Ogink (1988) performed flume experiments to study the influence of the dune form on the flow resistance. He found a considerable reduction of the form roughness for mild leeside slopes because the flow separation effect was less important.

Van Rijn (1989) reanalyzed the data of Ogink (1988) to determine the \(\gamma_d\)-parameter as a function of the leeside slope \((\lambda_r/\Delta)\). The results are shown in Figure 6.2.8.

Analysis of field data by Ogink (1988) showed that the leeside slopes of river dunes are much smaller than those of laboratory dunes. The leeside slopes of the river dunes were in the range of 1:5 to 1:7, yielding \(\gamma_d\)-values of about 0.7 (see Figure 6.2.8).

**Sand waves** are herein (see Chapter 5) defined as symmetrical bed forms with a length much larger than the water depth. The leeside slopes of symmetrical sand waves are relatively mild. Hence, flow separation will not occur. Therefore, the form roughness of symmetrical sand waves is assumed to be zero. Thus,

\[
k_{s,sw} = 0
\]

(6.2.17)

To show the applicability of Eq. (6.2.14), Van Rijn (1989) calculated the overall roughness of (artificial) dunes superimposed by ripples, as tested by Ogink (1988). Sand particles with a median size of 780 \(\mu\)m were coated on the bed forms to simulate grain roughness. The basic bed form dimensions are presented in Fig. 6.2.9. The grain roughness \((k_{s,e})\) was assumed to be \(k_{s,e} = 0.001\) m. The predicted and computed \(k_{s,e}\)-values are given in the following Table 6.1. The largest deviation is about 60% for test A, which seems rather large. However, the predicted and measured Chézy-coefficient do not differ more than 10%.
Figure 6.2.6 Form-roughness of ripples ($\gamma_r = 1$)

Figure 6.2.7 Form-roughness of dunes ($\gamma_d = 1$)
Figure 6.2.8 Form factor of dunes

Figure 6.2.9 Bed form geometry, Ogink (1988)
<table>
<thead>
<tr>
<th>Test</th>
<th>predicted $k_{s,e}$ (m)</th>
<th>measured $k_{s,e}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.018</td>
<td>0.027</td>
</tr>
<tr>
<td>B</td>
<td>0.025</td>
<td>0.029</td>
</tr>
<tr>
<td>C</td>
<td>0.022</td>
<td>0.025</td>
</tr>
<tr>
<td>D</td>
<td>0.024</td>
<td>0.019</td>
</tr>
<tr>
<td>E</td>
<td>0.018</td>
<td>0.014</td>
</tr>
</tbody>
</table>

**Table 6.1 Predicted and measured $k_{s,e}$-values for tests of Ogink (1988)**

Another approach was used by Engelund (1977) for dunes and Vanoni-Hwang (1967) for ripples and dunes, who related the form friction coefficient ($f^{''}$) to the bed form parameters.

Engelund: $f^{''} = 10 \frac{\Delta^2}{h\lambda} (e^{-2.5\Delta/h})$  \hspace{1cm} (6.2.18)

Vanoni-Hwang: $(f^{''})^{-0.5} = 3.5 \log\left(\frac{h}{e\Delta}\right)^{-2.3}$ \hspace{1cm} (6.2.19)

in which:

$f^{''}$ = form friction factor of Darcy-Weisbach

$\Delta$ = bed form height

$\lambda$ = bed form length

$h$ = water depth

$e$ = ratio of lee-slope area and total bed-form area

**6.2.4 Methods based on integral parameters**

Various methods based on integral parameters such as water depth, mean velocity, slope and bed material characteristics have been proposed: Einstein and Barbarossa (1953), Engelund and Hansen (1967), Alam and Kennedy (1969), Smith and McLean (1977), White et al (1979), and Brownlie (1981).

Herein, the methods of Engelund-Hansen (1967) and White et al (1979) are presented, because these two methods gave the best results in an extensive appraisal of available methods for river flow conditions as reported by White et al (1979).

The method of Brownlie (1981) and Smith and McLean (1977) are also presented, because of their simplicity.

It is noted that the depth ($h$) should be used for wide channels and the hydraulic radius of the bed ($R_b$) for narrow channels (see Appendix C).

**Engelund and Hansen (1967)**

They defined $\tau_{b,e} = \tau_{b,e}^' + \tau_{b,e}^{''}$ with:

$\tau_{b,e}^' = \rho gh T$ or $u_{*,e}^' = (gh T)^{0.5}$ \hspace{1cm} (6.2.20)

$\tau_{b,e}^{''} = \rho gh^{''} T$ or $u_{*,e}^{''} = (gh^{''} T)^{0.5}$ \hspace{1cm} (6.2.21)
Based on this, it can be derived that:

$$\left( \frac{u'_{*,c}}{u_{*,c}} \right)^2 = \frac{h'}{h} \quad \text{or} \quad \frac{\theta'}{\theta} = \frac{h'}{h}$$  \hspace{1cm} (6.2.22)

They assumed that $u'_{*,c}$ can be found from the following relationship:

$$\bar{u} = 2.5 \ u'_{*,c} \ln \left( \frac{12 \ h'}{2.5 \ d_{50}} \right)$$  \hspace{1cm} (6.2.23)

Using $\bar{u} = C(h')^{0.5}$ and $u'_{*,c} = (gh')^{0.5}$, Eq. (6.2.23) can be expressed as:

$$C = 2.5 \ g^{0.5} \left( \frac{h'}{h} \right)^{0.5} \ln \left( \frac{12h'}{2.5 \ d_{50}} \right)$$  \hspace{1cm} (6.2.24)

or

$$C = 2.5 \ g^{0.5} \left( \frac{\theta'}{\theta} \right)^{0.5} \ln \left( \frac{12h'}{2.5 \ d_{50}} \right)$$  \hspace{1cm} (6.2.25)

Data of movable bed experiments in flumes (Fort Collins data, USA) were analyzed by Engelund and Hansen. Knowing the basic flume data ($\bar{u}$, $h$, $I$, and $d_{50}$), the $u'_{*,c}$ and $h'$ parameters can be computed from Eqs. (6.2.23) and (6.2.20) yielding two equations with two unknown variables. Based on these results, the $\theta'$ and $\theta$-parameters were computed and plotted, as shown in Figure 6.2.10.

![Figure 6.2.10 Mobility parameters $\theta'$ and $\theta$ according to Engelund and Hansen (1967)](image_url)

0.09 < h < 0.33 m
0.25 < $\bar{u}$ < 1.60 m/s
190 < $D_{50}$ < 930 $\mu$m
Two regimes can be identified, as follows:

Lower regime: \[ \theta' = 0.06 + 0.4 \, \theta^2 \quad \text{for} \quad \theta \leq 0.7 \] \hspace{1cm} (6.2.26)

Upper regime: \[ \theta' = \theta \quad \text{for} \quad 0.7 < \theta < 1 \] \hspace{1cm} (6.2.27a)

\[ \theta' = (0.3 + 0.7 \, \theta^{-1.8})^{-0.56} \quad \text{for} \quad \theta \geq 1 \] \hspace{1cm} (6.2.27b)

Using Figure 6.2.10 or Eqs. (6.2.26), (6.2.27); the Chézy-coefficient can be determined. Two cases are possible:

Case I: \ h and I are known; find \( C \) and \( \bar{u} \)

1. compute \( u_{*,e} \) and \( \theta \)
2. determine \( \theta' \) from Fig. 6.2.10 or Eqs. (6.2.26), (6.2.27)
3. compute \( h' = (\theta'/\theta)h \)
4. compute \( C \) from Eq. (6.2.25)
5. compute \( \bar{u} = C(hI)^{0.5} \)

Case II: \ h and \( \bar{u} \) are known, find \( C \) and \( I \)

1. guess \( h' \) (start with \( h' = 0.33 \, h \))
2. compute \( u_{*,e} \) from Eq. (6.2.23), compute \( \theta' \)
3. determine \( \theta \) from Fig. 6.2.10 or Eqs. (6.2.26), (6.2.27)
4. compute \( h' = (\theta'/\theta)h \) and compare with initial value
5. repeat until \( h' \) = constant
6. compute \( C \) from Eq. (6.2.24)
7. compute \( I \) from \( \bar{u} = C(hI)^{0.5} \)

White et al., 1979

This method is based on the analysis of 1432 sets of flume data and 263 sets of field data of the lower regime.

Assuming the water depth \( h \), the slope \( I \) and the particle diameter \( d_{35} \) are known, the computational procedure is straightforward (otherwise iterative computations are necessary):

1. Compute particle parameter \( D_*, n, Y_{\alpha}, P \)

\[
D_* = d_{35} \left( \frac{(s-1)g}{v^2} \right)^{0.5} \] \hspace{1cm} (6.2.28)

\[
n = 1 - 0.56 \log(D_*) \quad \text{for} \quad 1 \leq D_* < 60 \]

\[
Y_{\alpha} = \frac{0.23}{(D_*)^{0.5}} + 0.14 \quad \text{for} \quad 1 \leq D_* < 60 \] \hspace{1cm} (6.2.29)

\[
n = 0 \quad \text{for} \quad D_* \geq 60 \]

\[
Y_{\alpha} = 0.17 \quad \text{for} \quad D_* \geq 60 \]

\[
P = \left[ \log(D_*) \right]^{1.7} \] \hspace{1cm} (6.2.30)

6.13
2. Compute bed-shear velocity, $u_{*c}$

$$ u_{*c} = \frac{g h H^{0.5}}{(s-1)g d_{50}^{0.5}} \quad (6.2.31) $$

3. Compute mobility parameter, $Y_{fg}$

$$ Y_{fg} = \frac{u_{*c}}{[(s-1)g d_{50}^{0.5}]} \quad (6.2.32) $$

4. Compute mobility parameter, $Y_{gs}$

$$ \frac{Y_{gr} - Y_{cr}}{Y_{fg} - Y_{cr}} = 1 - 0.76 \left[1 - e^p\right] \quad (6.2.33) $$

5. Compute average flow velocity, $\bar{u}$ from

$$ Y_{gr} = \frac{(u_{*c})^n}{[(s-1)g d_{50}^{0.5}]} \left[\frac{\bar{u}}{5.66 \log(10 h/d_{50})}\right]^{1-n} \quad (6.2.34) $$

6. Compute Chézy-coefficient, $C$

$$ C = \frac{g^{0.5} u}{u_{*c}} \quad (6.2.35) $$

*Brownlie (1981)*

Brownlie presented a method to predict the flow depth ($h$) as a function of the main flow variables. The method is based on dimensional analysis and data fitting using 344 flume data and 550 field data in the lower and upper flow regime. The following equations were proposed:

Lower regime: $h = 0.372 \ d_{50} \ Q_*^{0.654} \ I^{-0.254} \ \sigma_s^{0.105} \quad (6.2.36)$

Upper regime: $h = 0.284 \ d_{50} \ Q_*^{0.625} \ I^{-0.288} \ \sigma_s^{0.08} \quad (6.2.37)$

for $I \geq 0.006$ or $F_g \geq 1.74 \ I^{-1/3}$

in which:

$$ Q_* = \frac{Q}{b \ g^{0.5} d_{50}^{1.5}} $$

$$ F_g = \frac{Q}{b \ (s-1)g d_{50}^{0.5}} $$

$$ \sigma_s = \frac{1}{2} \left( \frac{d_{50}/d_{16}}{d_{50}/d_{50}} \right) $$
The Chézy-coefficient follows from: \[ C = \frac{Q}{b \ h^{1.5} \ I^{0.5}} \]

*Smith and McLean (1977)*

Based on the analysis of river data, they proposed:

\[ k_{c,e} = d_{50} \left[ 1 + 700 (\theta - \theta_{cr}) \right] \quad (6.2.38) \]

in which:

- \( k_{c,e} \) = effective current-related bed roughness
- \( \theta \) = \( \tau_b / ((\rho_s - \rho) g d_{50}) \) = mobility parameter
- \( \theta_{cr} \) = critical mobility parameter of Shields
- \( \tau_b \) = bed-shear stress
- \( d_{50} \) = median particle diameter of bed material

Equation (6.2.38) which is only valid in the lower regime (\( \theta < 0.7 \)) with dunes, yields \( k_{c,e} = d_{50} \) for \( \theta \leq \theta_{cr} \) and \( k_{c,e} = 400 \ d_{50} \) for \( \theta = 0.6 \) and \( \theta_{cr} = 0.05 \).

This method is straightforward when \( h, l \) and \( d_{50} \) are known; otherwise iterative computations are necessary.

Several deficiencies are common to all roughness-predictors:

- the effect of the water temperature on the bed configuration is not properly taken into account;
- the effect of the channel pattern on the overall roughness is not explicitly taken into account;
- the effect of the sediment discharge on the overall roughness is not explicitly taken into account.

Analysis of river data shows that the slope of a particular river does not change substantially for varying discharges. The slope is mainly imposed by the local bed slope. Consequently, methods which are exclusively based on slope as an input parameter are inherently inferior to methods which are based on mean velocity as input parameter.

Guidelines that can be provided are:

- use several different predictors;
- select and analyse data of similar streams (depth, velocity, width, bed material size, slope, temperature).

**6.2.5 Comparison of methods**

The methods of Engelund-Hansen, White et al and Van Rijn were compared for a large amount of flume and field data (Van Rijn, 1984b). The method of Van Rijn was based on the prediction of the dune height and length. The effective roughness was determined from Eqs. (6.2.5), (6.2.12) and (6.2.16) neglecting the form roughness reduction parameter (\( \gamma_d = 1 \)). Ripples were also neglected (\( \Delta_t = 0 \)).

To evaluate the accuracy of the three prediction methods, the percentage of the predicted C-values in the following error ranges was determined: \( C_{\text{measured}} \pm 10\% \), \( C_{\text{measured}} \pm 20\% \) and \( C_{\text{measured}} \pm 30\% \).
The results are presented in the following Table 6.2.

<table>
<thead>
<tr>
<th>Flume data (758)</th>
<th>Percentage of predicted C-values within</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{\text{measured}} \pm 10%$</td>
</tr>
<tr>
<td>1. Engelund-Hansen</td>
<td>37%</td>
</tr>
<tr>
<td>2. Van Rijn</td>
<td>34%</td>
</tr>
<tr>
<td>3. White et al</td>
<td>33%</td>
</tr>
<tr>
<td>Field data (786)</td>
<td>43%</td>
</tr>
<tr>
<td>1. Van Rijn</td>
<td>33%</td>
</tr>
<tr>
<td>2. White et al</td>
<td>25%</td>
</tr>
</tbody>
</table>

**Table 6.2 Comparison of predicted Chézy-coefficients**

For flume conditions the method of Engelund-Hansen produces the best results, whereas for field conditions the method of Van Rijn is superior.

Particularly, the method of Engelund-Hansen produces rather poor results for field conditions, probably because only flume data were used for calibration. For field data the method of Engelund-Hansen and White et al yield values which are, on the average, too large.

### 6.2.6 Examples and problems

1. A wide channel with a depth $h = 8 \text{ m}$ has a bed covered with dunes. Ripples are superimposed on the dunes. The dune dimensions are $\Delta_d = 1 \text{ m}$, $\lambda_d = 50 \text{ m}$. The ripple dimensions are $\Delta_r = 0.2 \text{ m}$, $\lambda_r = 3 \text{ m}$. The bed material characteristics are $d_{50} = 300 \mu\text{m}$, $d_{90} = 500 \mu\text{m}$.

   What is the effective bed roughness and the Chézy-coefficient?

   **Solution:**

   Grain roughness (lower regime): $k_s' = 3 \, d_{90} = 0.0015 \text{ m}$

   Ripple form roughness (take $\gamma_r = 0.7$): $k_{s,r}'' = 20 \, \gamma_r \, \Delta_r / \lambda_r$

   $= 20 \, 0.7 \, 0.2 / (0.2/3) = 0.187 \text{ m}$

   Dune form roughness (take $\gamma_d = 0.7$): $k_{s,d}'' = 1.1 \, \gamma_d \, \Delta_d(1 - e^{-25 \Delta_d / \lambda_d})$

   $= 1.1 \, 0.7 \, 1 \, (1 - e^{-0.5}) = 0.3 \text{ m}$

   Effective bed roughness: $k_{s,c} = k_s' + k_{s,r}'' + k_{s,d}'' = 0.49 \text{ m}$

   Chézy-coefficient: $C = 18 \, \log(12h / k_{s,c}) = 41.3 \text{ m}^{1/2}/\text{s}$

2. A wide channel has a depth of $h = 5 \text{ m}$. The depth-averaged velocity is $\bar{u} = 1 \text{ m/s}$. The bed material characteristics are $d_{50} = 500 \mu\text{m}$, $d_{90} = 1000 \mu\text{m}$.

   Other parameters are: $\rho = 1000 \text{ kg/m}^3$, $\rho_s = 2650 \text{ kg/m}^3$, $v = 1 \times 10^{-6} \text{ m}^2/\text{s}$.

6.16
What is the Chézy-coefficient and surface slope according to the method of Engelund and Hansen and the method of Van Rijn?

Solution:

**Method of Engelund-Hansen**

Guess $h' $ : $h' = 0.33 \ h = 1.67 \ m$

Compute $u'_{s,c}$ and $\theta'$

: $u'_{s,c} = \frac{\bar{u}}{2.5 \ \ln(12h'/2.5 \ d_{50})} = 0.0413 \ \text{m/s}$

: $\theta' = \frac{(u'_{s,c})^2}{(s \ 1)g \ d_{50}} = 0.21$

Compute $\theta$ from Eq. (6.2.26)

: $\theta = 0.614$

Compute $h' $:

: $h' = (\theta'/\theta)h = 1.7 \ m$

within 2% of initial guess, which is sufficiently accurate

Compute $C$ from Eq. (6.2.24)

: $C = 44.3 \ \text{m}^4/\text{s}$

Compute surface slope

: $I = (\bar{u})^2/(C^2h) = 1.02 \ 10^4$

**Method of Van Rijn**

Compute grain-related Chézy

: $C' = 18 \ \log(12h/3d_{90}) = 77.4 \ \text{m}^4/\text{s}$

Compute grain-shear stress

: $\tau'_{b,c} = \rho g(\bar{u}/C')^2 = 1.63 \ \text{N/m}^2$

Determine critical shear-stress

: $\tau_{b,cr} = 0.27 \ \text{N/m}^2$ (Fig. 4.1.5)

Compute $T$-parameter

: $T = (\tau'_{b} - \tau_{b,cr})/\tau_{b,cr} = 5$

Compute $D_*$-parameter

: $D_* = d_{50}(s-1)g/(\nu^2)^{1/3} = 12.6$

Type of bed forms

: dunes

Compute dune dimensions

: $\Delta_d = 0.11 \ h(d_{90}/h)^{0.3}(1-e^{-0.5 \ T})(25-T) = 0.64 \ m$

: $\lambda_d = 7.3 \ h = 36.5 \ m$

Compute effective bed roughness

: $k_{ac} = 3 \ d_{90} + 1.1 \ \gamma_d \Delta_d (1-e^{-25 \ \Delta_d \lambda_d})$

: $= 0.178 \ m \ (\gamma_d = 0.7)$

Compute Chézy-coefficient

: $C = 18 \ \log(12h/k_{ac}) = 45.5 \ \text{m}^4/\text{s}$

Compute surface slope

: $I = 9.7 \ 10^5$

Both methods yield similar results.
3. A wide channel has a water depth $h = 3$ m. The depth-averaged flow velocity is $\bar{u} = 1$ m/s. The bed material characteristics are $d_{50} = 300$ µm, $d_{90} = 500$ µm. The fluid density is $\rho = 1000$ kg/m$^3$, the sediment density is $\rho_s = 2650$ kg/m$^3$. The kinematic viscosity is $\nu = 1 \times 10^{-6}$ m$^2$/s.

What type of bed forms are present?
What are the bed form dimensions?
What is the effective bed roughness?

Solution: megaripples on dunes
$\Delta_e = 0.08$ m, $\lambda_e = 1.5$ m
$\Delta_d = 0.35$ m, $\lambda_d = 21.9$ m
$k_e = 0.15$ m, $C_e = 42.8$ m$^{1/4}$/s

4. A wide channel has a water depth of $h = 2$ m. The depth-averaged velocity is $\bar{u} = 0.7$ m/s. The bed material characteristics are $d_{50} = 200$ µm, $d_{90} = 400$ µm, $\rho = 1000$ kg/m$^3$, $\rho_s = 2650$ kg/m$^3$, $\nu = 1 \times 10^{-6}$ m$^2$/s.

What is the Chézy-coefficient according to the method of Engelund-Hansen and Van Rijn?

Solution: Engelund-Hansen : $C = 45.6$ m$^{1/4}$/s
Van Rijn : $C = 39.5$ m$^{1/4}$/s

5. A wide channel has a water depth of $h = 2$ m. The water surface slope is $I = 2 \times 10^{-4}$. The bed material characteristics are $d_{35} = 400$ µm, $d_{50} = 500$ µm, $d_{90} = 1000$ µm. Other parameters are $\rho = 1000$ kg/m$^3$, $\rho_s = 2650$ kg/m$^3$, $\nu = 1 \times 10^{-6}$ m$^2$/s.

What is the Chézy-coefficient and the depth-averaged velocity according to the methods of Engelund-Hansen, White et al and Smith-McLean?

Solution: Engelund-Hansen : $C = 38.4$ m$^{1/4}$/s, $\bar{u} = 0.77$ m/s
White et al : $C = 39.5$ m$^{1/4}$/s, $\bar{u} = 0.79$ m/s
Smith-McLean : $C = 39.5$ m$^{1/4}$/s, $\bar{u} = 0.79$ m/s

6. A wide channel has a water depth of $h = 2$ m. The flow velocity is 3 m/s over a plane bed in the upper regime. The bed material characteristics are $d_{50} = 400$ µm, $d_{90} = 1000$ µm. Other data are: $\rho = 1000$ kg/m$^3$, $\rho_s = 2600$ kg/m$^3$, $\nu = 1 \times 10^{-6}$ m$^2$/s.

What is the effective bed roughness according to Van Rijn and Wilson?

Solution: Van Rijn : $k_{e}' = 0.012$ m
Wilson : $k_{e,\infty}' = 0.007$ m

6.3 Wave-related bed roughness

6.3.1 Available method

The effective wave-related bed roughness is expressed as:

$$k_{e,w} = k_{e,w}' + k_{e,w}''$$

(6.3.1)
in which:

\[ k_{w}^{p} = \text{wave-related grain roughness height} \]
\[ k_{s,w} = \text{wave-related form roughness height} \]

The wave-related friction factor \( f_{w} \) for rough oscillatory flow is expressed by

\[
f_{w} = \exp[-6 + 5.2(\delta/\lambda_{s})^{-0.19}], \quad \text{with} \quad f_{w,\text{max}} = 0.3
\]  \hspace{1cm} (6.3.2)

Friction factors for the smooth and transition regime are presented in Chapter 2.

**Grain roughness**

Grain roughness is dominant when the bed is plane or when the peak orbital excursion at the bed is smaller than the ripple length \( \delta < \lambda_{s} \).

Based on laboratory experiments with a perfectly flat \textit{non-movable} bed \( d_{50} = 500, 2200, 12200 \) and \( 46000 \mu \text{m} \), Kamphuis (1975) proposed:

\[
k'_{s,w} = 2 \ d_{50}
\]

(6.3.3)

In natural conditions this value may be somewhat larger, because a perfectly arranged flat bed will not be present.

Grant and Madsen (1982) re-analyzed the movable bed data of Carstens et al (1969) and concluded that the grain roughness for a mobile bed should be related to the thickness of the moving bed-load layer. Based on data analysis, Grant and Madsen have proposed a function that yields:

\[
k'_{s,w} = \begin{cases} 10 \ d_{50} & \text{for } \theta' = 2 \ \theta_{cr} \\ 350 \ d_{50} & \text{for } \theta' = 20 \ \theta_{cr} \end{cases}
\]

in which:

\[ \theta' = \text{effective mobility parameter (based on } k'_{s,w} = d_{50}) \]
\[ \theta_{cr} = \text{critical mobility parameter} \]

The method of Grant and Madsen yields relatively large \( k'_{s,w} \)-values, which may be caused by the fact that ripple regime data have been analyzed by Grant and Madsen. This introduces the problem of estimating the ripple form roughness. Small errors in this latter parameter may lead to relatively large errors in the grain roughness.

High grain roughness values were also found by Raudkivi (1988) using the same data set of Carstens et al (1969). Raudkivi proposes an expression which yields \( k'_{s,w} = 0.16 \) m for \( \delta_{s} = 1 \) m/s.

According to Van Rijn, the effective grain roughness of a sheet flow bed is of the order of the sheet flow layer thickness or the boundary layer thickness \( k'_{s,w} \sim \delta_{w} \). The sheet flow layer is a high-concentration layer of bed material particles. The moving particles will reduce the near-bed velocities due to the presence of drag forces (particle velocity smaller than fluid velocity) which can be interpreted as a shear effect. The apparent or effective roughness of the sheet flow layer will be of the order of the thickness of the sheet flow layer. A good estimate for this is the thickness of the wave boundary layer. Thus \( k'_{s,w} \sim \delta_{w} \). An indication for this can be obtained from the experimental results of Horikawa et al (1982), who
measured the boundary layer thickness in a sheet flow experiment ($\delta_w = 0.02 \text{ m}$ for $\bar{U}_\delta = 1.27 \text{ m/s}$, $T = 3.6 \text{ s}$, $d_{so} = 200 \mu\text{m}$, $d_{sw} = 300 \mu\text{m}$). Using the boundary layer thickness equation: $\delta = \frac{\bar{U}_\delta}{(\bar{U}_\delta/k'_s)^{0.25}}$, it follows that $k'_s = 0.015 m$. This latter value is somewhat smaller than the observed boundary layer thickness of $\delta_w = 0.02 \text{ m}$. Since $d_{so} = 300 \mu\text{m}$, it follows that $k'_s = 50 d_{so}$ for the Horikawa-experiment.

Wilson (1989) found $k'_s$-values in the range of 1 to 40 $d_{so}$ for $\theta = \bar{u}/((\rho - \rho_s) g d_{so})$ in the range of 0.8 to 8. He proposed:

$$k'_s = 5 \theta d_{so} \quad (6.3.4)$$

Van Rijn (1989) proposed the following values (see Eq. (6.2.12)):

$$k'_s = \begin{cases} 3 d_{so} & \text{in the ripple regime} \\ 3.0 d_{so} & \text{in the sheet flow regime} \end{cases} \quad (6.3.5a)$$

in which:

$$\theta = \frac{(\bar{u}_{sw})^2}{(s-1)g d_{so}} = \text{peak mobility parameter}$$

$$\bar{u}_{sw} = \left[ \frac{1}{2} f_w \left(\bar{U}_\delta\right)^{0.5} \right] = \text{wave-related peak bed-shear velocity}$$

$$f_w = \exp \left( -6 + 5.2 \left( \frac{\bar{U}_\delta}{k'_s + 3.3 \nu_m/\bar{u}_{sw}} \right)^{-0.19} \right) = \text{friction factor transition regime}$$

$$\bar{U}_\delta = \text{near-bed peak orbital velocity}$$

$$\hat{A}_\delta = \text{near-bed peak orbital excursion}$$

$$\nu_m = \text{kinematic viscosity of fluid sediment mixture in near bed region} \left( \nu_m \approx 10^{-5} \text{ m}^2/\text{s} \right)$$

The friction factor for the transition regime can also be expressed as:

$$f_w = \exp \left( -6 + 5.2 \left( \frac{\bar{U}_\delta}{k'_s + 4.7 \nu_m/\bar{u}_{sw}^{0.5}} \right)^{-0.19} \right) \quad (6.3.6)$$

Equation (6.3.5b) is solved by iteration, see Example 1 of section 6.3.2. Equation (6.3.5) yields a value in the range of 3 $d_{so}$ to 30 $d_{so}$ for $\theta = 1$ to 10.

A perfectly plane bed will not exist in natural conditions. Small irregularities will always be present. Therefore, it is recommended to use a minimum value of $k'_s = 0.01 \text{ m}$ for the sheet flow regime in natural conditions.

**Form roughness**

Ripples are the dominant bed forms generated by oscillatory flow. Ripples may be present on a horizontal bed or superimposed on large sand waves. In the latter case the sand waves have no friction effect on the water waves, because the water waves experience the sand
waves as a gradual bottom topography. When the near-bed orbital excursion is larger than the ripple length, the ripples are the dominant roughness elements for the oscillatory motion. Sayao (1982) analyzed the experimental ripple data of Carstens et al (1969) and Lofquist (1980) and found $k_{sw}''$-values in the range of 2 to 4 times the ripple height. No influence of the ripple steepness was noticed ($0.1 < \Delta_r/\lambda_r < 0.25$). Thus,

$$k_{sw}'' = 3 \Delta_r \text{ for } 2 \leq \frac{\Delta_r}{\lambda_r} \leq 5$$  \hfill (6.3.7)

Sayao (1982) allowed $\Delta_r/\lambda_r \geq 5$ because eddy generation is dominant in this range. For $\Delta_r/\lambda_r < 2$, Equation (6.3.7) becomes less reliable because of the reduced importance of the eddy generation mechanism. For $\Delta_r/\lambda_r < 1$ the flow along the bed forms will be non-separating (potential) flow mainly affected by grain roughness (Honji et al, 1980 and Davies, 1985). Van Rijn (1989) proposed (see Eq. (6.2.15)):

$$k_{sw}'' = 20 \gamma_r \Delta_r \left( \frac{\Delta_r}{\lambda_r} \right)$$  \hfill (6.3.8)

in which:
- $\Delta_r$ = ripple height
- $\lambda_r$ = ripple length
- $\gamma_r$ = ripple presence factor ($\gamma_r = 1$ for a ripple covered bed, $\gamma_r = 0.7$ for ripples superimposed on sand waves)

Equation (6.3.8) yields values in agreement with those of Sayao (1982) for a ripple steepness in the range of 0.1 to 0.2 (see also Figure 6.2.6).

Other expressions available in the Literature are:

- Swart (1976) : $k_{sw}'' = 25 \Delta_r \left( \frac{\Delta_r}{\lambda_r} \right)$ \hfill (6.3.9)
- Grant-Madsen (1984) : $k_{sw}'' = 28 \Delta_r \left( \frac{\Delta_r}{\lambda_r} \right)$ \hfill (6.3.10)
- Raudkivi (1988) : $k_{sw}'' = 16 \Delta_r \left( \frac{\Delta_r}{\lambda_r} \right)$ \hfill (6.3.11)

6.3.2 Examples and problems

1. The bed of a coastal sea consists of sand with $d_{so} = 300 \mu m$, $d_{so} = 500 \mu m$. The water depth is $h = 4m$. The significant wave height is $H_s = 1.6 m$. The peak wave period is $T_p = 8s$. Other parameters are $\rho = 1025 kg/m^3$, $\rho_g = 2650 kg/m^3$, $v = 1.10^{-9} m^2/s$.

What is the effective bed roughness according to the method of Van Rijn?

Solution:

Compute wave length

$$L = 48m$$

$$k = 2\pi/L = 0.131$$
Compute peak orbital velocity at bed: 
\[ \dot{U}_b = \frac{\pi H_s}{T \sinh(kh)} = 1.14 \text{ m/s} \]

Compute peak orbital excursion: 
\[ \dot{A}_b = \frac{T_p}{2\pi} \dot{U}_b = 1.45 \text{ m} \]

Compute mobility parameter: 
\[ \psi = \frac{(\dot{U}_b)^2}{(s-1)g d_{so}} = 280 \]

Determine type of bed forms: sheet flow

Estimate \( k'_{s,w} \) and \( \nu_m \) : 
\[ k'_{s,w} = 0.005 \text{ m} \]
\[ \nu_m = 1.10^{-5} \text{ m}^2/\text{s} \]

Compute friction factor, Eq. (6.3.6): 
\[ f_w = 0.009 \]

Compute \( \dot{u}_{*,w} \) and \( \theta \) : 
\[ \dot{u}_{*,w} = [0.5 f_w \dot{U}_b]^{20.5} = 0.0764 \text{ m/s} \]
\[ \theta = \frac{\dot{u}_{*,w}^2}{((s-1)g d_{so})} = 1.25 \]

Compute \( k'_{s,w} \) from Eq. (6.3.5b): 
\[ k'_{s,w} = 0.0019 \text{ m} \]

Repeat procedure: 
\[ k'_{s,w} = 0.0025 \text{ m} \]

2. Same problem as 1., but \( H_s = 2 \text{ m} \).

What is the effective bed roughness according to Wilson?

Solution: 
\[ k'_{s,w} = 0.0044 \text{ m} \]

3. The bed of a coastal sea consists of sand with \( d_{so} = 250 \mu\text{m} \) and \( d_{so} = 500 \mu\text{m} \). The water depth is \( h = 5 \text{ m} \). The significant wave height is \( H_s = 1 \text{ m} \). The peak wave period is \( T_p = 8 \text{ s} \). Other parameters are: \( \rho_s = 2650 \text{ kg/m}^3 \), \( \rho = 1025 \text{ kg/m}^3 \), \( v = 1 \times 10^{-6} \text{ m}^2/\text{s} \).

What type of bed forms are present?
What are the bed form dimensions according to the method of Van Rijn?
What is the effective bed roughness according to the methods of Sayao, Swart, Grant-Madsen, Raudkivi and Van Rijn?

Solution: Bed forms = ripples

Dimensions: \( \Delta_z = 0.016 \text{ m}, \lambda_z = 0.3 \text{ m} \)
Bed roughness: 
\[ k'_{s,w} = 0.05 \text{ m}, \text{ Sayao} \]
\[ k'_{s,w} = 0.023 \text{ m}, \text{ Swart} \]
\[ k'_{s,w} = 0.025 \text{ m}, \text{ Grand-Madsen} \]
\[ k'_{s,w} = 0.015 \text{ m}, \text{ Raudkivi} \]
6.4 Bed roughness in combined currents and waves

6.4.1 Available method

The most important bed form regimes are:
• ripples in case of weak (tidal) currents and low waves,
• sand waves with ripples in case of (tidal) currents and low waves,
• plane bed with sheet flow in case of strong (tidal) currents and high waves (surf zone),
• sand waves with sheet flow in case of strong (tidal) currents and high waves (outside surf zone).

The $k_s$-parameter can be determined by summation of grain roughness and form roughness ($k_s = k_s' + k_s''$).

**Grain roughness**

Grain roughness is dominant for both the wave-related and current-related friction when the bed is plane.
When bed forms are present and the peak orbital excursion at the bed is smaller than the bed form length ($\Delta_p < \lambda_r$), the grain roughness is also dominant for the wave-related friction. In that case the bed forms act as topographic features for the waves.
Equation (6.3.5) is assumed to be valid for the wave motion. Equation (6.2.12) is assumed to be valid for the current motion.

**Form roughness**

When the bed is covered with ripples, the ripple roughness is dominant for the current-related friction. Ripple roughness is also dominant for the wave-related friction when the peak value of the orbital excursion at the bed is larger than the ripple length ($\Delta_p > \lambda_r$). Nap and Van Kampen (1988) performed flume experiments with combined currents and waves over a sediment bed of 100 μm. Ripple type bed forms (0.1 < $\Delta_r/\lambda_r$ < 0.2) were generated during the experiments. The effective roughness of the ripples was studied by generating a current (without waves) over the ripples and measuring the vertical distribution of the velocities and the water surface slopes. Based on the measured surface slope it was found that $k_{sc}'' = 3\Delta_r$, whereas analysis of the velocity profile data resulted in $k_{sc}'' = 7\Delta_r$. The former result probably is the most realistic result because it is based on the measured surface slope which is an integral parameter over the flume length, whereas velocity profile data represent local data at a particular cross-section.

A clear influence of the ripple steepness (in the range of 0.1 to 0.2) was not observed. Havinga (1992) found $k_{sc}''$-values in the range of 0.5 to 2$\Delta_r$.

Herein, it is proposed to use Eq. (6.2.15) to determine the ripple form roughness.

Thus:

$$k_{sc}'' = 20 \gamma_r \Delta_r \left( \frac{\Delta_r}{\lambda_r} \right)$$  \hspace{1cm} (6.4.1)

When sand waves with or without (mega or mini) ripples are present, the large-scale sand waves act as topographic features for the wave motion because the sand waves have a length ($\lambda$) much larger than the orbital excursion at the bed ($\Delta_p$). Thus, the wave-related friction factor is not determined by the large-scale sand wave dimensions, but by the small-scale ripples (if present) on the back of the sand waves. When the orbital velocities near the bed
become relatively large (in storm periods), the mega and mini ripples will be washed out resulting in a plane sloping bed with sheet flow on the back of the sand waves. In this latter case the wave-related friction factor is determined by grain roughness (Eq. 6.3.5b). The current related friction factor is determined by a combination of grain roughness and ripple-form roughness (if present). The sand waves do not contribute to the current-related roughness, because they act as topographic features for the current.

**Apparent roughness**

Basically, the \( k_c \) parameter expresses the physical roughness height of the bed forms. The overall roughness experienced by the current (in the presence of waves) may be considerably larger than the physical bed roughness \( (k_{s,c}) \), see Eq. (2.4.29). This roughness increase can be represented as an apparent roughness \( (k_a) \), which is related to the effect of the wave boundary layer on the current. The apparent roughness strongly depends on the relative strength of the wave and current motion. In case of a decreasing wave height and a constant current velocity the apparent roughness will approach to the effective bed roughness \( k_a \rightarrow k_{s,c} \) for \( \bar{U}_w \rightarrow 0 \). For an increasing wave height the apparent roughness will be considerably larger than the physical bed roughness (factor 10 to 100).

Lambrakos et al (1988) performed velocity measurements above a flat (rough) sea bed in the Strait of Juan de Fuca near Vancouver Island (USA). The water depth was 18 m. The bed was almost flat with rock and gravel sediments. Velocity measurements were made at 0.24, 0.61, 1.22, 1.83 and 3.6 m above the bed.

Analysis of the velocity data showed a \( k_{s,c} \)-value of about 0.001 to 0.005 m for current alone conditions \( (\bar{V} = 0.1 \text{ to } 0.5 \text{ m/s}) \). For combined current and wave conditions, the apparent roughness \( k_a \) was found to be 0.1 to 0.5 m for \( u_{rms} \bar{V} = 2 \) which means an increase of a factor 100. \( u_{rms} = \text{root mean square value of the oscillatory velocity} \).

**6.4.2 Examples and problems**

1. The water depth of a coastal sea is \( h = 20 \) m. The peak velocities of the tidal flood and ebb currents are 0.5 m/s. The significant wave height is \( H_s = 3 \) m. The peak wave period is \( T_p = 10 \) s. The angle between the wave and current direction is 90°. The bed is covered with large symmetrical sand waves with \( \Delta_{sw} = 3 \) m, \( \lambda_{sw} = 200 \) m. The crests of the sand waves are perpendicular to the tidal velocities. The sand waves are fully covered with three-dimensional wave-ripples, \( \Delta_r = 0.02 \) m and \( \lambda_r = 0.3 \) m. The bed material characteristics are \( d_{50} = 300 \mu \text{m} \) and \( d_{50} = 1000 \mu \text{m} \).

What is the physical current-related bed roughness?
What is the physical wave-related bed roughness?
What is the apparent roughness experienced by the current?

Solution:

Grain roughness (lower regime) \( \quad k'_{s,c} = 3 \ d_{90} = 0.003 \) m

Form roughness of wave ripples \( \quad k''_{s,c} = k''_{s,w} = 20 \ \gamma, \Delta_r (\Delta_r / \lambda_r) \)
\[ \quad = 0.027 \) m

Form roughness of sand waves \( \quad k''_{s,c} = 0 \)

Physical current-related bed roughness \( \quad k_{s,c} = 0.003 + 0.027 = 0.03 \) m

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Physical wave-related bed roughness : \( k_{s,w} = 0.003 + 0.027 = 0.03 \text{ m} \)

Compute peak orbital velocity at bed : \( \hat{U}_\delta = \frac{\pi H_s}{T_p \sinh(kh)} = 0.76 \text{ m/s} \)

Compute apparent roughness, Eq. (2.4.29) : \( \kappa = k_{s,c} e^{1.6 \hat{U}_\delta} = 0.34 \text{ m} \)

\( (\gamma = 1.6, \phi = 90^\circ) \)

2. Same problem as 1.

The bed is covered with large symmetrical sand waves with \( \Delta_{sw} = 3 \text{ m}, \lambda_{sw} = 200 \text{ m} \). The sand waves are covered with mega-ripples, \( \Delta_{mr} = 0.5 \text{ m}, \lambda_{mr} = 15 \text{ m} \). Ten mega-ripples are present on each sand wave. The width of the mega-ripples is 5 m. The crests of the sand waves and the mega-ripples are perpendicular to the tidal velocities. The mega-ripples are fully covered with three-dimensional wave-induced ripples, \( \Delta_r = 0.02 \text{ m}, \lambda_r = 0.3 \text{ m} \).

What is the physical current-related bed roughness?
What is the physical wave-related bed roughness?
What is the apparent roughness experienced by the current?

Solution: \( k_{s,c} = 0.28 \text{ m} \)
\( k_{s,w} = 0.03 \text{ m} \) \( (\hat{A}_\delta < 5 \text{ m}) \)
\( k_\alpha = 3.2 \text{ m} \)
REFERENCES


REFERENCES (continued)


REFERENCES (continued)


6.28
7. BED MATERIAL SUSPENSION AND TRANSPORT IN STEADY UNIFORM CURRENTS

7.1 Introduction

The transport of bed material particles by a flow of water can be in the form of bed-load and suspended load, depending on the size of the bed material particles and the flow conditions. The suspended load may also contain some wash load (usually, particles smaller than 50 μm), which is generally defined as that portion of the suspended load which is governed by the upstream supply rate and not by the composition and properties of the bed material. The wash load is mainly determined by land surface erosion (rainfall, no vegetation) and not by channel bed erosion. Although in natural conditions there is no sharp division between the bed-load transport and the suspended load transport, it is necessary to define a layer with bed-load transport for mathematical representation. The wash load will not be discussed in this chapter. The sediment transport in a steady uniform current is assumed to be equal to the transport capacity defined as the quantity of sediment that can be carried by the flow without net erosion or deposition, given sufficient availability of bed material (no armour layer).

Usually, three modes of particle motion are distinguished: (1) rolling and sliding motion or both; (2) saltation motion; and (3) suspended particle motion.

When the value of the bed-shear velocity just exceeds the critical value for initiation of motion, the particles will be rolling and sliding or both, in continuous contact with the bed. For increasing values of the bed-shear velocity, the particles will be moving along the bed by more or less regular jumps, which are called saltations. When the value of the bed-shear velocity exceeds the fall velocity of the particles, the sediment particles can be lifted to a level at which the upward turbulent forces will be comparable with or of higher order than the submerged weight of the particles and as result the particles may go in suspension.

Various types of formulae are available to predict the bed-load and the suspended load transport. The formulae can be divided in five main groups as defined by the relevant hydraulic parameter:
- fluid velocity, 
  \[ q_b = (\bar{u} - \bar{u}_s)^m \]
- bed shear stress, 
  \[ q_b = (\tau - \tau_s)^n \]
- probabilistic particle movement
- bed form celerity
- energetics (stream power), 
  \[ q_b = \tau \bar{u} \]

The n-coefficient is of the order of 1.5. The m-coefficient is in the range of 3 to 5.

Flume and field data show that the sand transport rate is most strongly related to the depth-averaged velocity, see Figure 7.1.1. The power of velocity is approximately 3.2 for this location in the Rio Grande river.

In Chapter 6 it has been shown that a certain value of the overall bed-shear stress (τ) can occur at two values of the mean velocity (\(\bar{u}\)) depending on the presence of bed forms or a flat bed. This means that the sand transport rate is not a unique function of the bed-shear stress. Sand transport formulae given in terms of the overall bed-shear stress are therefore less reliable.

In Section 7.2 the motion and transport of the bed-load particles are presented.
In Section 7.3 the suspended-load transport is described.
7.2 Bed-load transport

7.2.1 Introduction and definitions

Usually, the transport of particles by rolling, sliding and saltating is called the bed-load transport. For example, Baguold (1956) defines the bed-load transport as that in which the successive contacts of the particles with the bed are strictly limited by the effect of gravity, while the suspended-load transport is defined as that in which the excess weight of the particles is supported by random successions of upward impulses imported by turbulent eddies. Einstein (1950), however, has a somewhat different approach. Einstein defines the bed-load transport as the transport of sediment particles in a thin layer of 2 particle diameters thick just above the bed by sliding, rolling and sometimes by making jumps with a longitudinal distance of a few particle diameters. The bed layer is considered as a layer in which the mixing due to the turbulence is so small that it cannot influence the sediment particles, and therefore suspension of particles is impossible in the bed-load layer. Further, Einstein assumes that the average distance travelled by any bed-load particle (as a series of successive movements) is a constant distance of 100 particle diameters, independent of the flow condition, the transport rate and the bed composition. In the view of Einstein, the saltating particles belong to the suspension mode of transport, because the jump lengths of saltating particles are considerably larger than a few grain diameters.

Many formulae to predict the bed-load transport rate are described in the Literature. The earliest formula is that of Du Boys in 1879, who assumed that the sediment particles are moving along the bottom in layers of progressively decreasing velocities in vertical downward direction.

Figure 7.1.1 Bed sediment transport in Rio Grande river near Bernalillo (Nordin, 1964)
The first empirical formula was presented by Meyer-Peter and Mueller (1948). They performed flume experiments with uniform particles and with particle mixtures. Based on data analysis, a relatively simple formula was obtained, which is frequently used.

Kalinske (1947) and Einstein (1950) introduced statistical methods to represent the turbulent behaviour of the flow. Kalinske assumed a normal distribution of the instantaneous fluid velocity at grain level. Einstein gave a detailed but complicated statistical description of the particle motion in which the exchange probability of a particle is related to the hydrodynamic lift force and particle weight. Einstein proposed the $d_{50}$ as the effective diameter for particle mixtures and the $d_{85}$ as the effective diameter for grain roughness.

Frijlink (1952) had a very practical approach and made a simple fit of the formulae of Meyer-Peter-Mueller and that of Einstein.

Bagnold (1966) introduced an energy concept and related the sediment transport rate to the work done by the fluid.

Van Rijn (1984) solved the equations of motions of an individual bed-load particle and computed the saltation characteristics and the particle velocity as a function of the flow conditions and the particle diameter for plane bed conditions.

Basically, the bed load can be defined as the product of particle concentration, particle velocity and layer thickness, as follows:

$$q_b = c_b \ u_b \ \delta_b \quad (7.2.1)$$

in which:

$q_b$ = volumetric bed-load transport rate (m$^3$/s)
$c_b$ = volumetric concentration (-)
$u_b$ = particle velocity (m/s)
$\delta_b$ = thickness of bed load layer

The bed-load transport can also be defined as the product of the number of moving particles per unit area, the particle volume and the particle velocity, as follows:

$$q_b = N_b \ V_b \ u_b \quad (7.2.2)$$

in which:

$N_b$ = number of moving particle per unit area (m$^{-2}$)
$V_b$ = particle volume (m$^3$)
$u_b$ = particle velocity (m/s)

Defining the particle velocity as the ratio of the saltation length and the saltation period $u_b = \lambda/T$, it follows that:

$$q_b = N_b \ V_b \ \lambda/T = E \ \lambda = D \ \lambda \quad (7.2.3)$$

in which:

$E = D = N_b \ V_b/T =$ eroded or deposited volume of particles per unit area and time (m/s).
7.2.2 Characteristics of moving bed-load transport

1. Saltation characteristics

A shown by Bagnold (1954), the most typical motion of the bed-load particles is the saltation motion (or jumping motion). From detailed experiments of Francis (1973) and Abbott and Francis (1977), the general characteristics of particle saltations can be inferred. The saltation mode of transport is confined to a layer with a maximum thickness of about 10 particle diameters, in which the particle motion is dominated by gravitational forces, although the particle motion may be initiated by instantaneous turbulent impulses during upward bursts of fluid or just by the effect of shear in the sense that a body in sheared flow experiences a lift force due to the velocity gradient near the bed. The particles receive their momentum directly from the flow pressure and viscous skin friction. On the rising part of the trajectory, both the vertical component of the fluid drag force and the gravitational force are directed downwards. During the falling part of the trajectory, the vertical component of the fluid drag force opposes the gravitational force. The lift force is always directed upwards as long as the particle velocity lags behind the fluid velocity.

When a particle strikes the bed, it may either impact into the surface or rebound off the surface particles. During the impact of a particle with the bed, most of its momentum is dissipated by the particles of the bed in a sequence of more or less horizontal impulses which may initiate the rolling mode of transport known as surface creep.

A particle will alternate between periods of successive saltations interspersed with periods of rest on the bed. Saltation may cease when a particle is captured in an occasional bed depression.

2. Equations of motion

The trajectory of a saltating grain can be described by solving the equations of motion for a plane bed.

The models of Sekine and Kikkawa (1984), Anderson and Haff (1988) and Wiberg and Smith (1987, 1989) are based on a Lagrangian description of particle trajectory, which is obtained from a consideration of the dominant fluid forces acting on a grain. The effect of turbulence is neglected, so that the particle trajectory is deterministic once the initial conditions for each saltation are specified. These models all resort to a stochastic description of collision with the bed and subsequent rebound.

The model of Sekine and Kikkawa (1992) is also deterministic in the determination of the particle trajectory, but probabilistic in terms of bed collision which arises from the random structure of the bed.

Van Rijn (1984) used a deterministic model based on drag and lift forces for given initial particle velocities. Herein, the approach of Van Rijn (1984) is presented.

The forces acting on a saltating particle are a downward force due to its submerged weight \( \mathbf{F}_D \) and hydrodynamic fluid forces, which can be resolved into a lift force \( \mathbf{F}_L \), a drag force \( \mathbf{F}_D \), as shown in Fig. 7.2.1. The direction of the drag force is opposite to the direction of the particle velocity \( \mathbf{v}_p \) relative to the flow, while the lift force component is in the normal direction. Assuming that the particles are spherical and of uniform density, and that the forces due to fluid accelerations are of a second order, the equations of motions can be represented as:
\[
\begin{align*}
\mathbf{m} \ddot{\mathbf{x}} - \mathbf{F}_L \left( \frac{\ddot{z}}{v_t} \right) - \mathbf{F}_D \left( \frac{\ddot{u} - \dot{x}}{v_t} \right) &= 0 \\
\mathbf{m} \ddot{\mathbf{z}} - \mathbf{F}_L \left( \frac{\ddot{u} - \dot{x}}{v_t} \right) + \mathbf{F}_D \left( \frac{\ddot{z}}{v_t} \right) + \mathbf{F}_G &= 0
\end{align*}
\]

(7.2.4)

(7.2.5)

in which:

\( \mathbf{m} \) = particle mass and added fluid mass

\( v_t = [(u-\dot{x})^2 + (\dot{z})^2]^{0.5} \) = particle velocity relative to the flow

\( u \) = local flow velocity

\( \dot{x} \) and \( \dot{z} \) = longitudinal and vertical particle velocities

\( \ddot{x} \) and \( \ddot{z} \) = longitudinal and vertical particle accelerations

The total mass of the sphere can be represented by:

\[
\mathbf{m} = \frac{1}{6} (\rho_s + \alpha_m \rho) \pi d^3
\]

(7.2.6)

in which:

\( \alpha_m \) = added mass coefficient

\( d \) = particle diameter

Assuming potential flow, the added mass of a perfect sphere is exactly equal to half the mass of the fluid displaced by the sphere. When the flow is separated from the solid sphere, the added mass may be different.

The submerged particle weight is given by:

\[
\mathbf{F}_G = \frac{1}{6} \pi (\rho_s - \rho) g d^3
\]

(7.2.7)

The drag force, which is caused by pressure and viscous skin friction forces, can be expressed as:

\[
\mathbf{F}_D = \frac{1}{2} \mathbf{C}_D \rho A v_t^2
\]

(7.2.8)

in which:

\( \mathbf{C}_D \) = drag coefficient (see Morsi and Alexander, 1972)

\( A = \frac{1}{4} \pi d^2 \) = cross-sectional area of the sphere

The lift force in a shear flow is caused by the velocity gradient present in the flow (shear effect) and by the spinning motion of the particle (Magnus effect). For a sphere moving in a viscous flow (small Reynolds numbers), Saffman (1965) derived the following expression:

\[
\mathbf{F}_L(\text{shear}) = \alpha_L \rho v^{0.5} d^2 v_t \left( \frac{du}{dz} \right)^{0.5}
\]

(7.2.9)

in which:

\( \alpha_L \) = lift coefficient (= 1.6 for viscous flow)

\( du/dz \) = velocity gradient
Another expression of the lift force was given by Taylor (1917), who studied the motion of a solid body in a rotating flow system. The flow system and the solid body are defined to have the same angular velocity. This type of motion is not identical to that of a solid body in a uniform flow. The expression of Taylor reads as:

\[ F_L = C_L \rho V v_r \frac{du}{dz} \]  

(7.2.10)
in which:
\( V \) = particle volume
\( C_L \) = lift coefficient (\( \approx 0.5 \))

The lift force due to the spinning motion in a viscous flow was determined by Rubinow and Keller (1961):

\[ F_L(\text{spin}) = \alpha_L \rho d^3 v_r \omega \]  

(7.2.11)
in which:
\( \alpha_L \) = lift coefficient (\( = 0.4 \) for viscous flow)
\( \omega \) = angular velocity of the particle

Saffman (1965) showed theoretically that for a viscous flow the lift force due to the particle rotation (spin) is less by an order of magnitude than that due to the shear effect and may therefore be neglected.

To solve Eqs. (7.2.4) and (7.2.5), the following boundary conditions must be known:
• bed level position,
• initial velocities,
• flow velocity distribution.

The bed level is assumed to be at a distance of 0.25d below the top of the particles, as shown in Fig. 7.2.2. In its initial position a particle is supposed to be resting on a bed surface of close-packed identical particles. The most stable position will be that of a particle resting above one of the interstices formed by the top layer of the particles of the bed surface, which yields an initial position of about 0.6d above the bed level. It is evident that this schematization can not represent the movements of all the bed-load particles. The particles in the compacted bed can only be moved by the highest fluid velocities of the spectrum resulting in somewhat larger saltations. However, the majority of the particles is supposed to be moving over the surface of close-packed particles.

Measurements of particles in a water stream by Abbott and Francis (1977) indicate an average initial longitudinal and vertical velocity of approximately 2u* c. White and Schultz (1977) analyzed high-speed motion-picture films of saltating particles in air and observed a lift-off velocity varying from \( u_{c} \) to 2.5\( u_{c} \) and a lift-off angle varying from 30° to 70°.

The flow velocity distribution over the depth is described by (see Fig. 7.2.2):

\[ u(z) = \frac{u_{c}}{\kappa} \ln \left( \frac{z}{z_{o}} \right) \]  

(7.2.12)
in which:
\[ u_{*c} = \text{bed-shear velocity} \]
\[ \kappa = \text{constant of Von Karman} \ (\ k = 0.4) \]
\[ z_0 = 0.11(v/u_{*c}) + 0.03 \ k_{s,c} = \text{zero-velocity level above the bed} \]
\[ k_{s,c} = \text{equivalent roughness height of Nikuradse} \]

An experiment of Fernandez Luque and Van Beek (1976) was selected for calibration of the saltation model. Fernandez Luque and Van Beek carried out flume experiments on bed-load transport (plane bed). Advanced film techniques were used to measure the average particle velocity, the average saltation length and the average number of particles deposited per unit area and time (in water) as a function of the temporal mean shear stress. Fernandez Luque and Van Beek used four different bed materials: sand, gravel, magnetite and walnut grains. The experimental conditions were restricted to relatively low transport stages without bed forms (plane bed). The data of the experiment with gravel particles (d = 1800 \mu m) and a bed-shear velocity (u_{*c}) of about 0.04 m/s were used. The ratio of the bed-shear velocity and the particle fall velocity for this experiment was about 0.25. Under these conditions the sediment particles can only be transported as bed-load. Therefore, the trajectories measured by Fernandez Luque and Van Beek are considered to be trajectories of saltating particles, although the trajectories are somewhat wavy at certain locations (Fig. 7.2.3).

Two parameters were used to calibrate the model: the lift coefficient (\( \alpha_L \)) of Eq. (7.2.9) and the equivalent roughness of Nikuradse (\( k_s \)), see Eq. (7.2.12). As input data the following experimental results were used: d = particle diameter = 1800 \mu m (= 1.8 mm), \( \rho_s = \) density of sediment = 2650 kg/m\(^3\), \( u_{*c} = \) bed-shear velocity = 0.04 m/s, \( v = \) kinematic viscosity coefficient = 1.10\(^{-4}\) m\(^2\)/s. The initial longitudinal and vertical particle velocities were assumed to be equal to 2\( u_{*c} = 0.08 \) m/s.

Figure 7.2.3 shows measured and computed particle trajectories for various lift coefficients and equivalent roughness heights.

As can be observed, both calibration parameters have a strong influence on the computed trajectories. A reduced lift coefficient results in a reduction of the saltation length. Increasing the roughness height also reduces the saltation length considerably, due to its direct influence on the local flow velocity and thus on the lift and drag forces. As regards the average particle velocity, the "best" agreement between measured and computed values is obtained for \( \alpha_L = 20 \) and \( k_s/d = 2 \) to 3, which is a realistic value for plane bed conditions with active sediment transport (see Table 7.1).

<table>
<thead>
<tr>
<th>Particle characteristics</th>
<th>Measured</th>
<th>Computed (( \alpha_L = 20 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( k_s/d = 1 )</td>
</tr>
<tr>
<td>Particle velocity ( u_b/u_{*c} )</td>
<td>5.4</td>
<td>8.1</td>
</tr>
<tr>
<td>Saltation length ( \lambda_p/d )</td>
<td>21.24</td>
<td>23</td>
</tr>
<tr>
<td>Saltation height ( \delta_p/d )</td>
<td>2.3.5</td>
<td>3.5</td>
</tr>
</tbody>
</table>

**Table 7.1 Measured and computed saltation characteristics (Van Rijn, 1984)**

The measured saltation height and the measured particle velocity are "best" represented for \( \alpha_L = 20 \) and \( k_s/d = 2 \) to 3. For these values, however, a relatively large discrepancy between the measured and computed saltation lengths can be observed. Probably, the measured particle trajectories are influenced by turbulent motions resulting in a relatively large saltation length. Particularly, the wavy pattern of the measured trajectories indicates the influence of the upward fluid forces by turbulent action.
Figure 7.2.1 Definition sketch of saltating particle

Figure 7.2.2 Initial particle position and velocity profile

Figure 7.2.3 Measured and computed particle trajectories

7.8
Finally, some remarks are made with respect to the lift coefficient ($\alpha_l$). The value $\alpha_l = 20$, which is needed to represent the particle trajectories measured by Fernandez Luque, is rather large compared with the value $\alpha_l = 1.6$ for laminar flow (Eq. (7.2.9)). As the lift coefficient ($\alpha_l$) is used as a calibration parameter, it reflects all influences. For example, the fluctuating turbulent motions, additional pressure forces in the proximity of the wall and additional forces due to local fluid accelerations which are not taken into account by the model.

Applying the mathematical model, the saltation characteristics were computed for a range of conditions ($u_{*,c} = 0.04$ to 0.14 m/s) and particle diameters ($d = 100$ to 2000 $\mu$m) for a plane bed.

The input parameters were:

\[ k_s = 2d, \]
\[ \dot{x}_o = \dot{z}_o = 2u_{*,c}, z_o = 0.6d, \]
\[ \rho_s = 2650 \text{ kg/m}^3, \nu = 1.10^{-6} \text{ m}^2/\text{s}, \alpha_m = 0.5, \kappa = 0.4, \text{Re}_* = u_{*,c}d/\nu \]

\[ \alpha_L = 1.6 \quad \text{for} \quad \text{Re}_* \leq 5; \]
\[ \alpha_L = 20 \quad \text{for} \quad \text{Re}_* \geq 70; \]
\[ \alpha_L = 1.6 \text{ to } 20 \text{ (linear)} \quad \text{for} \quad 5 < \text{Re}_* < 70. \]

3. Saltation height

The computed saltation heights were related to a dimensionless bed-shear parameter $T$ and a dimensionless particle parameter $D_*$ as shown in Fig. 7.2.4. These curves can be approximated (inaccuracy = 10\%) by the following expression:

\[ \frac{\delta_b}{d} = 0.3 \left( D_* \right)^{0.7} T^{0.5} \quad (7.2.13) \]

in which:

\[ \delta_b \quad = \text{saltation height} \]
\[ d \quad = \text{median particle diameter} \]
\[ D_* \quad = d((s-1)g/\nu^2)^{1/3} = \text{dimensionless particle parameter} \]
\[ T \quad = (\tau_{b,c} - \tau_{b,cr})/\tau_{b,cr} = \text{dimensionless bed-shear stress parameters} \]
\[ \tau_{b,c} \quad = \text{effective current-related bed-shear stress} \quad (\tau_{b,c} \text{ in case of bed forms}) \]
\[ \tau_{b,cr} \quad = \text{critical bed-shear stress according to Shields} \]
\[ \nu \quad = \text{kinematic viscosity coefficient} \]
\[ s \quad = \rho_s/\rho = \text{relative density} \]

Experimental results have been presented by Williams (1970). Williams carried out flume experiments with bed-load transport ($d_{50} = 1350 \mu$m) in channels of different widths and depths. According to visual estimation, the bed-load particles moved within a zone of no more than about 8 particle diameters high at the largest transport stage (plane bed). The height of the bed-load layer was independent of the flow depth, but increased as the transport stage increased. Using the data of Williams ($u_{*,c} = 0.09$ m/s, $u_{*,cr} = 0.03$ m/s), Eq. (7.2.13) predicts a saltation height of about 10 particle diameters, which is remarkably close to the observed value.

Finally, it is remarked that Eq. (7.2.13) may predict relatively high values of the saltation height, because it is based on computations for individual particles, neglecting the influence of adjacent particles. In the case of collective motion of the particles, the actual saltation characteristics as well as the particle velocity, will be reduced by particle collisions in the bed-load layer.
4. Saltation height

The computed saltation lengths are shown in Fig. 7.2.5. At the same T-parameter a small particle performs a shorter saltation than a large particle, because the bed-shear velocity in the small particle case is much smaller.

The curves can be approximated (inaccuracy = 50%) by:

\[ \frac{\lambda_b}{d} = 3 \, D_s^{0.6} \, T^{0.9} \]  \hspace{1cm} (7.2.14)

Experimental support for the computed saltation lengths can be obtained from experiments concerning the sampling efficiency of bed-load samplers carried out by Poreh et al (1970). They observed saltation lengths in the range of 5d_{s0} to 40d_{s0} (d_{s0} = 1900 \, \mu m, u_{*,c} = 0.04 to 0.05 m/s). For these conditions, the mathematical model predicts a range from 10d_{s0} to 30d_{s0} (Fig. 7.2.5).

5. Particle velocity

The particle velocity on a (downward) sloping plane bottom can be derived from a force balance consisting of the drag force \( F_D \), the gravity force \( F_g \) and the friction force \( F_w \) exerted by the bottom on the particle during contact.

Thus,

\[ F_D + F_g - F_w = 0 \] \hspace{1cm} (7.2.15)

in which:

\[ F_D = \frac{1}{2} \, \rho \, C_D \, A \, (u_f - u_d)^2 \]
\[ F_g = (\rho_s - \rho) \, g \, V \, \tan \phi \cos \beta \]
\[ F_w = (\rho_s - \rho) \, g \, V \, \sin \beta \]
\[ A = \frac{1}{4} \pi \, d^2 = \text{particle area} \]
\[ V = \frac{1}{6} \pi \, d^3 = \text{particle volume} \]
\[ \phi = \text{angle of repose} \]
\[ \beta = \text{angle of local bottom slope} \]
\[ u_b = \text{particle velocity} \]
\[ u_f = \text{fluid velocity} \]

The force balance yields:

\[ u_b = u_f - \left( \frac{4(s-1)g \, d \, \tan \phi}{3 \, C_D} \frac{\sin(\phi - \beta)}{\sin \phi} \right)^{0.5} \] \hspace{1cm} (7.2.16)

Assuming \( u_f = \alpha_1 \, u_{*,c} \) and \( \frac{4}{3} \frac{\tan \phi}{C_D} = \alpha_2 \, \theta_{\sigma,o} \), it follows that

\[ \frac{u_b}{u_{*,c}} = \alpha_1 - \alpha_2 \left( \frac{\theta_{\sigma,o}}{\theta} \frac{\sin(\phi - \beta)}{\sin \phi} \right)^{0.5} \] \hspace{1cm} (7.2.17)

in which:

\( u_{*,c} \) = bed-shear velocity (\( u_{*,c}' \) in case of bed forms)
\( \theta_{\sigma,o} \) = critical mobility parameter of Shields on a horizontal bottom
\( \theta \) = mobility parameter
\( \alpha_1, \alpha_2 \) = coefficients
The term $\sin(\phi - \beta)/\sin\phi$ expresses the bottom slope effect on the particle velocity. Figure 7.2.6 shows computed particle velocities on a horizontal bottom according to the saltation model. The curves of Van Rijn can be approximated (inaccuracy = 10%) by:

$$\frac{u_b}{u_{*c}} = 9 + 2.6 \log(D_*) - 8 \left( \frac{\theta_{tr}}{\theta} \right)^{0.5} \quad (7.2.18)$$

For a particle mobility parameter ($\theta$) approaching the critical value, the particle velocity is supposed to approach zero. Figure 7.2.6 shows some data of Fernandez Luque and Francis. As regards the experiments of Fernandez Luque, only the sand ($d = 900 \ \mu m$) and the gravel ($d = 1800 \ \mu m$) experiments for a flat bed-surface slope ($\beta = 0$) were used. From the experiments of Francis only the gravel data were used. As can be observed, the (scarce) data do not confirm the influence of the $D_*$-parameter as expressed by the mathematical model. The computed velocities appear to be too high for $D_ *> 15$.

Equation (7.2.18) yields values in the range of 3 to 11 $u_{*c}$. A reasonable average value is about $u_b = 7 u_{*c}$.

The computational results can also be approximated (50% inaccuracy) by the following simple expression:

$$\frac{u_b}{[(s-1)g \ d]^{0.5}} = 1.5 \ T^{0.6} \quad (7.2.19)$$

Equation (7.2.19) yields smaller values for large particles compared to Eq. (7.2.18).

Using an expression similar to Eq. (7.2.17) and data fitting, Engelund and Fredsøe (1976) derived:

$$\frac{u_b}{u_{*c}} = 10 - 7 \left( \frac{\theta_{tr}}{\theta} \right)^{0.5} \quad (7.2.20)$$

Equation (7.2.20) is shown in Fig. 7.2.6.

The influence of particle shape was investigated by Francis (1973). His experiments show that angular particles travel slower than spherical particles. The spherical particles made violent rebounds from the bed and were lifted to higher levels where they experienced higher flow velocities than the angular particles, resulting in a relatively large average particle velocity.

---

**Figure 7.2.4** Computed saltation heights (Van Rijn, 1984)
Figure 7.2.5 Computed saltation lengths (Van Rijn, 1984)

Figure 7.2.6 Computed and measured particle velocities (Van Rijn, 1984)
7.2.3 Particle pick-up from the bed

1. Definitions

Usually, the pick-up rate of bed material particles is defined in terms of the number of particles \( N_p \) picked up from the bed per unit area and time, as:

\[
N_p = \eta \ n \ P_s = \frac{\eta}{\alpha_1 \ d^2} \ P_s
\]  

(7.2.21)

in which:

- \( \eta \) = fraction of susceptible particles per unit area exposed to the flow (-)
- \( n = 1/(\alpha_1 \ d^2) \) = number of particles (at rest) per unit area (-)
- \( d \) = particle diameter (m)
- \( \alpha_1 \) = shape constant (= \( \frac{1}{4} \pi \) for a sphere), (-)
- \( P_s \) = number of pick-ups per grain per unit time (s\(^{-1}\))

Various researchers have studied the pick-up of bed material particles. It is, however, not always clear what is meant by "pick-up", i.e. which events contribute to the pick-up rate and which do not.

Einstein (1950) assumes that a particle can be picked up only after a period of rest, which is large compared to the pick-up time scale. In this approach the total travel distance between two successive periods of rest may consists of several jumps or saltations.

Yalin (1977) assumes that a particle is picked-up whenever it leaves the bed surface to perform a jump. Hence, each particle jump involves a pick-up and a deposit event. It must be stated that the pick-up rate according to the definition of Yalin may be about 10 times as large as according to the definition of Einstein (assuming an average travel distance equal to about 10 jump lengths). Herein, the definition of Yalin is adopted. A detailed review was given by Van Rijn (1983).

2. Pick-up functions

**Einstein (1950)**

Based on experiments, Einstein assumed that the characteristic time scale of the pick-up process is proportional to the period necessary for a particle to settle over a distance equal to its diameter. According to Einstein, a particle will be eroded when the instantaneous lift force exceeds the submerged particle weight. The lift force is assumed to have a Gaussian distribution.

Based on this stochastic approach, Einstein found:

\[
E = \alpha \ \rho_s \ [(s-1)g \ d]^{0.5} \ P
\]  

(7.2.22)

in which:

- \( E \) = pick-up rate in mass per unit area and time
- \( P \) = fraction of time during which a sediment particle is picked-up by the flow (= probability of the instantaneous lift force to exceed the submerged particle weight), shown in Fig. 7.2.7.
- \( \alpha \) = coefficient
- \( d \) = median particle diameter
Yalin (1977)

Yalin assumed that the pick-up time scale is proportional to the ratio of the particle diameter and the bed-shear velocity. Using a stochastic approach, Yalin proposed:

\[ E = \alpha \rho_s u_s P \]  
(7.2.23)

in which:
\( u_s \) = bed-shear velocity
\( P \) = pick-up velocity, see Fig. 7.2.7

De Ruiter (1982, 1983)

According to De Ruiter, the pick-up time scale is equal to the time period during which the particle is moved from rest over a distance of half its diameter. De Ruiter has shown that this time scale is much smaller than the time period during which the instantaneous bed-shear stress exceeds the critical shear stress. Based on a stochastic approach, De Ruiter found:

\[ E = \alpha \rho_s F \left( \frac{\sigma - \rho}{\rho_s} \right) \left( g d \tan(\phi) \right)^{0.5} \]  
(7.2.24)

in which:
\( F \) = pick-up probability function (see Van Rijn, 1983)
\( \sigma \) = standard deviation of instantaneous bed-shear stress
\( \tau_{\alpha,0} \) = critical instantaneous bed-shear stress at horizontal bed
\( \phi \) = angle of repose
\( \alpha \) = coefficient (= 0.016)

Nagakawa and Tsujimoto (1980)

Based on experimental data (\( \theta = 0.03 \) to 0.2 and \( d_{50} = 3 \) to 13.5 mm), they proposed a deterministic expression:

\[ E = \alpha \rho_s [(s-1)d]^{0.5} \left[ 1 - \frac{0.035}{\theta} \right]^\theta \]  
(7.2.25)

in which:
\( \theta \) = particle mobility parameter
\( \alpha \) = coefficient (= 0.02)

The applied pick-up definition of the authors is not quite clear.

Fernandez Luque (1974)

Based on experimental data (\( \theta = 0.05 \) to 0.11, \( d_{50} = 0.9 \) to 1.8 mm), he proposed a deterministic function.

\[ E = \alpha \rho_s [(s-1)g d]^{0.5} \left[ \theta - \theta_{\alpha} \right]^{1.5} \]  
(7.2.26)

in which:
\( \alpha \) = coefficient (= 0.02)
\( \theta_{\alpha} \) = critical mobility parameter according to Shields
3. **Experimental results**

Van Rijn (1983, 1984, 1986) performed experiments to determine the pick-up rate of particles in the range of 130 to 1500 μm. A sediment lift was designed by which the sediment particles were moved upwards at a constant rate through a circular opening in the flume bottom, as shown in Fig. 7.2.8.

The particles were pushed upwards by use of a piston connected to an electromotor. The upward piston speed could be set at different (constant) values by means of a mechanical drive system.

The sediment lift was installed in the center line of the test section of a small flume (length = 30 m, width = 0.5 m and depth = 0.7 m). To establish a uniform flow, the flume was equipped with a false floor supported by a jack system which was given (by trial and error) a pre-set slope equal to the expected water surface slope in each experiment. The false floor was covered with prefabricated wooden plates coated with sediment particles of a size equal to those on the sediment lift. In all, five types of almost uniform sand material were used. For each type of sand, a series of tests was performed with mean flow velocities in the range of 0.5 to 1.0 m/s. The flow depth was kept constant at a value of 0.25 m.

Prior to each run the sediment lift was filled with sediment particles by using a (perspex) tube with a length larger than the flow depth and placed in the opening of the sediment lift. Using this method, the lift could be filled without emptying the flume. The total mass to be poured into the lift tube was computed from the volume of the lift tube, assuming a porosity factor of 0.4. Some mechanical stirring was necessary to fill the tube with all available material.

At the beginning of a run, the opening of the sediment lift was covered to prevent initial pick-up of sediment particles. The (constant) piston speed of the lift was set at a value slightly higher than the expected pick-up rate of the flow (trial and error). Then the flow (constant discharge) was started. After an adjustment period of about 10 min to establish equilibrium flow conditions, the cover plate was removed and the sediment lift was started. When the surface of the movable particles was observed (visually) to rise above the surrounding surface of fixed particles, the lift was stopped for a short period and restarted when the over-height of particles had disappeared. The flow was stopped when all particles were removed. To reduce the subjective element in this procedure, different observers were used to operate the sediment lift. The experiments with a relatively large flow velocity, and therefore a relatively large pick-up rate, were repeated several times to increase the accuracy of the results. The maximum variation between similar tests using different observers was about 20%.

The pick-up rate was determined as:

\[
E = \frac{M}{A \Delta T}
\]

(7.2.27)

in which:
- \( E \) = pick-up rate in mass per unit area and time (kg/sm²)
- \( M \) = total sediment mass (kg)
- \( A \) = area of movable surface (m²)
- \( \Delta T \) = measuring period (s)

Analysis of the experimental data yielded the following empirical pick-up function:

\[
E = 0.00033 \rho_s [(s-1)g]^{0.5} D_s^{-10.4} T^{1.5}
\]

(7.2.28)

in which:
- \( D_s \) = \( d_{50} [(s-1)g/v^2]^{1/3} \)
- \( T = (\tau_b - \tau_{b,cr})/\tau_{b,cr} \)
Figure 7.2.7 Pick-up probability according to Einstein (1950) and Yalin (1977)

Figure 7.2.8 Empirical pick-up function of Van Rijn (1984d)
Figure 7.2.9 Pick-up rate according to Mastbergen (1987)
Figure 7.2.10 Measured and computed pick-up rates for $d_{50} = 130 \, \mu m$, $190 \, \mu m$ and $360 \, \mu m$
Figure 7.2.11 Measured and computed pick-up rates for $d_{50} = 790$ $\mu$m and 1500 $\mu$m
Equation (7.2.28) is shown in Fig. 7.2.8. The relative standard error is about 30% (dashed lines).

Mastbergen (1987) used the data of Van Rijn and data from high-concentration \( c_{\text{max}} = 0.4 \) flows to determine the pick-up rate for a large range of mobility parameters \( \theta \). The results are shown in Fig. 7.2.9 (\( \tan \beta = \) bottom slope, \( \tan \phi = 0.6 \)).

4. Evaluation of pick-functions

The existing pick-up functions were compared to available experimental results by fitting of the \( \alpha \)-coefficients. The best-fitted \( \alpha \)-values and the originally proposed \( \alpha \)-values are shown in Table 7.2. Einstein and Yalin did not propose an \( \alpha \)-constant because their pick-up functions were an integral part of a bed-load transport theory.

<table>
<thead>
<tr>
<th>Sediment</th>
<th>Einstein</th>
<th>Yalin</th>
<th>De Ruiter</th>
<th>Nagakawa</th>
<th>Fernandez Luque</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Best fitted</td>
<td>Proposed</td>
<td>Best fitted</td>
<td>Proposed</td>
<td>Best fitted</td>
</tr>
<tr>
<td>130 ( \mu )m</td>
<td>0.0077</td>
<td>0.0088</td>
<td>0.0092</td>
<td>0.0150</td>
<td>0.0121</td>
</tr>
<tr>
<td>190 ( \mu )m</td>
<td>0.0077</td>
<td>0.0118</td>
<td>0.0089</td>
<td>0.0331</td>
<td>0.0224</td>
</tr>
<tr>
<td>360 ( \mu )m</td>
<td>0.0094</td>
<td>0.0167</td>
<td>0.0141</td>
<td>0.0642</td>
<td>0.0436</td>
</tr>
<tr>
<td>790 ( \mu )m</td>
<td>0.0067</td>
<td>0.0157</td>
<td>0.0182</td>
<td>0.144</td>
<td>0.0854</td>
</tr>
<tr>
<td>1500 ( \mu )m</td>
<td>0.0050</td>
<td>0.0125</td>
<td>0.020</td>
<td>0.228</td>
<td>0.138</td>
</tr>
</tbody>
</table>

**Table 7.2 \( \alpha \)-values of pick-up functions**

The "best-fitted" \( \alpha \)-coefficients of Einstein and Yalin are not very much dependent on the particle size. The "best-fitted" \( \alpha \)-coefficient of De Ruiter is weakly dependent on the sediment size, and is in good agreement with the proposed \( \alpha \)-coefficient, particularly for the larger particle sizes.

The "best-fitted" \( \alpha \)-coefficients of Nagakawa-Tsujimoto and Fernandez Luque are strongly dependent on the particle size. As regards the largest particle sizes, the "best-fitted" \( \alpha \)-coefficients are about 10 times too large. For the smaller particle sizes the "best-fitted" \( \alpha \)-coefficients are in relatively good agreement with the proposed values, which is a remarkably good result because the proposed values are based on experiments with rather coarse bed material particles.

Figures 7.2.10 and 7.2.11 show that the variation (trend) of the measured pick-up rates with the dimensionless particle mobility parameter \( \theta \) is best represented by the curves of Fernandez Luque and Nagakawa-Tsujimoto, while also the curve of De Ruiter shows good results for the larger particle sizes (790 and 1500 \( \mu \)m).

7.2.4 Deterministic bed load transport formulas

1. Meyer-Peter Mueller (1948)

Extensive experimental work has carried out by Meyer-Peter and Mueller at the "Eidgenössische Technische Hochschule" (ETH) in Switzerland. The experiments were performed in a laboratory flume with a cross-section of 2 x 2 m² and a length of 50 m. Uniform bed material as well as particle mixtures were used in the experiments (\( d = 0.4 \) to 29 mm, slope 1 = 0.0004 to 0.02, depth = 0.1 to 1.2 m). The bed-load transport rate is expressed (see Fig. 7.2.12):

\[
\phi_b = 8(\mu \theta - 0.047)^{1.5}
\]

(7.2.29)
Figure 7.2.12 Bed load transport rates
in which:

\[ \phi_b = \frac{q_{b,c}}{(s-1)^{0.5} \rho^{0.5} d_m^{1.5}} = \text{dimensionless bed-load transport rate} \]

\[ \theta = \frac{\tau_{b,c}}{(\rho_s - \rho) g d_m} = \text{dimensionless particle mobility parameter} \]

\[ c_{b,c} = \text{volumetric bed load transport rate (m}^3/\text{s}) \]

\[ \tau_{b,c} = \rho g h I = \text{current-related bed-shear stress (N/m}^2) \]

\[ d_m = \text{mean particle diameter (m)} \]

\[ \mu = (C/C')^{1.5} = \text{bed-form factor or efficiency factor} \]

\[ C = 18\log(12h/k_{s,c}) = \text{overall Chézy-coefficient (m}^{1/2}/\text{s}) \]

\[ C' = 18\log(12h/d_{so}) = \text{grain-related Chézy-coefficient (m}^{1/2}/\text{s}) \]

\[ h = \text{water depth (m)} \]

\[ I = \text{energy gradient (-)} \]

\[ k_{s,c} = \text{effective bed roughness (m)} \]

\[ \rho_s = \text{relative density (-)} \]

The 0.047-value of Eq. (7.2.29) can be interpreted as the critical mobility parameter \( (\theta_{cr}) \). Since, the formula is related to coarse material, the authors preferred to use a constant value of 0.047.

Meyer-Peter Mueller proposed to use the mean particle diameter \( d_m \) as the characteristic particle diameter, defined as: \( d_m = \sum p_i d_i \). The \( d_m \)-parameter is about 1.1 to 1.3 times larger than the \( d_{so} \)-parameter for nearly uniform material.

Equation (7.2.29), however, is only weakly dependent on the particle diameter. Therefore, the median particle diameter \( d_{so} \) may also be used. The grain-roughness is related to the \( d_{so} \)-diameter.

**Influence of particle diameter**

The effect of the particle diameter on the bed-load transport is shown by a computation example.

**Given** : bed-shear stress \( \tau_b = 5 \text{ N/m}^2 \), bed form factor \( \mu = 0.5 \).

**Compute** : bed load transport for \( d_m = 600, 700, 800, 900 \) and 1000 \( \mu \text{m} \).

**Solution** : see Table 7.3.

<table>
<thead>
<tr>
<th>( d_m ) (( \mu \text{m} ))</th>
<th>( \theta ) (-)</th>
<th>( \mu \theta ) (-)</th>
<th>( \phi_b ) (-)</th>
<th>( q_b ) (m(^2)/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0.51</td>
<td>0.255</td>
<td>0.76</td>
<td>4.5 ( 10^{-5} )</td>
</tr>
<tr>
<td>700</td>
<td>0.44</td>
<td>0.220</td>
<td>0.58</td>
<td>4.3 ( 10^{-5} )</td>
</tr>
<tr>
<td>800</td>
<td>0.39</td>
<td>0.195</td>
<td>0.46</td>
<td>4.1 ( 10^{-5} )</td>
</tr>
<tr>
<td>900</td>
<td>0.34</td>
<td>0.170</td>
<td>0.35</td>
<td>3.8 ( 10^{-5} )</td>
</tr>
<tr>
<td>1000</td>
<td>0.31</td>
<td>0.155</td>
<td>0.28</td>
<td>3.6 ( 10^{-5} )</td>
</tr>
</tbody>
</table>

*Table 7.3* Bed-load transport rates according to Meyer-Peter-Mueller
Thus, a 25\% -variation of the particle diameter \( d_{pm} = 800 \pm 200 \mu m \) results in a 10\% -variation of the transport rate \( q_b = 4.1 \pm 0.4 \times 10^{-7} \text{ m}^2/\text{s} \).

2. Frijlink (1952)

The formula of Frijlink essentially is an approximation of the formula of Meyer-Peter and Mueller (1948) and that of Einstein (1950), as shown in Fig. 7.2.12.

The formula, which is given here because of its simplicity, reads as:

\[
q_{b,c} = 5 \mu 0.5 u_{*c} d_{50} e^{0.27/(\mu \theta)} \tag{7.2.30}
\]

The parameters are identical to those of Meyer-Peter-Mueller (1948).


The forces per unit area moving the bed-load particles are (see Fig. 7.2.13):

- the applied bed-shear stress: \( \tau_b = \rho g h I \) \tag{7.2.31}
- the gravity component: \( \tau_g = \left( \frac{\rho_s - \rho}{\rho_s} \right) g m \sin \beta \) \tag{7.2.32}

in which:

- \( m = \rho_s V_b \) = mass of bed-load particles per unit area (kg/m\(^2\))
- \( \beta = \) slope angle
- \( V_b = \) solid volume of bed-load particles per unit area (m)
- \( \rho_s = \) sediment density (kg/m\(^3\))

The moving grains exert a normal stress on the bed,

\[
\sigma_s = (\rho_s - \rho) V_b g \cos \beta = \left( \frac{\rho_s - \rho}{\rho_s} \right) g m \cos \beta \tag{7.2.33}
\]

The tangential stress at the bed resisting the moving bed-load grains is:

\[
\tau_s = \left( \frac{\rho_s - \rho}{\rho_s} \right) g m \cos \beta \tan \phi \tag{7.2.34}
\]

in which:

- \( \tan \phi = \) dynamic friction coefficient

The applied bed-shear stress (at the base of moving bed-load layer) consists of a dispersive grain-shear stress \( (\tau_{b,s}) \) and an intergranular fluid bed-shear stress \( (\tau_{b,f}) \).

\[
\tau_b = \tau_{b,s} + \tau_{b,f} \tag{7.2.35}
\]

The dispersive stress \( (\tau_{b,s}) \) is a shear stress which is transferred by grain to grain interaction. Since, the moving grains have a velocity smaller than the local fluid velocity, the grains
receive their momentum from the fluid motion. Thus, the grains by receiving momentum resist the fluid motion. The $\tau_{bf}$-parameter is the residual fluid shear stress exerted directly on the bed by drag and skin friction. The $\tau_{bf}$-parameter is negligible small in the near-bed layer where the grain concentration is high. According to Bagnold, grains are set in motion only by the fluid-shear stress ($\tau_{bf}$). As many layers of bed-load particles will be eroded as necessary to develop a dispersive grain shear stress which just keeps the fluid-shear stress at the immobile bed below the critical bed shear stress for initiation of motion. Thus, the bed load layer acts as a protective layer to obtain a stable bed. When there is no bed-load layer, each successive layer of bed particles will be eroded and suspended because the fluid bed shear stress remains always larger than the critical shear stress.

Equilibrium of shear stress at the bed requires (see Fig. 7.2.13):

$$\tau_{b,\alpha} + \tau_g = \tau_s$$  \hspace{1cm} (7.2.36)

$$\tau_{b,\alpha} + (\rho_s - \rho)g V_b \sin\beta - (\rho_s - \rho)g V_b \cos\beta \tan\phi$$  \hspace{1cm} (7.2.37)

$$\tau_{b,\alpha} = (\rho_s - \rho)g V_b \cos\beta (\tan\phi - \tan\beta)$$  \hspace{1cm} (7.2.38)

The work ($W_r$) required to be done by the grain-shear stress in moving the bed-load particles is:

$$W_r = \tau_{b,\alpha} u_b = (\rho_s - \rho)g V_b u_b \cos\beta (\tan\phi - \tan\beta)$$  \hspace{1cm} (7.2.39)

Defining the volumetric bed-load transport (m$^2$/s) as $q_{b,n,c} = V_b u_b$, it follows that:

$$W_r = (\rho_s - \rho)g q_{b,c} \cos\beta (\tan\phi - \tan\beta)$$  \hspace{1cm} (7.2.40)

The available fluid energy per unit area and time is:

$$W_s = \tau_b \bar{u} = \rho g h I \bar{u}$$  \hspace{1cm} (7.2.41)

Bagnold assumed that: $W_r = e_b W_s$, yielding:

$$q_{b,c} = \frac{e_b \tau_b \bar{u}}{(\rho_s - \rho)g \cos\beta (\tan\phi - \tan\beta)}$$  \hspace{1cm} (7.2.42)

in which:

$q_{b,c}$ = volumetric bed-load transport (m$^2$/s)
$\tau_b$ = $\rho g h I$ = overall bed-shear stress (N/m$^2$)
$\bar{u}$ = depth-averaged velocity (m/s)

7.24
I = energy gradient (-)
$e_b$ = efficiency factor (0.1-0.2)
$\tan \phi = 0.6$ = dynamic friction coefficient (-)
$\tan \beta = I_b$ = bed slope (-)
$h$ = water depth (m)
$g$ = gravity acceleration (m/s$^2$)
$V_b$ = volume of bed-load per unit area (m$^3$/m$^2$)

It should be noted that:
- the bed load transport is independent of the particle diameter ($d$),
- the transport rate is related to the overall bed-shear stress ($\tau_u$) and not to the effective stress ($\tau^*_u$); the effect of the bed forms is not taken into account.

4. Van Rijn (1984a)

Four different methods were used by Van Rijn to compute the bed load transport rate.

A. Van Rijn followed the approach of Bagnold (1954) assuming that the motion of the bed-load particles is dominated by particle saltations (jumps) under the influence of hydrodynamic fluid forces and gravity forces. The saltation characteristics have been determined by solving the equations of motions for an individual bed-load particle. The bed load transport rate ($q_{b,c}$) is defined as the product of the particle velocity ($u_b$), the saltation height ($\delta_b$) and bed-load concentration ($c_b$) resulting in $q_{b,c} = u_b \delta_b c_b$.

The bed-load concentration ($c_b$) was calculated from measured bed-load transport rates ($q_{b,c}$) using Eqs. (7.2.13) and (7.2.18). In all, 130 flume experiments with particle diameters ($d_{so}$) ranging from 200 to 2000 $\mu$m, water depths larger than 0.1 m and a Froude number smaller than 0.9 were selected from the Literature. The influence of side-wall roughness was eliminated by using the method of Vanoni and Brooks (1957). The form roughness was eliminated by using a bed form factor $\mu = (C/C')^2$.

Analysis of the results showed that the bed-load concentration can be represented by (see Fig. 7.2.14):

$$\frac{c_b}{c_o} = 0.18 \frac{T}{D^*} \quad (7.2.43)$$

in which:
- $c_b$ = volumetric bed-load concentration (-)
- $c_o$ = maximum volumetric concentration = 0.65

Using Eqs. (7.2.13), (7.2.19) and (7.2.43), the bed-load transport rate for particles in the range of 200 to 2000 $\mu$m can be derived from $q_{b,c} = \delta_b u_b c_b$, resulting in:

$$q_{b,c} = 0.053 (s-1)^{0.5} g^{0.5} d_{so}^{1.5} D^*^{-0.3} T^{2.1} \quad (7.2.44a)$$

Equation (7.2.44a) was found to overpredict (factor 2) the transport rates for $T \geq 3$. Therefore, a modified expression is proposed for this range:

$$q_{b,c} = 0.1 (s-1)^{0.5} g^{0.5} d_{so}^{1.5} D^*^{-0.3} T^{1.5} \quad \text{for} \quad T \geq 3 \quad (7.2.44b)$$

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in which:

\( q_{b,c} \quad = \quad \text{volumetric bed load transport rate (m}^2/\text{s)} \)

\( T \quad = \quad (\tau_{b,c}^{'} - \tau_{b,cr})/\tau_{b,cr} \quad = \quad \text{dimensionless bed-shear parameter} \)

\( \tau_{b,c}^{'} \quad = \quad \rho g (\bar{u}/C^{'})^2 \quad = \quad \text{effective bed-shear stress (N/m}^2) \)

\( C^{'} \quad = \quad 18 \log(12h/3d_{90}) \quad = \quad \text{grain-related Chézy-coefficient (m}^{1/2}/\text{s)} \)

\( h \quad = \quad \text{water depth (m)} \)

\( d_{50}, d_{90} \quad = \quad \text{particle diameters (m)} \)

\( \bar{u} \quad = \quad \text{depth-averaged velocity (m/s)} \)

\( \tau_{b,cr} \quad = \quad \text{critical bed-shear stress according to Shields (N/m}^2) \)

\( D_* \quad = \quad d_{50}[(s-1)g\nu^2]^{1/3} \quad = \quad \text{dimensionless particle parameter (-)} \)

\( s \quad = \quad \rho_s/\rho \quad = \quad \text{relative density (-)} \)
\[ \rho_s = \text{sediment density (kg/m}^3) \]
\[ \rho = \text{fluid density (kg/m}^3) \]
\[ \nu = \text{kinematic viscosity coefficient (m}^2/\text{s}) \]
\[ g = \text{acceleration of gravity (m/s}^2) \]

The effective bed-shear stress \( (\tau_{b,c}) \) can also be expressed as \( \tau'_{b,c} = \mu \cdot \tau_{b,c} \) with \( \mu = (C/C')^2 = \text{bed form factor, } C = \text{overall Chézy-coefficient, } \tau_{b,c} = \rho g \left( \frac{\bar{u}}{C'} \right)^2 = \text{overall bed-shear stress.} \)

B. A more simple formula can be obtained from Eqs. (7.2.13), (7.2.43) and assuming \( u_b = 7 \cdot u'_{*,c} \), yielding:

\[ q_{b,c} = 0.25 \cdot d_{50} \cdot u'_{*,c} \cdot D_\ast^{0.3} \cdot T^{1.5} \quad (7.2.45) \]

in which:

\[ u'_{*,c} = \left( \frac{\tau_{b,c}}{\rho} \right)^{0.5} = g^{0.5} \frac{\bar{u}}{C'} = \text{effective bed-shear velocity (m/s)} \]

C. Equation (7.2.44) can be approximated by the following formula based on the independent variables \( \bar{u}, \bar{u}_{\ast}, h \) and \( d_{50} \):

\[ q_{b,c} = 0.005 \bar{u} \cdot h \left( \frac{\bar{u} - \bar{u}_{\ast}}{(s-1)g \cdot d_{50}^{0.5}} \right)^{2.4} \left( \frac{d_{50}}{h} \right)^{1.2} \quad (7.2.46) \]

in which:

\[ \bar{u} = \text{depth-averaged flow velocity (m/s)} \]
\[ \bar{u}_{\ast} = \text{critical depth-averaged flow velocity based on Shields (m/s)} \]
\[ h = \text{water depth (m)} \]
\[ \bar{u}_{\ast} = 0.19 \cdot (d_{50})^{0.1} \cdot \log(12h/3d_{90}) \quad \text{for } 0.0001 \leq d_{50} \leq 0.0005 \quad (7.2.47) \]
\[ \bar{u}_{\ast} = 8.50 \cdot (d_{50})^{0.6} \cdot \log(12h/3d_{90}) \quad \text{for } 0.0005 \leq d_{50} \leq 0.002 \]
\[ d_{50}, d_{90} = \text{particle diameters of bed material (in metres)} \]

D. The bed-load transport can also be calculated by multiplication of the pick up rate \( (E) \) and the saltation length \( (\lambda_b) \) resulting in \( q_{b,c} = E \cdot \lambda_b \), as shown by Van Rijn (1986), see also Eq. (7.2.3).

\textit{Influence of particle diameter}

The effect of the particle diameter on the bed-load transport is shown by means of a computation example.

Given : bed-shear stress \( \tau_b = 5 \text{ N/m}^2 \), bed form factor \( \mu = 0.5, \nu = 1.1 \times 10^{-6} \text{ m}^2/\text{s}, \rho_s = 2600 \text{ kg/m}^3, \rho = 1000 \text{ kg/m}^3 \)

Compute : bed load transport rate for \( d_{50} = 600, 700, 800, 900 \) and \( 1000 \mu \text{m using Eq. (7.2.44) and (7.2.45)} \)

Solution : \( \tau_b = \mu \cdot \tau_b = 2.5 \text{ N/m}^2, u_{\ast} = (\tau_{b,c}/\rho)^{0.5} = 0.05 \text{ m/s, results see Table 7.4.} \)
<table>
<thead>
<tr>
<th>d_{50} (µm)</th>
<th>D.</th>
<th>τ_{b,c} (N/m²)</th>
<th>T</th>
<th>q_b (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>15.2</td>
<td>0.30</td>
<td>7.3</td>
<td>5.2 \times 10^{-5}</td>
</tr>
<tr>
<td>700</td>
<td>17.7</td>
<td>0.35</td>
<td>6.1</td>
<td>4.7 \times 10^{-5}</td>
</tr>
<tr>
<td>800</td>
<td>20.2</td>
<td>0.40</td>
<td>5.3</td>
<td>4.5 \times 10^{-5}</td>
</tr>
<tr>
<td>900</td>
<td>22.8</td>
<td>0.47</td>
<td>4.3</td>
<td>3.8 \times 10^{-5}</td>
</tr>
<tr>
<td>1000</td>
<td>25.3</td>
<td>0.55</td>
<td>3.5</td>
<td>3.2 \times 10^{-5}</td>
</tr>
</tbody>
</table>

**Table 7.4 Bed-load transport rates according to Van Rijn**

According to Equation (7.2.44), a 25%-variation of the particle diameter (d_{50} = 800 ± 200 µm) results in a 25%-variation in the transport rate (q_b = 4.3 ± 1.1 \times 10^{-5} m²/s). Equation (7.2.45) yields a 35%-variation of the transport rates. The computed transport rates are in reasonable agreement with those according to the Meyer-Peter-Mueller method (see Table 7.3).

Information of the influence of the particle diameter on measured bed-load transport rates can be obtained from the data of Guy et al (1966). Figure 7.2.15 shows measured bed-load transport rates as a function of bed-shear stress for two grain sizes. As can be observed, the transport rate is inversely proportional to the grain size diameter. A decrease of the grain size by a factor 2 yields an increase of the transport rate by a factor 2. This value is similar to that according to the method of Van Rijn (see Table 7.4), but larger than that according to the method of Meyer-Peter-Mueller (see Table 7.3).

**Figure 7.2.15 Influence of grain size on bed-load transport rate**
7.2.5 Bed load transport at low shear stress

Extensive measurements of low bed-load transport rates were performed and analyzed by Paintal (1971). Based on the experimental results, Paintal proposed:

\[ \phi_b = \alpha \theta^\beta \]  \hspace{1cm} (7.2.48)

in which:

\[ \alpha = 6.56 \times 10^4 \]
\[ \beta = 16 \]
\[ \phi_b = \frac{q_b}{[(s-1)^0.5 g^{0.5} d_{50}^{1.5}]} \text{ = dimensionless bed-load transport} \]
\[ \theta = \frac{u_2^2}{[(s-1)g d_{50}]} \text{ = bed-shear stress parameter} \]

Equation (7.2.48) which is valid for \( \theta < 0.06 \) and \( d_{50} = 1 \text{ to } 25 \text{ mm} \) is shown in Figure 7.2.16. Application of Eq. (7.2.48) will lead to large errors in the transport rate as a result of small errors in the bed-shear velocity.

![Figure 7.2.16 Bed-load transport at low bed shear stress](image)

7.2.6 Bed-load transport at steep slopes

1. Longitudinal bed slopes

Most bed-load transport formulae were derived for nearly horizontal beds \( (\text{bed slope} = \tan \beta \leq 10^\circ) \). Experimental data of steep channels (Smart, 1984) show a strong increase of the transport rates at steep bed slopes.

It appears that the bed slope may affect the transport rate in three ways, as follows:
- the bed slope will influence the local near-bed flow velocity;
- the bed slope will change the threshold conditions;
- the bed slope may change the transport rate once the sediment is in motion.

The first effect is a matter of hydrodynamics and will not be discussed herein. The second effect (see Section 4.1.2) can be taken into account properly by the Scholkitsch-factor (Eq. (4.1.20)).

The third effect was studied by Bagnold (1966). Based on his work (see Eq. (7.2.42)), a relationship between the transport rate on a sloping bed and on a horizontal bed can be derived, which reads as:

\[ q_{b,\text{slope}} = \alpha_s q_{b,\text{o}} \]  \hspace{1cm} (7.2.49)
in which:

\( q_{b,\text{slope}} \) = bed-load transport on a sloping bed
\( q_{b,\text{h}} \) = bed-load transport on a horizontal bed

\[
\alpha_s = \frac{\tan \phi}{\cos \beta (\tan \phi + \tan \beta)} = \text{Bagnold slope factor (see Fig. 7.2.17),}
\]

\( + \) for upsloping flow, \(-\) for downsloping flow

\( \phi \) = angle of repose \( (\tan \phi = 0.6) \)
\( \beta \) = local bed slope (longitudinal direction)

The Bagnold slope factor was studied by Hardisty and Whitehouse (1988) for aeolian sand transport. The Bagnold factor was found to give a significant underprediction of the measured results.

Smart (1984) measured bed-load transport rates in steep channels (bed slopes \( \tan \beta = 0.03 \) to 0.2, \( d = 0.002 \) to 0.0105 m, depths \( h = 0.01 \) to 0.09 m, plane bed). Based on his data and the old data of Meyer-Peter-Mueller, he proposed the following formula for plane bed \( (d \geq 400 \ \mu m, \ I_b = 0.0004 \) to 0.2):

\[
\phi_b = 4 \sigma I_b^{0.6} C g^{-0.5} \theta^{0.5} (\theta - \theta_{cr})
\]  
(7.2.50)

in which:

\( \phi_b = q_b/[(s-1)g \ d_m^3] \) = dimensionless bed-load transport rate
\( \theta = (h I_b)/[(s-1)d_m] \) = dimensionless mobility parameter
\( q_b \) = volumetric bed-load transport rate \((m^2/s)\)
\( I_b \) = longitudinal bed slope
\( h \) = water depth
\( s = \rho_s/\rho \) = relative sediment density
\( C \) = Chézy-coefficient
\( g \) = acceleration of gravity
\( \sigma = (d_{90}/d_{30})^{0.2} \) = sediment gradation coefficient
\( d_m = \sum p_i d_i \) = mean particle diameter
\( d_{90}, d_{30} \) = characteristic bed material sizes
\( \theta_{cr} \) = critical mobility parameter of Shields corrected for the slope effect \( (\theta_{cr} = k_{\beta} \theta_{cr,o}) \), see Section 4.1.2

Comparing Eq. (7.2.50) to the original formula of Meyer-Peter-Mueller (1948) for plane bed represented as \( \phi_b = 8(\theta - \theta_{cr})^{1.5} \), yields an expression for the slope factor:

\[
\alpha_s = \frac{1}{2} g^{-0.5} \sigma C I_b^{0.6} \left( \frac{\tau_b}{\tau_{b,cr}} \right)^{0.5}
\]  
(7.2.51)

in which:

\( \alpha_s \) = slope factor \( \alpha_s \geq 1 \)
\( \tau_b \) = bed-shear stress
\( \tau_{b,cr} \) = critical bed-shear stress of Shields corrected for the slope effect \( (\tau_{cr} = k_{\beta} \tau_{cr,o}) \).
Equation (7.2.51) which is only valid for downsloping flow, is shown in Fig. 7.2.17 for \( \tau_b / \tau_{b,cr} = 1.5, 2.5, 5 \) and \( \infty \), \( C = 35 \) and \( 70 \text{ m}^3/\text{s} \), \( d_{90} / d_{30} = 4 \) and plane bed. Figure 7.2.17 expresses that the Meyer-Peter-Mueller formula considerably underpredicts the transport rates at steep slopes (factor 2 to 10). The largest effect does occur for \( \tau_b \) close to \( \tau_{b,cr} \). The effect is smallest for \( \tau_b / \tau_{b,cr} = \infty \). The slope factor becomes important (\( \alpha_s > 1 \)) for a slope of about 0.01. For steep slopes the increase of the bed-load transport rate may be a factor 2 to 10, depending on the C-value. This effect is additional to the reduction of the critical bed-shear stress effect taken into account by the \( k\beta \) -factor (see Chapter 4). The slope factor of Bagnold appears to be much too small (see Fig. 7.2.17).

Rickermann (1991) studied the influence of fine sediments (wash load with volume concentrations up to 22%) on the bed-load transport of coarse materials at steep slopes. The wash load had no influence on the bed roughness in the turbulent flow regime. The water and sediment mixture behaved as a slurry with a different rheological behaviour than that of a clear water flow.

For a given bed slope and flow discharge, the clear-water bed-load transport rate showed a considerable increase (factor 2 to 3) for increasing slurry concentration (increasing fluid density), particularly at steeper slopes. This change did occur at almost the same depth and velocity implying that flow resistance did not significantly change. The increase of the bed-load transport was found to be mainly related to the decreasing relative density \( s = \rho_s / \rho_m \), \( \rho_s \) = sediment density, \( \rho_m \) = mixture density).

At slurry (clay) concentrations above 17% \( (\rho_m < 1280 \text{ kg/m}^3) \), a decrease of the bed-load transport rate was observed with further increase of the slurry concentration. This decrease was found to occur at a particle Reynolds number smaller than about 10, which means a viscous sublayer thickness larger than the grain diameter.

![Figure 7.2.17 Ratio of bed-load transport rate over sloping bed (q_{b,slope}) and over horizontal bed (q_{b,0})](image)

7.31
Figure 7.2.18 Definition sketch for a particle moving in a curved channel with transverse slope $\gamma$.

Figure 7.2.19 Ratio of bed-load transport in transverse direction ($q_{b,n}$) and in longitudinal direction ($q_{b,s}$).
2. Transverse bed slopes

Bed-load transport on transverse slopes was studied by Engelund (1974), and by Ikeda (1982, 1988).

Herein, the approach of Ikeda is followed. Based on analysis of the equations of motion in transverse \((n)\) and longitudinal direction \((s)\), the particle velocity in \(n\)-direction (see Fig. 7.2.18) was found to be:

\[
\frac{v_{p,n}}{v_p} = \frac{u_{b,n}}{u_b} + \varepsilon \left( \frac{u_{b,cr}}{u_b} \right) \tan \gamma \quad (7.2.52)
\]

in which:
- \(v_{p,n}\) = particle velocity in \(n\)-direction
- \(v_p\) = particle velocity vector
- \(u_{b,n}\) = near-bed fluid velocity in \(n\)-direction
- \(u_b\) = near-bed fluid velocity vector
- \(u_{b,cr}\) = critical near-bed fluid velocity (threshold)
- \(\gamma\) = transverse slope angle \((\tan \gamma = -\partial z_b/\partial y)\)
- \(\varepsilon\) = \((1 + \alpha \mu/\mu)\) = coefficient
- \(\alpha\) = ratio of lift and drag coefficient
- \(\mu\) = dynamic coefficient of Coulomb friction (0.5-0.8)

Assuming: \(v_p = v_{p,s}\), \(u_b = u_{b,s}\), \(u_{b,n}/u_{b,s} = \tan \delta\),

\[
\frac{u_{b,cr}}{u_b} = \left( \frac{\tau_{b,cr}}{\tau_b} \right)^{0.5} \quad \text{and} \quad q_{b,n} = \frac{v_{p,n}}{v_{p,s}} q_{b,s} \quad (7.2.54)
\]

It follows that:

\[
q_{b,n} = \left[ \tan \delta + \varepsilon \left( \frac{\tau_{b,cr}}{\tau_b} \right)^{0.5} \tan \gamma \right] q_{b,s} \quad (7.2.55)
\]

in which:
- \(q_{b,s}\) = bed-load transport in \(s\)-direction
- \(\tau_b\) = bed-shear stress in \(s\)-direction
- \(\tan \delta = u_{b,n}/u_{b,s}\) (\(\tan \delta = 0\) for transverse slopes in straight channels)

Based on measured \(q_{b,n}\)-values for transverse slopes along a straight channel in a wind-tunnel \((d_{50} = 260 \mu m\) and \(420 \mu m\)), the \(\varepsilon\)-factor was found to be about \(\varepsilon = 1.5\).

The ratio of \(q_{b,n}\) and \(q_{b,s}\) is shown in Figure 7.2.19 for \(\tan \delta = 0\), \(\varepsilon = 1.5\) and \(\tau_b/\tau_{b,cr} = 1.5\), 2, 5 and 10.

7.2.7 Bed-load transport of non-uniform material

1. Bed load transport

The bed material in natural conditions consists of non-uniform sediment particles. The effect of the non-uniformity of the bed material will result in selective transport processes which was studied by Einstein (1950), Egiazarov (1965), Samaga et al. (1986), Ribberink (1987) and others.
Generally, the approach is to divide the bed material in a number of size fractions and to apply an existing formula for each size fraction with a correction factor \( \xi_i \) to account for the non-uniformity effects. The correction is necessary because the coarser particles are more exposed to the flow than the finer particles which are somewhat sheltered by the coarse particles. This correction can be effectuated by increasing the critical shear-stress of the finer particles and decreasing the critical value of the coarser particles.

Using the formula of Meyer-Peter and Mueller (1948), the bed-load transport rate integrated over the fractions reads as:

\[
q_b = 8(s-1)^{0.5} \cdot g \cdot \sum_{i=1}^{N} p_i \cdot d_i^{1.5} \cdot (\mu \theta_i - \xi_i \theta_c)^{1.5}
\]  
(7.2.56)

in which:

- \( q_b \) = bed-load transport rate integrated over \( N \) size classes (fractions)
- \( p_i \) = percentage of size class \( i \) of the bed material
- \( d_i \) = particle diameter of size class \( i \)
- \( s \) = specific density (= 2.65)
- \( g \) = acceleration of gravity
- \( \mu \) = bed form factor
- \( \theta_i \) = mobility parameter of size class \( i \)
- \( \xi_i \) = correction factor of size class \( i \)
- \( \theta_c \) = critical mobility parameter based on the average diameter \( d_m \)
- \( d_m = \sum p_i \cdot d_i \) = average diameter of bed material
- \( N \) = number of size fractions

Equation (7.2.56) should be used when \( d_{50}/d_{50} \geq 5 \).

Generally, the correction factor \( \xi_i \) is assumed to be equal to unity for \( d_i = d_m \). Thus \( \xi_i = 1 \) for \( d_i = d_m \).

Egiazarov (1965) proposed:

\[
\xi_i = \left( \frac{\log(19)}{\log(19 \cdot d_i/d_m)} \right)^2
\]  
(7.2.57)

Equation (7.2.57) yields \( \xi_i = 5 \) for \( d_i/d_m = 0.2 \) and \( \xi_i = 0.4 \) for \( d_i/d_m = 5 \).

The particle diameter of the transported bed-load particles can be expressed as:

\[
d_b = \sum_{i=1}^{N} \left( \frac{q_{b,i}}{q_b} \right) d_i
\]  
(7.2.58)

Ribberink has carried out sensitivity computations applying the formula of Meyer-Peter and Mueller. Figure 7.2.20 shows the ratio \( q_{b,N}/q_b \) as a function of the number of size fractions \( N \) for 3 different gradation coefficients \( \sigma_g = 1.23, 1.96 \) and 2.45. The bed material was assumed to have a log-normal distribution. It can be observed that approximately 5 fractions are necessary to obtain a constant ratio \( q_{b,N}/q_b \). Furthermore, it appears that the transport rate is hardly affected by using the size-fraction method (maximum variation \( = 10 \% \)).

Klaassen (1991) performed flume experiments with uniform and non-uniform sand materials (\( d_{50} = 770 \mu m, \sigma_g = 1.06 \) and \( 5_{50} = 660 \mu m, \sigma_g = 2.34 \)).
The measured dune heights for non-uniform material were approximately 20% larger than for uniform material. The dune lengths were approximately 50% larger than for uniform material. The Chézy-coefficient was found to be larger for non-uniform material because of the decrease of the dune steepness (reduced form roughness). The bed-load transport rates for non-uniform material were approximately 50% larger than for uniform material.

Selective transport processes are most typical for gravel-bed rivers, because of the rather wide size distribution of the bed material (1 to 100 mm). Gravel-bed rivers are generally found near the head of a river system (upland regions) near the primary source of sediments. Sediment supply events tend to be episodic related to the presence of high flows. The most typical feature of a gravel bed is a surface cover that is relatively coarse in comparison with the bulk sediment mixture beneath it ($d_{50, \text{surface layer}} \approx d_{90, \text{subsurface layer}}$). The surface layer (thickness $d_{sp}$) is termed pavement or armour layer. The lower layer is termed subpavement, substrate or subsurface layer.

During low flows the finer particles are winnowed from the surface leaving a surface of coarser particles; which will act as an armour layer. During high flows the surface layer may become mobile depending on the magnitude of the applied bed-shear stress compared to the critical bed-shear stress. Even in high flows particle motion is sporadic; only a small fraction of the surface grains of a given size will be in motion. Fine grains may be eroded from the subsurface layer by eddies generated in the holes created by coarse grains just eroded by the flow. Laboratory experiments show that grains of all size classes are in motion at high flows (Parker and Klingemann, 1982; Parker et al, 1982; Thorne et al, 1987).

Parker and Klingemann (1982) proposed a transport formula (based on $d_{50}$ of subsurface layer) for gravel-bed rivers.

Ashiq et al (1992) found bad agreement between measured and computed transport rates for a gravel-bed river using the $d_{50}$ as characteristic diameter in the formulae of Meyer-Peter-Mueller (1948) and Parker et al (1982). The computed values were much too small.

---

**Figure 7.2.20** Influence of size fractions on bed-load transport (Ribberink, 1987)
2. Vertical sorting

Vertical sorting takes place in the upper layer of the bed which generally is covered with bed forms. The particles that are (occasionally) taking part in the transportation process are present in a layer with a thickness equal to the largest bed form height. Flume and field observations show a vertical sorting effect with finer particles dominating in the upper layers and coarser particles dominating in the lower layers at the trough level of the bed forms. This sorting effect can be explained by the transportation process. After the particles have passed the bed form crest, they roll and slide down the leeside slope of the bed form. As a result of gravity the coarsest particles have the largest probability to reach the deepest part of the bed form trough. Armouring of the bed occurs when an immobile fraction is present in the top layer of the bed. The finer particles will be carried away by the flow and the immobile particles will eventually form an armour layer.

3. Horizontal sorting (armour layer)

In non-uniform bed material an armour layer will be formed when the coarser particles of bed material have critical stresses which are larger than the mean bed-shear stress ($\tau_c$), whereas the finer particles have critical stresses smaller than the mean bed-shear stress. In that case the finer particles will be eroded and transported in downstream direction, but the coarser particles remain stable (immobile) and form an armour layer against erosion of sediment material of the lower layers. If the flow velocity increases to a point where the smaller particles of the armour layer can be disturbed, some of the finer particles become exposed and will be carried downstream over and around the (stable) coarser particles of the armour layer. Whenever, the protective armour particles are moved and carried downstream, the process of armouring with respect to the particles of the lower layer is repeated.

Coarsening of the bed was observed downstream of the Lake Mead reservoir in the USA (Lane et al, 1949). Livesey (1965) observed a flat armour layer consisting of one layer of gravel downstream of Fort Randall Dam in the Missouri river in the USA. The median size of the bed sediment before closure of the dam was about 170 $\mu$m. This material was still present below the armour layer.

Armouring may occur when there is no supply of sediment from upstream (sediments trapped in reservoirs) or when the supply from upstream consists of finer material than that of the local bed.

Gessler (1970, 1973) proposed a method to determine the composition of the armour layer and the eroded material.

The fraction ($p_{a,i}$) of particles of size class $i$ of the armour layer is given by:

$$p_{a,i} = \frac{r_i p_{o,d}}{\sum r_i p_{o,i}}$$  \hspace{1cm} (7.2.59)

The fraction ($p_{e,i}$) of particles of size class $i$ of the eroded material is given by:

$$p_{e,i} = \frac{(1-r_i) p_{o,d}}{\sum (1-r_i) p_{o,i}}$$  \hspace{1cm} (7.2.60)

in which:

$p_{o,i}$ = fraction of size class $i$ of original bed material

$r_i$ = probability that a particle from size class $i$ will remain stable
The probability $r_i$ was derived from experimental results and is shown in Figure 7.2.21 as a function of the ratio $\tau_{cr,i}/\tau_b$ with $\tau_{cr,i}$ = critical bed-shear stress of particle with diameter $d_i$ from size class $i$, and $\tau_b$ = mean bed-shear stress ($= \rho gh_l$).

Measured and computed particle size distributions show good agreement (see Fig. 7.2.22).

The $d_{50}$ of the armour layer was found to be approximately equal to the $d_{85}$ of the original bed material. The erosion depth above the armour layer was found to be about 1 to 2 $d_{85}$.

**Figure 7.2.21** Probability that a grain will be stable (Gessler, 1970)

**Figure 7.2.22** Particle size distributions (Gessler, 1970)
7.2.8 Comparison of bed-load transport formulae

A large amount (500) of bed-load transport rates ($d_{so}$ in the range of 110 to 1500 $\mu$m) measured in flume and field conditions have been selected from the literature to verify the formulae of Van Rijn, Meyer-Peter and Mueller, and Frijlink (see Van Rijn, 1986). The results are presented in terms of a discrepancy ratio ($r$) defined as the ratio of the computed and measured bed load transport rate. The percentage of $r$-values of all data falling in the ranges $0.75 \leq r \leq 1.5$; $0.5 \leq r \leq 2$ and $0.33 \leq r \leq 3$ are given in Table 7.5.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Percentage of $r$-values in range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0.75 \leq r \leq 1.5$</td>
</tr>
<tr>
<td>Van Rijn</td>
<td>34%</td>
</tr>
<tr>
<td>Meyer-Peter-Mueller</td>
<td>31%</td>
</tr>
<tr>
<td>Frijlink</td>
<td>31%</td>
</tr>
</tbody>
</table>

Table 7.5 Comparison of measured and computed transport rates

The results of the three methods are about the same. The formulae of Engelund-Hansen and Ackers-White (Section 7.3) produce similar values for large particle sizes ($d_{so} \geq 500 \mu$m). Finally the accuracy of the measured transport rates is discussed. A comparison of flume experiments performed under similar flow conditions (equal depth, velocity, particle size, temperature) by different research workers shows deviations of the transport rates up to a factor 2. Thus, even under controlled flume conditions, the accuracy of the measured values is rather low, which may be caused by the influence of the applied width-depth ratio, the applied adjustment period to establish uniform flow conditions and the applied experimental method (sand feed or recirculating flume).

Based on this, it is stated that it is hardly possible to predict the transport rate with an inaccuracy less than factor 2.

The formula of Meyer-Peter and Mueller (1948) and that of Frijlink (1952) are also shown in Figure 7.2.12 for a specific case ($C = 63 \text{ m}^{2}/\text{s}$, $h/d_{so} = 1000$ and $\mu = 0.76$).

7.2.9 Stochastic bed-load transport formulae

1. Introduction

Physically, the stochastic approach is more realistic than the deterministic approach because the stochastic nature (turbulence) of the fluid forces on the grains is taken into account. The proposed equations are, however, not easy applicable because of the statistics involved. Furthermore, for uniform flow the predicted transport rates based on stochastic theories are not more accurate than those based on deterministic theories. Both approaches need experimental data for calibration.

In strongly decelerating flow (adverse pressure gradient) the deterministic formulae do not give realistic results because the mean bed-shear stress will approach zero ($\tau_b \to 0$) in case of flow separation and reattachment. Hence, the computed transport rate will also approach zero. Physically, this is not realistic because at the location where $\tau_b$ approaches zero, the turbulent fluctuations usually are large and may generate bed-load transport. These effects can be represented by applying a stochastic approach, as introduced by Kalinske (1974), Einstein (1950), Yalin (1977) and by Van Rijn (1987). The stochastic approach may also give more accurate results for the computation of transport rates close to initiation of motion.
2. Kalinske (1947)

He assumed that the grains are transported in a layer with thickness equal to the particle diameter (d).
The instantaneous particle velocity is taken as:

\[ U_p = U - U_{cr} \quad (7.2.61) \]

in which:
- \( U \) = instantaneous fluid velocity at particle level
- \( U_{cr} \) = critical fluid velocity (threshold velocity)

The instantaneous fluid velocity is assumed to have a normal distribution with average value (u) and standard deviation (\( \sigma \)), as follows:

\[ f(U) = \frac{1}{(2\pi)^{1/2} \sigma} e^{-(U-u)^2/2\sigma^2} \quad (7.2.62) \]

The mean particle velocity is given by:

\[ U_p = \int_{U_{cr}}^{\infty} (U-U_{cr}) f(U) \, dU \quad (7.2.63) \]

The volumetric bed load transport is defined as:

\[ q_b = N \, V \, u_p \quad (7.2.64) \]

in which:
- \( N \) = number of moving grains per unit area (\( = p/(\frac{1}{4} \pi d^2) \))
- \( V \) = volume of a grain
- \( p \) = fraction of moving solid material per unit area (\( p = 0.35 \))

This leads to:

\[ q_b = \frac{2}{3} \, p \, d \, u_p \quad (7.2.65) \]

Equation (7.2.65) is shown in Figure 7.2.12 for a specific case (\( C = 63 \, m^{1/2}/s, \, h/d_{s0} = 1000, \, \mu = 0.76 \)).

3. Einstein (1950)

The most detailed theory of particle motion and transport was presented by Einstein (1950). His approach is based on the assumption that in steady uniform flow there is equilibrium between the number of particles eroded and deposited per unit area and time (\( F = D \)).

The erosion or pick-up rate is given by (see Eq. (7.2.22) and Figure 7.2.7):

\[ E = \alpha_1 \, [(s-1)g \, d]^{0.5} \, p \quad (7.2.66) \]
in which:
\[ p = \text{probability of erosion} \]
\[ \alpha_1 = \text{coefficient} \]

The deposition rate is defined as (see Eq. (7.2.3)):
\[ D = \frac{q_b}{\lambda} \quad (7.2.67) \]

in which:
\[ \lambda = \text{average travel distance of a particle} \]

Using Eqs. (7.2.66), (7.2.67) and \( E = D \), it follows that:
\[ q_b = \alpha_1 [(s-1)g d]^{0.5} \lambda^{-1} p \quad (7.2.68) \]

The average travel distance of a particle was found to be:
\[ \lambda = \frac{\alpha_2 d}{1-P} \quad (7.2.69) \]

in which:
\[ 1-P = \text{probability of deposition} \]
\[ \alpha_2 = \text{coefficient (}= 100) \]
\[ d = \text{particle diameter} \]

Using Eqs. (7.2.69) in Eq. (7.2.68), yields:
\[ q_b = \alpha_3 [(s-1)g]^{0.5} d^{1.5} \left( \frac{p}{1-P} \right) \]

or,
\[ \phi_b = \frac{1}{A_s} \left( \frac{p}{1-P} \right) \quad (7.2.70) \]

in which:
\[ \phi_b = \text{dimensionless bed-load transport} \]
\[ A_s = \text{empirical coefficient (}= 43.5) \]
\[ P = \text{probability of erosion} \]

The probability of erosion \( P \) was expressed as a function of the particle mobility parameter \( \theta' \):
\[ P = F \left( \frac{1}{\alpha \theta'} \right) \quad (7.2.71) \]
in which:

\[ P = \text{probability of erosion (see Fig. 7.2.7 for uniform material)} \]
\[ \theta' = \frac{\tau_b}{[(\rho_s - \rho)g d]} \text{ mobility parameter} \]
\[ \tau_r' = \rho gh' I = \text{effective bed-shear stress} \]
\[ h' = \text{effective water depth (see Einstein and Barbarossa, 1952)} \]
\[ I = \text{energy gradient} \]
\[ \alpha = \xi y \frac{\beta}{\beta} \frac{\alpha^2}{2} = \text{coefficient} \]

For graded bed material the transport is obtained by summation of the transport rate per fraction \( (\phi = \sum \phi_i) \). The \( \alpha \)-coefficient represents various effects related to the behaviour of particles in a graded mixture such as the hiding of smaller particles between the larger particles \( (\xi) \), the variation of lift coefficient \( (y) \) and the variation of particle positions \( (\beta/\beta) \)^2.

For almost uniform bed material it was proposed to use the \( d_{50} \) as the characteristic particle diameter related to particle mobility and the \( d_{50} \) as the effective grain roughness diameter. The \( \alpha \)-coefficient is unity \( (\alpha = 1) \) for uniform material. Figure 7.2.7 shows the probability of erosion \( (P) \) for uniform material. Equation (7.2.70) is shown in Figure 7.2.12 for a specific case.

Einstein (1950) described a very detailed theory taking many physical phenomena of the particle motions into account. Despite all physical details, the scatter of the data around the proposed function is not significantly less than that of other more simple methods. The justification of the Einstein approach is given by its detailed representation of physical phenomena involved, more than by its accuracy.

**Example**

**Given** : \( d_{50} = 800 \mu m, \tau_b = 1.3 \text{ N/m}^2, \) plane bed

**Compute** : \( \phi \) and \( q_b \)

**Solution** : \( \theta = \frac{\tau_b}{[(\rho_s - \rho)g d_{50}]} = 0.1 \)
\( P = 0.8, \) see Figure 7.2.7
\( \phi_b = (1/43.5) (0.8/0.2) = 0.092 \)
\( q_b = 0.092 (1.65 9.81)^{0.5} (0.0008)^{1.5} = 8.4 \times 10^{-6} \text{ m}^2/\text{s} \)

4. **Van Rijn (1987)**

It is assumed that the instantaneous bed-load transport rate is related to the instantaneous T-parameter, as follows (see Eq. (7.2.44)):

\[ q_b = \alpha (S - 1)^{0.5} g^{0.5} d_{50}^{1.5} D_{50}^{0.5} T_m^{0.5} \]  

\( 7.2.72 \)

in which:
\( T_m = (\tau'_b - \tau_{b,cr})/\tau_{b,cr} = \text{instantaneous shear stress parameter} \)
\( \tau'_b = \mu \tau_b = \text{instantaneous effective bed-shear stress} \)
\( \tau_{b,cr} = \text{instantaneous critical bed-shear stress} \)
\( \mu = \text{efficiency factor} \)
The effective bed-shear stress \( \tau'_b \) is assumed to have a normal distribution (mean value \( \overline{\tau}'_b \), standard deviation \( \sigma \)), as follows:

\[
P(\tau'_b) = \frac{1}{(2\pi)^{0.5} \sigma} e^{-\frac{(\tau'_b - \overline{\tau}'_b)^2}{2\sigma^2}}
\]  

(7.2.73)

The probability that \( \tau'_b > \tau_{b,cr,1} \) (positive in flow direction) or \( -\tau'_b < -\tau_{b,cr,2} \) (negative against flow direction) is:

\[
P(\tau'_b > \tau_{b,cr} \text{ or } -\tau'_b < -\tau_{b,cr}) = \frac{1}{(2\pi)^{0.5} \sigma} \left[ \int_{\tau_{b,cr,1}}^{\infty} e^{-\frac{(\tau'_b - \overline{\tau}'_b)^2}{2\sigma^2}} d\tau'_b + \int_{-\tau_{b,cr,2}}^{-\infty} e^{-\frac{(\tau'_b - \overline{\tau}'_b)^2}{2\sigma^2}} d\tau'_b \right]
\]

(7.2.74)

which can be represented as the shaded areas in Figure 7.2.23.

Applying Equation (7.2.74), the time-averaged shear stress parameter \( \langle T_m \rangle^{2.1} \) can be derived, as follows:

\[
\langle T_m \rangle^{2.1} = \frac{1}{(2\pi)^{0.5}} \left[ \left( \frac{\sigma}{\tau_{b,cr,1}} \right)^{2.1} I_3 - \left( \frac{\sigma}{\tau_{b,cr,2}} \right)^{2.1} |I_4| \right]
\]

(7.2.75)

in which:

\[
I_3 = \int_{-\infty}^{\infty} (x)^{2.1} e^{-\frac{1}{2}(x-r)^2} dx, \text{ see Fig. 7.2.23}
\]

\[
I_4 = \int_{-\infty}^{0} (x)^{2.1} e^{-\frac{1}{2}(x-p)^2} dx, \text{ see Fig. 7.2.23}
\]

\[
r = \frac{(+\tau'_b - \tau_{b,cr,1})}{\sigma}
\]

\[
p = \frac{(-\tau'_b - \tau_{b,cr,2})}{\sigma}
\]

\[
\tau'_b = \overline{\tau}' + \chi \sigma
\]

The \( I_3 \)-integral (\( I_3 = -I_4 \)) is shown in Figure 7.2.24. It is noted that the \( I_4 \)-integral has a negative sign, representing the transport of bed-load particles against the flow direction due to negative velocity fluctuations (against flow direction). Because the transport of material is related to a particular direction, the bed-load transport against the flow direction has to be subtracted from the value in the flow direction.

Based on the work of De Ruiter (1983), the instantaneous critical bed-shear stress was found to be:

\[
\tau_{b,cr} = 1.5 \overline{\tau}_{b,cr}
\]

(7.2.76)
The standard deviation of the effective bed-shear stress in uniform flow was found to be (De Ruiter, 1983):

$$\sigma = 0.4 \overline{\tau}_b$$  \hspace{1cm} (7.2.77)

Using Eqs. (7.2.75), (7.2.76) and (7.2.77), Equation (7.2.72) was recalibrated for uniform flow, yielding:

$$q_b = 0.1 (s-1)^{0.5} g^{0.5} d_{s0}^{1.5} D_*^{-0.3} T_m^{2.1}$$  \hspace{1cm} (7.2.78)

To apply Eq. (7.2.78), the values of $\sigma$, $\tau_{b,cr,1}$ and $\tau_{b,cr,2}$ must be known.

**Example**

**Given**: bed material $d_{s0} = 800 \mu m$, $D_* = 20$, plane bed, bed-shear stress $\tau_b = 1.3$ N/m$^2$, critical bed-shear stress $\tau_{b,cr} = 0.4$ N/m$^2$

**Compute**: bed-load transport using stochastic and deterministic method

**Solution**:

**Stochastic method, Eq. (7.2.78)**

$$\tau_{b,cr,1} = 1.5 \overline{\tau}_{b,cr} = (1.5) (0.4) = 0.6 \text{ N/m}^2$$

$$\tau_{b,cr,2}$$

$$\sigma = 0.4 \overline{\tau}_b = (0.4) (1.3) = 0.52 \text{ N/m}^2$$

$$r = (1.3 - 0.6)/0.52 = 1.35 \rightarrow I_s = 7 \hspace{1cm} \text{(Fig. 7.2.23)}$$

$$p = (-1.3 - 0.6)/0.52 = -3.65 \rightarrow I_\tau = 10^4 \hspace{1cm} \text{(Fig. 7.2.23)}$$

$$\overline{T_m}^{2.1} = \frac{1}{(2\pi)^{0.5}} \left[ \left( \frac{0.52}{0.6} \right)^{2.1} 7 - \left( \frac{0.52}{0.6} \right)^{2.1} 10^{-4} \right] = 2.07$$

$$q_b = (0.1) (1.65^{0.5}) (9.81^{0.5}) (0.0008^{1.5}) (20^{-0.3}) (2.07) = 7.7 \times 10^{-6} \text{ m}^2/\text{s}$$

**Deterministic method, Eq. (7.2.44)**

$$T = (\tau_b - \tau_{b,cr})/\tau_{b,cr} = (1.3 - 0.4)/0.4 = 2.25$$

$$q_b = (0.053) (1.65^{0.5}) (9.81^{0.5}) (0.0008^{1.5}) (20^{-0.3}) (2.25^{2.1}) = 10.8 \times 10^{-6} \text{ m}^2/\text{s}$$

**7.2.10 Examples and problems**

1. A wide river has a depth of $h = 3$ m. The depth-averaged velocity $\ddot{u} = 1.2$ m/s. The energy slope $I = 1.67 \times 10^{-4}$. The mean particle size of the bed material is $d_m = 2000 \mu m$, $d_{s0} = 1800 \mu m$, $d_{s0} = 3000 \mu m$.

Compute the bed-load transport according to Meyer-Peter-Mueller, Bagnold and Van Rijn?
Solution:

Meyer-Peter-Mueller:
\[ C = \bar{u}(h)^{0.5} = 5.36 \text{ m}^{1/2}/s \]
\[ C' = 18 \log(12h/d_{50}) = 73.4 \text{ m}^{1/4}/s \]
\[ \mu = (C/C')^{1.5} = 0.623 \]
\[ \theta' = \mu \theta = \frac{\mu \cdot h}{(s-1)d_m} = 0.0946 \]
\[ \phi_b = 8(\theta' - 0.047)^{1.5} = 0.083 \]
\[ q_b = \phi_b [(s-1)g]^{0.5} d_m^{1.5} = 3 \times 10^{-5} \text{ m}^2/\text{s} \]

Bagnold:
\[ \tau_b = \rho \cdot g \cdot h \cdot I = 4.91 \text{ N/m}^2 \]
\[ \tan \beta = I = 1.67 \times 10^{-4} \]
\[ \cos \beta = 1 \]
\[ \tan \phi = 0.6, \ c_b = 0.1 \]
\[ q_b = \frac{(0.1) \ (4.91) \ (1.2)}{(1650) \ (9.81) \ (1) \ (0.6)} = 6.1 \times 10^{-5} \text{ m}^2/\text{s} \]

Van Rijn (Eq. (7.2.44)):
\[ C' = 18 \log(12h/3d_{50}) = 64.8 \text{ m}^{1/4}/s \]
\[ \tau'_b = \rho g (\bar{u}/C')^2 = 3.36 \text{ N/m}^2 \]
\[ \tau_{b,cr} = 1.2 \text{ N/m}^2 \text{ (Shields)} \]
\[ T = (\tau'_b - \tau_{b,cr})/\tau_{b,cr} = 1.8 \]
\[ D_c = d_{50} [(s-1)g/v^2]^{1/3} = 45.5 \]
\[ q_b = 1.8 \times 10^{-5} \text{ m}^2/\text{s} \]

2. A wide river has the following characteristics: \( h = 2 \text{ m}, \ \bar{u} = 1.4 \text{ m/s}, \ I = 8 \times 10^{-4}, \ d_m = 800 \mu\text{m}, \ d_{50} = 700 \mu\text{m}, \ d_{90} = 1500 \mu\text{m}, \ v = 1 \times 10^{-6} \text{ m}^2/\text{s}. \)

Compute the bed-load transport rate according to Meyer-Peter-Mueller, Van Rijn and Bagnold?

Solution:
Meyer-Peter-Mueller:
\[ q_b = 1.4 \times 10^{-4} \text{ m}^2/\text{s} \]
Bagnold:
\[ q_b = 2.3 \times 10^{-4} \text{ m}^2/\text{s} \]
Van Rijn, Eq. (7.2.44):
\[ q_b = 1.1 \times 10^{-4} \text{ m}^2/\text{s} \]

3. A wide river has a depth of \( h = 3 \text{ m}. \) The depth-averaged velocity is \( \bar{u} = 1.2 \text{ m/s}. \) The energy gradient is unknown. The bed material characteristics are: \( d_{50} = 1800 \mu\text{m}, \ d_{90} = 3000 \mu\text{m}. \) The bed is plane (no bed forms). Other data are: \( v = 1.1 \times 10^{-6} \text{ m}^2/\text{s}, \rho_s = 2650 \text{ kg/m}^3. \)
Figure 7.2.23 Gaussian shear stress distribution

Figure 7.2.24 I-integrals
Compute the saltation height and length of the particles, the mean particle velocity according to Engelund-Fredsøe and Van Rijn, and the fluid velocity at the saltation height?

Solution:

Saltation height (Van Rijn) : \[ C' = C = 18 \log(12h/3d_{50}) = 64.8 \text{ m}^{1/4}/\text{s} \]
\[ D_* = 45.5 \text{ (see example 1)} \]
\[ T' = 1.8 \text{ (see example 1)} \]
\[ \delta_b = 0.3 \cdot d_{50}^{0.7} \cdot T^{0.5} = 0.0105 \text{ m} \]

Saltation length (Van Rijn) : \[ \lambda_b = 3 \cdot d_{50}^{0.5} \cdot T^{0.9} = 0.0906 \text{ m} \]

Particle velocity (Engelund) : \[ u_* = (g^{0.5}/C) \bar{u} = 0.058 \text{ m/s} \]
\[ \theta = u_*^2/[(s-1)g \cdot d_{50}] = 0.115 \]
\[ \theta_{cr} = 0.041 \text{ (Shields)} \]
\[ u_b = u_*/[10-7(\theta_{cr}/\theta)^{0.5}] = 5.8 \ u_* = 0.34 \text{ m/s} \]

Particle velocity (Van Rijn) : \[ u_b = 1.5 [(s-1)g \cdot d_{50}^{0.5}] \cdot T^{0.6} = 0.37 \text{ m/s} \]

Fluid velocity : \[ z = \delta_b = 0.0105 \text{ m}, \ u_* = 0.058 \text{ m/s} \]
\[ k_s = 3 \cdot d_{50} = 0.009 \text{ m}, \ u_*k_s/\nu = 520 \text{ (rough flow)} \]
\[ u = (u_*/\kappa) \ln(30z/k_s) = 0.52 \text{ m/s} \]

4. Same question as example 3. Use \( h = 3 \text{ m}, \ \bar{u} = 1.5 \text{ m/s}, \ l = 5 \times 10^4, \ d_{50} = 1800 \mu\text{m}, \ d_{90} = 3000 \ \mu\text{m}, \) dunes are present.

Compute saltation characteristics, particle velocity and fluid velocity at the saltation level?

Solution: use \( u_*' \) to estimate grain roughness

saltation height : 0.0144 m
saltation length : 0.16 m
particle velocity : 0.48 m/s (Engelund)
: 0.53 m/s (Van Rijn)
fluid velocity : 0.70 m/s

5. The bed of a river consists of non uniform material with grain sizes between 100 and 6000 \( \mu\text{m} \). The size fractions are given in Table 7.6. The \( d_{50} \) of the bed material is 980 \( \mu\text{m} \); the \( d_{90} \) is 3600 \( \mu\text{m} \). The water depth is \( h = 5 \text{ m} \). The bed slope is \( I_b = 10^{-4} \). The \( \text{Chézy-coefficient} \) is \( C = 50 \text{ m}^{1/2}/\text{s} \). Other data are: \( \rho_s = 2650 \text{ kg/m}^3, \rho = 1000 \text{ kg/m}^3, \) \( v = 1 \times 10^{-6} \text{ m}^2/\text{s} \).

Compute the bed-load transport according to Meyer-Peter-Mueller, using the mean particle diameter and the size-fraction method?
Solution:

a. based on mean particle diameter

mean grain diameter : \( d_m = \sum p_i d_i = 1460 \mu m \)

grain roughness : \( C' = 18 \log(12h/d_{sp}) = 76 \text{ m}^2/\text{s} \)

bed-form factor : \( \mu = (C/C')^{1.5} = 0.533 \)

bed-shear stress : \( \tau_b = \rho g h I = 4.9 \text{ N/m}^2 \)

mobility parameter : \( \theta = \tau_b/[(\rho_s-\rho)g d_m] = 0.207 \)

\( \mu \theta = 0.11 \)

bed load transport : \( \phi_b = 8(\mu \theta-0.047)^{1.5} = 0.127 \)

\( q_{b,c} = 0.127(s-1)^{0.5} g^{0.5} d_m^{1.5} = 2.8 \times 10^4 \text{ m}^2/\text{s} \)

b. based on size-fraction method

critical mobility parameter : \( D_s = d_m[(s-1)g/\nu^2]^{1/3} = 37 \)

\( \theta_{cr} = 0.037 \text{ (Shields)} \)

The bed load transport rate per fraction is given in Table 7.6.

<table>
<thead>
<tr>
<th>size class ((\mu m))</th>
<th>(d_i) ((\mu m))</th>
<th>(p_i)</th>
<th>(\xi_i)</th>
<th>(\mu \theta_i)</th>
<th>(\lambda = p_i(\mu \theta_i-\xi_i \theta_{cr})^{1.5})</th>
<th>(q_{bd} = 8(s-1)^{0.5} g^{0.5} d_i^{1.5} \lambda)</th>
<th>(q_{bd}) ((\text{m}^2/\text{s}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>100-500</td>
<td>300</td>
<td>0.30</td>
<td>4.67</td>
<td>0.538</td>
<td>0.0662</td>
<td>11.0 \times 10^4</td>
<td></td>
</tr>
<tr>
<td>500-1000</td>
<td>750</td>
<td>0.22</td>
<td>1.67</td>
<td>0.215</td>
<td>0.0132</td>
<td>8.7 \times 10^4</td>
<td></td>
</tr>
<tr>
<td>1000-2000</td>
<td>1500</td>
<td>0.23</td>
<td>0.98</td>
<td>0.108</td>
<td>0.0044</td>
<td>8.2 \times 10^4</td>
<td></td>
</tr>
<tr>
<td>2000-3000</td>
<td>2500</td>
<td>0.10</td>
<td>0.71</td>
<td>0.065</td>
<td>0.0008</td>
<td>3.1 \times 10^4</td>
<td></td>
</tr>
<tr>
<td>3000-4000</td>
<td>3500</td>
<td>0.08</td>
<td>0.59</td>
<td>0.046</td>
<td>0.0003</td>
<td>2.0 \times 10^4</td>
<td></td>
</tr>
<tr>
<td>4000-5000</td>
<td>4500</td>
<td>0.05</td>
<td>0.52</td>
<td>0.036</td>
<td>0.0001</td>
<td>1.1 \times 10^4</td>
<td></td>
</tr>
<tr>
<td>5000-6000</td>
<td>5500</td>
<td>0.02</td>
<td>0.48</td>
<td>0.029</td>
<td>0.0002</td>
<td>0.3 \times 10^5</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>34.4 \times 10^4 m^2/s</td>
<td></td>
</tr>
</tbody>
</table>

**Table 7.6 Bed-load transport based on size fraction method**

The size-fraction method yields a value which is only 20% larger than that based on the mean particle diameter.

6. The bed material of a river consists of non-uniform material with 20% between 100 and 500 \( \mu m \), 30% between 500 and 1000 \( \mu m \), 20% between 1000 and 2000 \( \mu m \), 20% between 2000 and 3000 \( \mu m \) and 10% between 3000 and 4000 \( \mu m \). Other data see example 5.

Compute bed-load transport using mean particle size and size fraction method (Meyer-Peter-Mueller formula)?
Solution: mean particle size \( q_{b,c} = 2.7 \times 10^4 \) m²/s
size fraction method \( q_{b,c} = 3.3 \times 10^2 \) m²/s

7. A reservoir has been built in a river. The bed of the river section downstream of the reservoir consists of non-uniform material with a size distribution according to Table 7.1.

What is the composition of the armour layer which will be formed downstream of the reservoir, when the flow passing this section exerts a bed-shear stress of \( \tau_b = 2.5 \) N/m²?

Solution: use method of Gessler, see Table 7.7

\[ \tau_{b,cr,i} \] according to Shields
\[ r_i \] according to Gessler

<table>
<thead>
<tr>
<th>size class</th>
<th>( d_i ) (µm)</th>
<th>( p_{a,i} )</th>
<th>( \tau_{cr,i} ) (N/m²)</th>
<th>( \frac{\tau_{cr,i}}{\tau_b} )</th>
<th>( r_i )</th>
<th>( r_i p_{a,i} )</th>
<th>( p_{a,i} )</th>
<th>( \sum p_{a,i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100-500</td>
<td>300</td>
<td>0.30</td>
<td>0.18</td>
<td>0.07</td>
<td>0.05</td>
<td>0.015</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>500-1000</td>
<td>750</td>
<td>0.25</td>
<td>0.36</td>
<td>0.14</td>
<td>0.07</td>
<td>0.018</td>
<td>0.09</td>
<td>0.17</td>
</tr>
<tr>
<td>1000-2000</td>
<td>1500</td>
<td>0.20</td>
<td>0.85</td>
<td>0.34</td>
<td>0.15</td>
<td>0.030</td>
<td>0.16</td>
<td>0.33</td>
</tr>
<tr>
<td>2000-3000</td>
<td>2500</td>
<td>0.10</td>
<td>1.7</td>
<td>0.68</td>
<td>0.30</td>
<td>0.030</td>
<td>0.16</td>
<td>0.49</td>
</tr>
<tr>
<td>3000-4000</td>
<td>3500</td>
<td>0.08</td>
<td>2.2</td>
<td>1.12</td>
<td>0.55</td>
<td>0.044</td>
<td>0.23</td>
<td>0.72</td>
</tr>
<tr>
<td>4000-5000</td>
<td>4500</td>
<td>0.05</td>
<td>3.6</td>
<td>1.44</td>
<td>0.75</td>
<td>0.038</td>
<td>0.19</td>
<td>0.91</td>
</tr>
<tr>
<td>5000-6000</td>
<td>5500</td>
<td>0.02</td>
<td>4.6</td>
<td>1.84</td>
<td>0.92</td>
<td>0.018</td>
<td>0.09</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\[ \sum p_{a,i} = 0.193 \]

**Table 7.7 Armour layer composition**

Based on the \( d_i \) and \( \sum p_{a,i} \)-columns of Table 7.7, the characteristic diameters of the armour layer are: \( d_{50} = 2500 \) µm, \( d_{90} = 4500 \) µm.

8. Same question as that of example 7.
Use the bed material composition of example 6.
The applied bed-shear stress downstream of the reservoir is \( \tau_b = 2 \) N/m².

Solution: armour layer \( d_{50} = 2600 \) µm, \( d_{90} = 3700 \) µm
\[ p_{a,i} = 0.05, 0.10, 0.16, 0.36, 0.33 \]

9. A wide mountain river has a bed slope of \( I = 10^{-1} \).
The water depth is \( h = 0.1 \) m. The bed is plane. The bed material characteristics are \( d_m = 0.02 \) m, \( d_{30} = 0.012 \) m, \( d_{x0} = 0.03 \) m. Other data are \( \rho_s = 2650 \) kg/m³, \( \rho = 1000 \) kg/m³, angle of repose \( \phi = 30^\circ \).

What is the dimensionless bed-load transport according to Meyer-Peter-Mueller and according to Smart?

Solution: Meyer Peter-Mueller \( \phi_b = 1.1 \)
Smart \( \phi_b = 1.6 \)
7.3 Suspended load transport

7.3.1 Introduction

Usually, these modes of motions of the bed material particles are distinguished:

• rolling and sliding;
• saltation, and
• suspension.

When the value of the bed-shear velocity just exceeds the critical value for initiation of motion, the particles will start rolling and sliding in continuous contact with the bed. For increasing values of the bed-shear stress the particles will perform more or less regular jumps which are called saltations.

When the value of the bed-shear velocity exceeds the particle fall velocity, the particles can be lifted to a level at which the upward turbulent forces will be comparable to or higher than the submerged particle weight and as a result of the particle with the bed in the suspension mode is occasional and random. The particle velocity in longitudinal direction is almost equal to the fluid velocity. Usually, the behaviour of the suspended sediment particles is described in terms of the sediment concentration, which is the solid volume (m$^3$) per unit fluid volume (m$^3$) or the solid mass (kg) per unit fluid volume (m$^3$).

The hydraulic conditions at initiation of suspension are described in Section 4.4. According to Bagnold (1966): suspension will occur for $u_\ast > w_s$. According to Van Rijn (1984b) suspension will start at considerably smaller bed-shear velocities.

Observations show that the suspended sediment concentrations decrease with distance up from the bed. The rate of decrease depends on the ratio of the fall velocity and the bed-shear velocity ($w_s/u_\ast$).

The depth-integrated suspended-load transport ($q_{s,c}$) is herein defined as the integration of the product of velocity ($u$) and concentration ($c$) from the edge of the bed-load layer ($z=a$) to the water surface ($z=h$), yielding (see Figure 7.3.1):

\[
q_{s,c} = \int_a^h uc \, dz
\]  

(7.3.1)

or

\[
q_{s,c} = c_a \overline{u} \frac{h}{h} \int_a^h \frac{u}{u} \frac{c}{c_a} \, dz = c_a \overline{u} h F
\]  

(7.3.2)

with

\[
F = \int_a^h \frac{u}{u} \frac{c}{c_a} \, d(z/h)
\]  

(7.3.3)
in which:
\[ q_{bc} \] - volumetric suspended load transport (\( \text{m}^2/\text{s} \))
\[ u \] = fluid velocity at height \( z \) above bed
\[ c \] = sediment concentration (volume) at height \( z \) above bed
\[ u \] = depth-averaged fluid velocity
\[ c_a \] = reference concentration at height \( z = a \) above bed
\[ h \] = water depth
\[ F \] = dimensionless shape factor (see Van Rijn, 1984b, Dyer and Soulsby, 1988 and McLean, 1991b)

Application of Eq. (7.3.1) requires information of the velocity profile, concentration profile and a known concentration \( (c_v) \) close to the bed \( (z = a) \). These latter parameters are referred to as the reference concentration and the reference level, \( c_a \) at \( z = a \).

Sometimes, the suspended load transport is given as a mean volumetric concentration defined as the ratio of the volumetric suspended load transport (= sediment discharge) and the flow discharge:

\[ c_{\text{mean}} = \frac{q_{bc}}{q} \quad (7.3.4) \]

The mean concentration \( (c_{\text{mean}}) \) is approximately equal to the depth-averaged concentration \( (\bar{c}) \) for fine sediments.

The concentration can be expressed as a weight concentration \( (c_p) \) in \( \text{kg/m}^3 \) or as a volume concentration \( (c_v) \) in \( \text{m}^3/\text{m}^3 \). Both are related through: \( c_p = \rho_s c_v \). Sometimes the volume concentration is expressed as a volume percentage after multiplying with 100%.

In the American literature the sediment concentration often is expressed as a concentration \( (c_p) \) in parts per million by weight. In this latter case the sediment mass in a sample is multiplied by \( 10^6 \) and divided by the total mass (sediment and fluid) of the same sample. The \( c_p \), \( c_v \) and \( c_a \)-values are related by:

\[ c_p = \frac{10^6 \cdot s \cdot c_v}{1 + (s-1)c_v} = \frac{10^6 \cdot s \cdot c_v}{\rho_s + (s-1)c_a} \quad (7.3.5) \]

with \( s = \rho_s/\rho \) = specific density = 2.65.

For low concentrations \( c_v < 10 \text{ kg/m}^3 \) it follows that \( c_p = 10^6 s c_v \) or \( c_p = \left(\frac{10^6}{\rho} \right) c_v \).

Some rivers carry very high concentrations of fine sediments (particles < 50 \( \mu \text{m} \)), usually referred to as the wash load.

Experience shows that the presence of fines enhances the suspended sand transport rate because the fluid viscosity and density are increased by the fine sediments. As a result the fall velocity of the suspended sand particles will be reduced with respect to that in clear water and hence the suspended sand transport capacity of the flow will increase.

A drop in temperature will have the same effect, Lane et al (1949) reported an increase of the measured sand transport rate by a factor 2 from summer to winter (temperature drop of about 15°C) at the same discharge.

Many analytic models are available to describe the sediment concentration distribution over the water depth.

7.50
These models can be divided into:
- diffusion models,
- energy models (see Bogardi, 1974),
- stochastic models (see Ashida-Fujita, 1986).

Herein, only diffusion models and energy models (Bagnold, 1966) are discussed.

![Figure 7.3.1 Definition sketch suspended load transport](image1)

**Figure 7.3.1 Definition sketch suspended load transport**

![Figure 7.3.2 Definition sketch vertical sediment diffusion](image2)

**Figure 7.3.2 Definition sketch vertical sediment diffusion**

### 7.3.2 Mass balance equation suspended sediment (diffusion model)

A complete derivation of the 3D-mass balance for the suspended sediment particles is given in section 12.4.2. Here, a simple derivation for suspended sediment particles in a steady and uniform turbulent flow is presented. The particles are assumed to have a uniform size (d) and fall velocity (w).

The instantaneous sediment concentration at a certain level 1 is assumed to be $c + c'$. The instantaneous sediment concentration at level 2 is $c - c'$ (see Figure 7.3.2), with $c =$ time-averaged concentration and $c' =$ concentration fluctuation (by definition $\overline{c'} = 0$ in steady uniform flow).
The instantaneous upward sediment velocity at level 1 is \( w' - w_s \), with \( w' = \) vertical velocity fluctuation (by definition \( \bar{w}' = 0 \) in steady uniform flow). The instantaneous downward sediment velocity at level 2 is \( w' + w_s \).

The time-averaged upward transport is:

\[
q_u = \frac{(w' - w_s)(c + c')}{c} = -cw_s + \overline{c'w'}
\]  
(7.3.6)

The time-averaged downward transport is:

\[
q_d = \frac{(w' + w_s)(c - c')}{c} = cw_s - \overline{c'w'}
\]  
(7.3.7)

In steady uniform flow the time-averaged upward and downward transport rates are equal, giving \( q_u = q_d \):

\[
cw_s - \overline{c'w'} = 0
\]  
(7.3.8)

The \( cw_s \) term represents the downward transport of sediment by gravity. The \( \overline{c'w'} \) term represents the upward transport by the turbulent velocity fluctuations.

Adopting the diffusion model (also known as the law of Fick), the vertical transport of sediment by turbulence \( \overline{c'w'} \) is proportional to the vertical concentration gradient \( (dc/dz) \). The proportionality coefficient is called the diffusion or mixing coefficient \( (\varepsilon_s) \). Thus:

\[
\overline{c'w'} = -\varepsilon_s \frac{dc}{dz}
\]  
(7.3.9)

Combining Eqs. (7.3.8) and (7.3.9), yields:

\[
cw_s + \varepsilon_s \frac{dc}{dz} = 0
\]  
(7.3.10)

in which:

- \( c \) = time averaged sediment concentration at height \( z \) above bed
- \( w_s \) = particle fall velocity (constant)
- \( \varepsilon_s \) = mixing coefficient at height \( z \) above bed

Equation (7.3.10) is valid for low concentrations because the fall velocity is assumed to be constant. In mixtures of high concentrations the fall velocity is not constant but dependent on the concentration because the particles are hindering each other during settling. The falling particles are affected by the return flow of the displaced fluid but also by additional effects such as particle collisions, particle induced turbulence, modified drag coefficients and group effects. The overall effect can be represented by (see Section 3.2.5):

\[
w_{s\text{mix}} = (1 - c)^\gamma w_s
\]  
(7.3.11)
in which:

\( w_{s,m} \) = particle fall velocity in a mixture

\( w_* \) = particle fall velocity in clear still fluid

\( \gamma \) = coefficient (4 to 5 for particles of 50 to 500 \( \mu \)m)

7.3.3 Fluid and sediment mixing coefficient

Various distributions of the fluid mixing coefficient \( \epsilon_f \) can be found in the Literature. Herein, the following distributions are given (see Figure 7.3.3):

\[
\text{constant} \quad : \quad \epsilon_f = \frac{1}{\alpha_1} \kappa u_* h \quad (7.3.12)
\]

\[
\text{linear} \quad : \quad \epsilon_f = \frac{1}{\alpha_2} \kappa u_* h \frac{z}{h} \quad (7.3.13)
\]

\[
\text{parabolic} \quad : \quad \epsilon_f = \kappa u_* h \frac{z}{h} \frac{(1 - z)}{h} \quad (7.3.14)
\]

\[
\text{parabolic-constant} \quad : \quad \begin{align*}
\epsilon_f &= \kappa u_* h \frac{z}{h} \frac{(1 - z)}{h} \text{ for } \frac{z}{h} \leq 0.5 \\
&= 0.25 \kappa u_* h \text{ for } \frac{z}{h} \geq 0.5
\end{align*} \quad (7.3.15)
\]

in which:

\( u_* \) = current-related bed-shear velocity

\( h \) = waterdepth

\( z \) = vertical coordinate

\( \kappa \) = Von Karman constant (0.4)

\( \alpha_1, \alpha_2 \) = coefficient

The parabolic distribution is most satisfactory in a physical sense because it is based on a linear shear stress distribution and a logarithmic velocity profile. A disadvantage of the parabolic distribution is that it yields a zero-concentration at the water surface. Mixing coefficients based on the analysis of measured concentration profiles (Coleman, 1970) indicate a parabolic-constant distribution rather than a parabolic one, as shown in Figure 7.3.4.

Usually, the mixing or diffusion of the sediment particles is related to the fluid mixing coefficient for a clear fluid \( (\epsilon_f) \), as follows:

\[
\epsilon_s = \beta \phi \epsilon_f \quad (7.3.16)
\]
Figure 7.3.3 Fluid mixing coefficients

Figure 7.3.4 Sediment mixing coefficient according to Coleman (1970)
\( \beta \)-factor

The \( \beta \)-factor describes the difference in the diffusion of a fluid "particle" (or small coherent fluid structure) and a discrete sediment particle. Herein, the \( \beta \)-factor is assumed to be constant over the flow depth.

Carstens (1952) concluded that \( \beta < 1 \) because the sediment particles cannot fully respond to the turbulent velocity fluctuations, which were assumed to be one dimensional. Singamsetti (1966) found \( \beta > 1 \) for two-dimensional eddy motion because of the presence of centrifugal forces acting on the particles (higher density) causing the particles to be thrown to the outside of the eddies with a consequent increase of the effective mixing length.

Information of the \( \beta \)-values can also be obtained from the experimental results of Coleman (1970) who measured concentration profiles and fall velocities in flume and field conditions. Based on Eq. (7.3.10), Coleman computed the sediment mixing coefficient from the measured variables: \( \varepsilon_c, \theta_c/dz \) and \( w_a \), see Figure 7.3.4. Van Rijn (1984b) used the results of Coleman to determine the \( \beta \)-factor, defined as \( \beta = \varepsilon_{s,\text{max}}/\varepsilon_{f,\text{max}} \). The \( \varepsilon_{f,\text{max}} \) value was computed from Equation (7.3.14) for \( z/h = 0.5 \). The \( \varepsilon_{s,\text{max}} \)-value was determined as the average value of the \( \varepsilon_s \)-values in the upper half of the flow (as given by Coleman, Figure 7.3.4) where the concentrations are relatively small. The \( \beta \)-factors can be represented by the following function:

\[
\beta = 1 + 2 \left( \frac{w_s}{u_*} \right)^2 \quad \text{for} \quad 0.1 < \frac{w_s}{u_*} < 1 \quad (7.3.17)
\]

Equation (7.3.17) specifies a value larger than unity indicating a dominant influence of the centrifugal forces which cause the particles to be thrown to the outside of the eddies with a consequent increase of the effective mixing length. Given the limited knowledge of the physical processes involved, it is not advisable to use a \( \beta \)-factor larger than 2.

\( \phi \)-factor

The \( \phi \)-factor expresses the influence of the sediment particles on the turbulence structure of the fluid. This effect is extremely important in the upper regime with high concentrations (\( \bar{c} > 10 \, \text{kg/m}^3 \)) because this results in stratification and hence damping of turbulence (see Section 7.3.9).

7.3.4 Concentration profiles

Expressions for the sediment concentration profiles in the lower regime (low concentrations \( c < 10 \, \text{kg/m}^3 \)) can be obtained by integration of the convection-diffusion equation (Eq. 7.3.10) using relevant expressions for the mixing coefficient. As bed-boundary condition a reference concentration \( \bar{c}_a \) at a small distance \( \alpha \) above the mean bed is applied (integration constant).

Hindered settling effects and turbulence damping effects can be neglected in the lower regime \( (w_{s,m} = w_s \text{ and } \phi = 1) \).

Integration of Eq. (7.3.10) yields:

\[
\text{constant} \quad \varepsilon_f : \quad \frac{c}{c_a} = e^{-\alpha \left( w_s/\left( \beta \times u_* \right) \right)/(\alpha - \alpha)/h} \quad (7.3.18)
\]

\[
\text{linear} \quad \varepsilon_f : \quad \frac{c}{c_a} = \left( \frac{a}{z} \right)^{a z/\left( \beta \times u_* \right)} \quad (7.3.19)
\]

7.55
parabolic \[ \varepsilon_f : \frac{c}{c_a} = \left( \frac{h-z}{z} \cdot \frac{a}{h-a} \right)^{w_s/(\beta \times u_*)} \] (7.3.20)

parabolic-constant \[ \varepsilon_f : \frac{c}{c_a} = \left( \frac{h-z}{z} \cdot \frac{a}{h-a} \right)^{w_s/(\beta \times u_*)} \quad \text{for} \quad \frac{z}{h} < 0.5 \] (7.3.21)

\[ \frac{c}{c_a} = \left( \frac{a}{h-a} \right)^{w_s/(\beta \times u_*)} \quad (e)^{-4 \times (w_s/(\beta \times u_*))(z/h-0.5)} \quad \text{for} \quad \frac{z}{h} \geq 0.5 \]

in which:
- \( c \) = concentration at height \( z \) above the mean bed
- \( c_a \) = reference concentration at height \( z - a \) above bed
- \( h \) = water depth
- \( w_s \) = fall velocity in clear water
- \( u_* \) = bed-shear velocity

The parameter \( Z = w_s/(\beta \times u_*) \) is termed the suspension number.

Figure 7.3.5 shows the above-given concentration profiles for \( w_s/u_* = 0.2 \), \( \beta = 1 \), \( \kappa = 0.4 \) giving \( Z = 0.5 \).

Equation (7.3.20) also known as the Rouse concentration profile and Eq. (7.3.21) yield the best agreement with measured concentrations. Hunt (1953) proposed a concentration profile slightly different from the Rouse concentration profile. Figures 7.3.6 and 7.3.7 show Eq. (7.3.20) for different values of \( Z \) in comparison with measured concentrations (Vanoni, 1946). Equation (7.3.21) yields a finite concentration at the water surface, while Eq. (7.3.20) gives a zero concentration at the water surface which is less realistic.

Chien (1954) applied Eq. (7.3.20) to determine the \( Z \)-parameter from measured concentration profiles. The measured \( Z \)-values \( (Z = w_s/\beta \times u_*) \) were plotted against computed \( Z \)-values \( (Z = w_s/\kappa \times u_*) \), as shown in Figure 7.3.8. The results show smaller measured \( Z \) values. This can be interpreted as a \( \beta \) factor larger than 1 (\( \beta > 1 \)).

Based on Figure 7.3.7, the relative importance of the suspended sediment load can be determined, see Table 7.8.

<table>
<thead>
<tr>
<th>( Z = \frac{w_s}{\beta \times u_*} )</th>
<th>( \frac{u_*}{w_s} ) (( \kappa = 0.4 ), ( \beta = 1 ))</th>
<th>Suspended sediment distribution over the depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.5</td>
<td>suspended sediment in near-bed layer ( (z &lt; 0.1 \ h) )</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>suspended sediment up to mid depth ( (z &lt; 0.5 \ h) )</td>
</tr>
<tr>
<td>1</td>
<td>2.5</td>
<td>suspended sediment up to water depth</td>
</tr>
<tr>
<td>0.1</td>
<td>25</td>
<td>suspended sediment almost uniformly distributed over depth</td>
</tr>
</tbody>
</table>

*Table 7.8 Suspended sediment distribution over the depth*
Figure 7.3.5 Concentration profiles

<table>
<thead>
<tr>
<th>Curve</th>
<th>Symbol</th>
<th>$z$</th>
<th>Run number</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>□</td>
<td>1.46</td>
<td>17</td>
</tr>
<tr>
<td>B</td>
<td>◊</td>
<td>1.03</td>
<td>15,16</td>
</tr>
<tr>
<td>C</td>
<td>▲</td>
<td>0.89</td>
<td>14</td>
</tr>
<tr>
<td>D</td>
<td>◊</td>
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<td>19,22</td>
</tr>
<tr>
<td>E</td>
<td>◊</td>
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<td>18</td>
</tr>
<tr>
<td>F</td>
<td>◊</td>
<td>0.34</td>
<td>21,21</td>
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</table>

Figure 7.3.6 Concentration profiles based on parabolic mixing coefficients, Vanoni (1946)
Figure 7.3.7 Concentration profiles based on parabolic mixing coefficient ($Z = 1/32$ to 4)

Figure 7.3.8 Measured and computed $Z$-values according to Chien (1954)
Influence of bed forms

When bed forms are present, it is not fully clear whether the grain-shear velocity \( u' \) or the overall bed shear velocity \( u_* \) should be used to compute the Z-parameter. Einstein (1950), later followed by Engelund and Fredsøe (1976), proposed to use the \( u' \)-value. Thus, they neglect the influence of the vortex zone (downstream of the crest) on the mixing process of the suspended sediments. As shown in Figure 5.2.11b, the vortex zone has a large effect on the turbulence structure over a major part of the dune. According to Van Rijn (1984b), these effects cannot be neglected and it was proposed to use the overall \( u_* \)-value to compute the Z-parameter.

Ikeda (1980) and Ikeda-Asaeda (1983) measured concentration profiles in experiments with and without bed forms. Their experimental results show a large effect of the bed forms on the concentration profiles. He observed that the concentration profiles measured in experiments with bed forms were much more uniform than those measured in an experiment with a plane bed at the same hydraulic conditions \( w/u_* \approx 0.6 \). The bed forms seem to cause a more intensive mixing of suspended particles.

7.3.5 Velocity profiles in lower regime

In the lower transport regime the sediment concentrations are relatively low \( (c < 10 \text{ kg/m}^3) \) and do not influence the turbulence structure of the flow (no stratification effect).

In case of hydraulic rough flow over a flat bed the velocity profile can be described by a logarithmic distribution, as follows:

\[
\frac{u}{u_*} = \frac{1}{\kappa \ln \left( \frac{z}{z_o} \right)}
\]  

(7.3.22)

in which:

\( u \) = current velocity at height \( z \) above mean bed level
\( u_* \) = current-related bed-shear velocity
\( z_o \) = zero-velocity level \( (= 0.033 \text{ k}) \)
\( k_s \) = equivalent or effective roughness height of Nikuradse
\( \kappa \) = Von Karman constant \( (= 0.4) \)

In case of a dune-covered bed, which is the most common condition in the lower regime, the flow basically is non uniform showing acceleration effects upstream of the crests and deceleration effects downstream of the crests (see Figure 5.2.11a). Figure 7.3.9 shows the measured velocity profiles (as presented in Figure 5.2.11a) in one plot. The layer influenced by the dunes has a thickness of about 1.5 \( \Delta_d \) with respect to the mean bed level.

The bed form also has a considerable effect on the turbulence structure, as presented in Figure 5.2.11b. The turbulence intensities in and around the vortex zone downstream of the crest show a considerable increase.

A simple method describing the non-uniform behaviour of the velocity profiles is not available. Detailed representation of these types of disturbed velocity profiles requires the application of a hydrodynamic model in combination with a higher order turbulence model (K-Epsilon model, see Rodi, 1980).

Space-averaged velocity profiles can to a certain extent (above the crest level) be represented by the method of Nikuradse which is based on the application of an equivalent roughness height \( (k_s) \), see Eq. (7.3.22). Figure 7.3.9 shows Eq. (7.3.22) for \( k_s = 0.063 \text{ m} \) which was derived from the known Chézy-coefficient \( (C = 32 \text{ m}^{1/2}/\text{s}) \). As can be observed, the deviations
between measured and computed velocities are large in the region below the crest level, but are considerably smaller above the crest level (maximum relative error of about 20%). McLean (1991) proposed a two-layer model for spatially-averaged flow over dunes; logarithmic functions were used to describe the velocity profiles in both layers. This method also yields inaccurate near-bed velocities because the horizontal non-uniformity is not taken into account.

\[ u = 0.53 \text{ m/s} \]
\[ h = 0.33 \text{ m} \]
\[ \Delta = 0.08 \text{ m} \]
\[ \lambda = 1.60 \text{ m} \]

<table>
<thead>
<tr>
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<th>distance from crest (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
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</tr>
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<td>1.27</td>
</tr>
<tr>
<td>10</td>
<td>1.58</td>
</tr>
</tbody>
</table>

**Figure 7.3.9** Velocity profiles in case of a dune-covered bed
7.3.6 Reference concentration and reference level

1. Flat bed

The most logical assumption for the reference level \((z = a)\) of the sediment concentration profile is the upper edge of the bed-load layer \((z = \delta_b)\). The reference concentration \((c_a)\) is defined to be equal to the bed load concentration \((c_b)\). Thus,

\[
c_a - c_b \text{ at } z = a - \delta_b
\]  \hspace{1cm} (7.3.23)

A good estimate of the thickness of the bed-load layer in case of a flat bed can be obtained from Eq. (7.2.13) which expresses the saltation height of the bed-load particles (Van Rijn, 1984a). Equation (7.2.13) yields a value in the range of \(\delta_b = 2 - 10 \ d_{50}\). The layer-averaged sediment concentration \((c_b)\) in the bed-load layer was found to be (see Eq. (7.2.43), Van Rijn (1984a)):

\[
c_b = 0.18 \ c_o \frac{T}{D_s} \text{ at } z = \delta_b = 0.3 \ d_{50} \ D_s^{0.7} \ T^{0.5}
\]  \hspace{1cm} (7.3.24)

in which:
- \(c_b\) = volumetric bed-load concentration
- \(c_o\) = maximum volume concentration \((= 0.65)\)
- \(T\) = dimensionless bed-shear stress parameter, see Eq. (7.2.44)
- \(D_s\) = dimensionless particle size parameter, see Eq. (7.2.44)
- \(\delta_b\) = thickness of bed-load layer, see Eq. (7.2.13)

The concentration is given as a volume concentration; multiplying by \(\rho_s\) yields the concentration by weight (in kg/m\(^3\)).

Einstein (1950) assumed that the thickness of the bed-load layer was equal to \(2 \ d_{55}\). The average particle velocity was found to be \(u_b = 11.6 \ u_*\). Using the definition \(q_b = c_b \ u_b \ \delta_b\), the bed-load concentration was found to be:

\[
c_b = \frac{q_b}{23.2 \ u_* \ d_{35}} = \frac{\phi_b}{23.2 \ \theta^{0.5}} \text{ at } z = 2 \ d_{35}
\]  \hspace{1cm} (7.3.25)

in which:
- \(q_b\) = volumetric bed-load transport rate, Eq. (7.2.70)
- \(u_*\) = bed-shear velocity
- \(\phi_b\) = dimensionless bed-load transport rate

Engelund and Fredsøe (1976) assumed a bed-load layer thickness equal to \(2 \ d_{50}\). The volumetric bed concentration was found to be:

\[
c_b = 0.65 \ (1 + \lambda^{-1})^{-3} \text{ at } z = 2 \ d_{50}
\]  \hspace{1cm} (7.3.26)

with:

\[
\lambda = 4.3 \left(\frac{\theta - \theta_{\varphi} - 0.26 \ p}{\theta}\right)^{0.5}
\]  \hspace{1cm} (7.3.27a)

\[
p = \left(1 + \left(\frac{0.26}{\theta - \theta_{\varphi}}\right)^4\right)^{-0.25}
\]  \hspace{1cm} (7.3.27b)

7.61
in which:
\[ \theta = \text{particle mobility parameter} = \frac{u_*^2}{((s-1)g d_{s0})} \]
\[ \theta_{cr} = \theta_{cr} = \text{particle mobility parameter at initiation of motion (Shields)} \]
\[ u_* = \frac{\bar{u}}{(6 + 2.5 \ln(h/2.5 d_{s0}))} = \text{bed-shear velocity} \]

Zyserman and Fredsoe (1993) proposed an empirical function defined at \( z = a = 2d_{s0} \) (flat bed \( \theta' = \theta \)):
\[
c_b = \frac{0.331(\theta' - \theta_{cr})^{1.75}}{1 + 0.72(\theta' - \theta_{cr})^{1.75}} \tag{7.3.28}
\]

Smith and McLean (1977) assumed that the thickness of the bed-load layer was equal to the zero-velocity level \( z_0 \). They found \( k_s' = 3d_{s0} = \text{grain roughness} \):
\[
a = k_s' + 26.3(\theta - \theta_{cr})d_{s0} \tag{7.3.29}
\]

The average sediment concentration (volume) in the bed-load layer was proposed to be:
\[
c_h = 0.004c_s\left(\frac{S}{1 + 0.004 S}\right) \text{ at } z = \delta_h \tag{7.3.30}
\]
in which:
\[ c_s = \text{maximum (volume) concentration (} = 0.6 \]
\[ S = (\theta - \theta_{cr})/\theta_{cr} = \text{transport stage parameter} \]
\[ \delta_h = \text{thickness of bed-load layer, Eq. (7.3.29)} \]

The results of the proposed methods are compared in Figure 7.3.10 for a specific case (plane bed, \( d_{s0} = 300 \mu m, D_* = 7.6 \)). The methods of Einstein, Engelund-Fredsoe and Van Rijn yield results which are in reasonably good agreement. The method of Engelund and Fredsoe predicts an almost constant bed-load concentration for \( \theta > 0.6 \). The method of Smith and McLean yields considerably smaller bed-load concentration values, because it is defined at a higher level.

Using the proposed methods, the reference concentration is prescribed at a reference level close to the bed \( a = 2 \) to \( 10 d_{s0} \). This may easily lead to relatively large errors of the concentrations at higher levels, as will be shown by a computation example (see, also Van Rijn, 1984b).

Equation (7.3.10) was solved numerically applying Eqs. (7.3.11) to represent the fall velocity and Eq. (7.3.15) to represent the parabolic-constant mixing coefficient. The suspension number \( Z = w_s/(\beta \kappa u_*) \) was taken as 0.625. Because of errors in \( w_s, \beta, \kappa \) and \( u_* \), the \( Z \)-value also has a certain error. The error of the \( Z \)-parameter was assumed to be \( \pm 25 \% \). Thus, \( Z = 0.625 \pm 0.175 \). Figure 7.3.11 shows concentration profiles for \( Z = 0.45, 0.625 \) and \( 0.8 \). As can be observed, an error of about 25\% in the \( Z \)-parameter leads to a concentration error of a factor 3 at a level of \( z = 0.5 h \) (mid depth) and an error of a factor 2 at a level of \( z = 0.1 h \).
Figure 7.3.10 Bed load concentrations

Figure 7.3.11 Influence of error in Z-parameter on concentration profile
Based on these results, it is evident that the application of a reference level at the upper edge of the bed-load layer is not very attractive for reasons of accuracy. It is recommended to use a minimum value of $a_{\text{min}} = 0.01\ h$.

In stead of a prescribed concentration at the bed it is also possible to apply a prescribed pick-up rate as bed-boundary condition (Van Rijn, 1984d). Pick-up rate functions are given in Section 7.2.3.

2. Bed forms

As shown in Section 7.3.5 and in Figure 7.3.9, the flow over a dune-covered bed is strongly non-uniform.

The bed-load particles are transported close to the bed by rolling, sliding and saltating at the upsloping part of the dunes. Arriving at the top of the dune, the majority of the bed-load particles jumps over the edge and rolls down the leeside slope of the dune towards the dune trough where they are buried waiting for a new (transport) cycle.

The suspended load particles are entrained from the bed-load layer developing at the upsloping part of the dune. The entrainment rate is maximum at the dune top (large velocities). Above the dune trough the suspended particles are transported further upwards by turbulence mixing produced in the shear layers of the vortex generated in the dune trough.

A detailed mathematical representation of the afore-mentioned processes requires the application of a sophisticated hydrodynamic model and a higher order turbulence model such as the K-E model (Rodi, 1980) in combination with a convection-diffusion model for the suspended sediment particles. The horizontal grid size should be much smaller than the dune length ($\Delta x < \lambda$); the reference level should be applied at the upper edge of the bed-load layer ($a < \Delta$). To get a first understanding of the phenomena involved, a relatively simple two-dimensional vertical suspended sediment model was used by Van Rijn (1985) to compute the concentration profiles for the flow over dunes.

Figure 7.3.12 shows computed concentration profiles at the dune top and at the dune trough representing two extreme profiles; all other profiles are within the indicated variation range. As can be observed, the near-bed concentrations can vary over a range of nearly a factor 10. In vertical direction the concentration variation is confined to a near-bed region of about 0.3 h. The concentrations in the upper layers are not noticeably affected. Atkins et al (1989) show measured concentrations along a sand dune in an estuary.

Generally, the application of mathematical models for estuarine and coastal problems requires a schematization on a macro-scale (limited computer facilities) applying a horizontal grid size much larger than the bed-form length. Consequently, a detailed representation of the concentration profiles and also the velocity profiles is not feasible. The effect of the dunes on the flow field, usually, is represented by introducing an effective roughness parameter ($k_v$) as shown in Figure 7.3.12.

To represent the effect of the dunes on the concentration field, it is herein proposed:
- to represent the concentration profiles by a schematized concentration profile being an estimate of the spatially-averaged (over the dune length) concentration profiles,
- to specify an effective concentration ($c_e$) at a level through the top of the dunes ($a = 0.5\Delta$ or $a = k_v$ when the dune dimensions are unknown),
- to represent the transport of particles below the reference level as bed-load transport using a simple formula.
The effective reference concentration at the top of the bed forms was proposed to be (Van Rijn, 1984b):

\[ c_a = 0.015 \frac{d_{50}}{a} \frac{T^{1.5}}{D_{s0}^{0.3}} \]  \( (7.3.31) \)

in which:
- \( c_a \) = reference concentration (volume)
- \( a \) = reference level above the mean bed (\( a = k_s \) or \( a = \frac{1}{2} \Delta \))
- \( D_{s0} \) = dimensionless particle parameter
- \( T \) = dimensionless bed-shear parameter

The constant in Eq. (7.3.31) is based on calibration using flume and field data with velocities in the range of 0.4 to 1.6 m/s, depths in the range of 0.1 to 25 m and mean particle size in the range of 180 to 700 \( \mu \)m (Van Rijn, 1984). Voogt al et al (1991) showed that Eq. (7.3.31) is valid for velocities up to 3 m/s. The reference level is assumed to be equal to half the bed form height (\( a = \frac{1}{2} \Delta \)) or equal to the effective bed roughness (\( a = k_s \)) when the bed form height is unknown. The \( k_s \)-value can be computed from the Chezy-coefficient.

Based on Eq. (7.3.31), Van Rijn (1987) presented a stochastic method to compute the reference concentration for strongly non uniform flow.

A different approach was used by Einstein (1950), Engclund-Fredsoe (1976) and Smith-McLean (1977). They assume that the bed-load layer develops at the up-sloping part of the bed forms. The reference concentration is given by Eqs. (7.3.25), (7.3.26) and (7.3.30) using the effective mobility parameter \( \theta^f = \frac{\tau_b}{[(\rho_s - \rho)g d_{50}] r_{b^f}} \) with \( \tau_b \) = = grain-related bed-shear stress.

An appraisal of existing methods to compute the reference concentration was presented by Garcia and Parker (1991). The methods of McLean and Van Rijn were found to give the best results.

### 7.3.7 Suspended sediment size in case of non-uniform bed material

In natural conditions the bed material is non-uniform. Observations in flume and field conditions have shown that the bed material and the suspended sediment material have different particle size distributions. Usually, the suspended sediment particles are considerably smaller than the bed material particles. Hubbel and Matejka (1959) found that the median size of the total load (measured in a special contracted section) in the Middle Loup river was about 250 \( \mu \)m, whereas the median size of the bed material was about 350 \( \mu \)m. Basically, it is possible to compute the suspended load for any known type of bed material and flow condition by dividing the bed material into a number of size fractions and assuming that the size fractions do not influence each other (see Section 7.2.7).

Van Rijn (1984) has applied the size fraction method to determine a representative suspended sediment diameter (\( d_1 \)) which accounts for the non-uniformity effects. Using the size fraction method (as proposed by Einstein, 1950), the total suspended load transport was computed for various conditions, after which by trial and error the representative (suspended) particle diameter (\( d_1 \)) was determined that gave the same value for the suspended load transport as according to the size-fraction method.
Figure 7.3.12 Concentration profiles along a dune

Figure 7.3.13 Representative particle diameter of suspended sediment (Van Rijn, 1984b)
In all, six computations were performed using two types of bed material with a geometric standard deviation: $\sigma_s = \frac{1}{2}(d_{54}/d_{50} + d_{50}/d_{16}) = 1.5$ and 2.5. The $d_{50}$ of the bed material was equal to 250 μm. The mean flow velocities were 0.5 and 1.5 m/s. The flow depth was assumed to be 10 m. The concentration profile was computed by applying Eq. (7.3.21) with $\beta = 1$, $\delta = 1$ and $\kappa = 0.4$. The reference concentration according to Eq. (7.3.31) was applied at a level of $z = 0.05$ h. The flow velocity profile was assumed to be logarithmic.

The computed $d_s$-values were found to be related to $\sigma_s$-parameter and to the dimensionless shear-stress parameter $T$, as follows:

$$d_s = [1 + 0.011(\sigma_s - 1)(T - 25)]d_{50} \quad \text{for } 0 < T < 25$$

$$d_s = d_{50} \quad \text{for } T \geq 25$$

(7.3.32)

in which:

$d_s$ = representative particle diameter of suspended sediment

$d_{50}$ = median particle diameter of bed material

$\sigma_s$ = geometric standard deviation of bed material $= \frac{1}{2}(d_{54}/d_{50} + d_{50}/d_{16})$

$T$ = dimensionless bed shear stress parameter, Eq. (7.2.44)

Equation (7.3.32) and experimental data of Guy et al. (1966) are shown in Figure 7.3.13. The scatter of the data is too large to detect a clear influence of the $\sigma_s$-parameter. In an average sense the agreement between measured and computed values ($\sigma_s = 2.5$) is quite good.

Using this method, a better representation of the suspended sediment in case of graded bed material can be obtained than by taking a particular particle diameter such as the $d_{10}$, $d_{35}$ or $d_{50}$ of the bed material.

Morse and Townsend (1989) also addressed the modelling of sediment concentration profiles in river flows carrying mixed sediments. The grain size distribution of the bed material was divided into several size ranges (fractions), which were then characterized by their respective geometric means. Numerical errors introduced through this procedure were quantified. Depth-averaged concentrations can be under-estimated by a factor 10 when the concentration gradients are large (Z-parameter is large). Estimates were found to be particularly sensitive when the chosen reference level was small (close to bed) and the fraction width was large. Fall velocity coefficients were introduced to correct the discrepancy between the actual representative fall velocity of a fraction and that of the geometric mean size of the fraction.

### 7.3.8 Suspended load transport rates

Herein, the methods of Einstein (1950), Bagnold (1966), Bijker (1971) and Van Rijn (1984b) are described.

#### 1. Einstein (1950)

The method of Einstein is based on a parabolic distribution of the fluid mixing coefficient and a logarithmic distribution of the velocity.

The suspended sediment transport rate can be expressed as:

$$q_{s,c} = 11.6 \, u'_c \, c_s \, [I_2 + I_1 \, \ln(30.2 \, c_h/d_{50})]$$

(7.3.33)

$$I_1 = 0.216 \, \frac{A^{Z-1}}{(1-A)^Z} \int_A^1 \left(\frac{1-X}{X}\right)^Z \, dX$$

(7.3.34)
\[ I_2 = 0.216 \frac{A Z-1}{(1-A)^2} \int_{A}^{1} \left( \frac{1-X}{X} \right)^Z \ln(X) \ dX \quad (7.3.35) \]

in which:
- \( q_{s,c} \) = volumetric current-related suspended load transport (m²/s)
- \( u_* \) = current-related bed-shear velocity due to the grains (m/s)
- \( c_a \) = reference concentration (volume) = \( q_{s,c} / (11.6 \ u_* \ a) \)
- \( a \) = reference level (= 2 d), (m)
- \( h \) = water depth (m)
- \( d \) = particle diameter (m)
- \( A \) = \( a/h \) = dimensionless reference level (-)
- \( X \) = \( z/h \) = dimensionless vertical coordinate (-)
- \( Z \) = \( w_s/(\kappa \ u_*) \) = suspension number (-)
- \( e \) = correction factor (-)

According to Einstein, the suspended load transport is related to the grain-shear velocity\( (u_*) \) and not to the overall shear velocity \( (u_0) \).

The reference concentration is determined from the bed-load transport, assuming a bed-load layer thickness equal to 2 particle diameters.

For almost uniform bed material the \( d_{50} \) is taken as the representative particle diameter. For graded bed material the size fraction method should be used.

The \( I_1 \) and \( I_2 \)-integrals can be determined graphically (see Einstein, 1950) or numerically.

The method of Einstein is not given in full detail here, because of its complexity. Furthermore, an extensive verification study of White et al (1973) has shown that the predicting ability of the Einstein method is much less than other (more simple) methods.

2. Bagnold (1966)

The method of Bagnold is based on an energy balance concept relating the suspended load transport to the work done by the fluid.

The suspended load per unit area is supported by a fluid stress \( (\sigma) \) normal to the bed

\[ \sigma = (\rho_s - \rho) g V_s \cos \beta \]

in which: \( V_s \) = solid volume of suspended load per unit area (m³/m²).

Per unit time the suspended load sinks over a vertical distance equal to the fall velocity \( w_s \). To keep the load in suspension (all particles remain statistically at the same level above the bed), the normal stress \( (\sigma) \) must do work (per unit time and area). The work required is:

\[ W_r = (\rho_s - \rho) g V_s w_s \cos \beta \quad (7.3.36) \]

The volumetric suspended load transport is defined as \( q_{s,c} = V_s \bar{u}_c \), giving:

\[ W_r = (\rho_s - \rho) g q_{s,c} \frac{w_s}{\bar{u}_c} \cos \beta \quad (7.3.37) \]

The work done per unit time and width by the longitudinal gravity force component to keep the fluid in motion is \( \tau_b \bar{u} \). Part of this energy is available to transport the suspended load, yielding.
\[ W_{a,1} = c_a(1-e_b) \tau_b \bar{u} \quad (7.3.38) \]

in which:
\( \bar{u} \) = depth-averaged velocity
\( \tau_b \) = bed-shear stress
\( e_a \) = efficiency factor related to suspended load (\( = 0.01 \) to \( 0.02 \))
\( e_b \) = efficiency factor related to bed load (\( = 0.1 \) to \( 0.2 \))

Per unit time and width the suspended load is moved in longitudinal direction over a distance equal to \( \bar{u} \). The work done by the longitudinal gravity component during the unit time period is:

\[ W_{a,2} = (\rho_s - \rho) V_s \bar{u} \sin\beta = (\rho_s - \rho) q_{a,c} \sin\beta \quad (7.3.39) \]

This quantity is fully available for transportation of the suspended load.

The suspended load transport rate can now be derived from the energy balance
\[ W_r = W_{a,1} + W_{a,2}, \] yielding:

\[ q_{a,c} = \frac{c_a(1-e_b) \tau_b \bar{u}}{(\rho_s - \rho) g \cos\beta [(w_s/\bar{u}) - \tan\beta]} \quad (7.3.40) \]

in which:
\( q_{a,c} \) = volumetric current-related suspended load transport (m\(^2\)/s)
\( w_s \) = fall velocity of sediment (m/s)
\( \bar{u} \) = depth-averaged velocity (m/s)
\( \tau_b \) = overall current-related bed shear stress (N/m\(^2\))
\( \tan\beta \) = bottom slope (-)
\( \rho_s \) = sediment density (kg/m\(^3\))
\( \rho \) = fluid density (kg/m\(^3\))
\( g \) = gravity acceleration (m/s\(^2\))
\( e_b \) = efficiency factor of bed load (\( = 0.1 \) to \( 0.2 \))
\( e_a \) = efficiency factor of suspended load (\( = 0.01 \) to \( 0.02 \))
\( \beta \) = bottom angle with horizontal datum

Kachel and Sternberg (1971) have shown that the efficiency factors (\( e_a \) and \( e_b \)) are not constant, but are strongly related to the bed-shear stress and the particle diameter. Basically, these results show that the validity of the Bagnold concept is questionable.

3. **Bijker (1971)**

Based on the concept of Einstein (1950), Bijker (1971) proposed:

\[ q_{a,c} = 1.83 q_{b,c} [I_2 + I_1 \ln(33 \ h/k_s)] \quad (7.3.41) \]
in which:

\[ q_{b,c} = \text{suspended sediment transport rate (m}^2/\text{s}) \]
\[ q_{b,c} = \text{bed load transport rate (m}^2/\text{s}) \]
\[ I_1 = \text{integral according to Eq. (7.3.34)} \]
\[ I_2 = \text{integral according to Eq. (7.3.35)} \]
\[ a = \text{reference level} \]

The current-related bed-load transport rate (m²/s) is expressed as:

\[ q_{b,c} = b \ u_\ast \ d_{50} \ e^{-0.27(\mu)} \]  \hspace{1cm} (7.3.42)

in which:

\[ u_\ast = \text{overall bed shear velocity} \]
\[ \theta = \text{mobility parameter} \]
\[ \mu = (C/C')^{1.5} = \text{bed form factor} \]
\[ C = \text{overall Chézy coefficient} \]
\[ C' = \text{grain-related Chézy coefficient} = 18\log(12h/d_{50}) \]
\[ b = \text{coefficient (} = 1 \text{ to } 5) \]

Because the reference level is assumed to be equal to the bed roughness height (a = k₁), the ratio q₉/c/q₉ can be expressed as a function of the parameter Z and k₉/h, as shown in Figure 7.3.14.

4. Van Rijn (1984b)

The suspended load transport can be computed from Eq. (7.3.2), yielding:

\[ q_{s,c} = F \ \bar{u} \ h \ c_s \]  \hspace{1cm} (7.3.43)

Using Eqs. (7.3.21) and (7.3.22), the F-factor is:

\[ F = \frac{u_\ast}{\kappa \ \bar{u}} \left( \frac{a}{h-a} \right)^{Z} \left( \int_{a/h}^{0.5} \left( \frac{h-z}{z} \right)^{z'} \ln(z/z_0) \ d(z/h) \right) + \int_{0.5}^{1} e^{-4(z'^h-0.5)} \ln(z/z_0) \ d(z/h) \]  \hspace{1cm} (7.3.44)

Equation (7.3.44) cannot be integrated analytically. An approximate solution accurate to about 25% for 0.3 ≤ Z' ≤ 3 and 0.01 ≤ a/h ≤ 0.1 is given by:

\[ F = \frac{(a/h)^{z'} - (a/h)^{12}}{(1-a/h)^{z'} (1.2 - Z')} \]  \hspace{1cm} (7.3.45)

The F-factor for a/h = 0.01, 0.05 and 0.1 is shown in Figure 7.3.15.
Figure 7.3.14 Ratio of suspended load and bed load transport, Bijker (1978)

Figure 7.3.15 Shape factor $F$
The complete method reads as (see also Appendix A):

\[ q_{s,c} = F \, \bar{u} \, h \, c_a \]

in which:

\[ c_a = 0.015 \frac{d_{50}}{D_0^{0.3}} T^{1.5} = \text{reference concentration (-)} \]

\[ F = \frac{(a/h)^{Z'} - (a/h)^{1.2}}{(1 - a/h)^{Z'} (1.2 - Z') } = \text{shape factor (-)} \]

\[ D_* = d_{50} \left( \frac{(s-1) g}{\nu^2} \right)^{1/3} = \text{particle parameter (-)} \]

\[ T = \frac{\tau_b'}{\tau_{b,cr}} = \text{bed-shear stress parameter (-)} \]

\[ \tau_b' = \rho g \left( \frac{\bar{u}}{C'} \right)^2 = \text{current-related effective bed-shear stress (N/m²)} \]

\[ u_* = \frac{g^{0.5}}{C} \bar{u} = \text{current related overall bed shear velocity (m/s)} \]

\[ C' = 18 \log \left( \frac{12h}{3d_{90}} \right) = \text{grain-related Chézy coefficient (m^{0.5}/s)} \]

\[ C = 18 \log \left( \frac{12 h}{k_s} \right) = \text{overall Chézy coefficient (m^{0.5}/s)} \]

\[ \tau_{b,cr} = (\rho_s \rho g) d_{50} \theta_{cr} = \text{critical bed-shear stress (N/m²)} \text{ according to Shields} \]

\[ Z' = Z + \psi = \text{suspension number (-)} \]

\[ Z = \frac{w_s}{\beta \kappa u_*} = \text{suspension number (-)} \]

\[ \psi = 2.5 \left( \frac{w_s}{u_*} \right)^{0.8} \left( \frac{c_a}{c_o} \right)^{0.4} = \text{stratification correction, Eq. (7.3.86)} \]

\[ \beta = 1 + 2 \left( \frac{w_s}{u_*} \right)^2 = \text{ratio of sediment and fluid mixing coefficient (} \beta_{\text{max}} = 2) \]

\[ d_s = [1 + 0.011 (\sigma_s - 1) (T-25)]d_{50} = \text{representative particle size of suspended sediment (m)} \]

\[ d_s = d_{50} \text{ for } T \geq 25 \]

7.72
\( \dot{q}_{sc} \) = volumetric current-related suspended load transport \((m^2/s)\)
\( \bar{u} \) = depth-averaged velocity \((m/s)\)
\( h \) = water depth \((m)\)
\( a \) = reference level \((m)\), \( a = \frac{1}{2} \Delta \) or \( a = k_s \)
\( k_s \) = overall roughness height \((m)\)
\( \Delta \) = bed form height \((m)\)
\( d_{s0} \) = median particle diameter of bed material \((m)\)
\( d_{16}, d_{50}, d_{90} \) = characteristic diameter of bed material \((m)\)
\( \dot{w}_{s} \) = fall velocity of suspended sediment \((based on \ d_{s} \ value) \) using Fig. 3.2.6 or Eqs. (3.2.21), (3.2.22), \((m/s)\)
\( d_s \) = representative particle size of suspended sediment \((m)\)
\( \sigma_s \) = geometric standard deviation of bed material, \( \sigma_s = \frac{1}{2}(d_{84}/d_{50} + d_{50}/d_{16}) \)
\( c_o \) = maximum concentration \((= 0.65)\)
\( s \) = specific density \((= \rho_s/\rho)\)
\( \rho_s \) = sediment density \((= 2650 \text{ kg/m}^3)\)
\( \rho \) = fluid density \((\text{kg/m}^3)\)
\( \nu \) = kinematic viscosity coefficient \((m^2/s)\)
\( \kappa \) = constant of Von Karman \((= 0.4)\)
\( g \) = acceleration of gravity \((m/s^2)\)

The \( \beta \)-factor yields an increase of the suspended load transport rate, whereas the \( \psi \)-factor yields a decrease of the transport rate. In the lower regime both effects cancel out \((\beta = 1, \ \psi = 0)\).

A simplified method was given by Van Rijn (1984b). This method is based on computer computations (using the detailed method, see above) in combination with a roughness predictor (see Chapter 6). Using regression analysis, the computational results for a depth range of 1 to 20 m, a velocity range of 0.5 to 2.5 m/s and a particle range of 100 to 2000 \( \mu \text{m} \) were represented by a simple power function, as follows:

\[
\frac{\dot{q}_{sc}}{\bar{u} h} = 0.012 \left( \frac{\bar{u} - \bar{u}_{cr}}{(s-1)g_d_{50}^{0.5}} \right)^{2.4} \left( \frac{d_{50}}{h} \right) \left( \frac{1}{D_s} \right)^{0.6} \tag{7.3.46}
\]

in which:
\( \dot{q}_{sc} \) = volumetric suspended load transport \((m^2/s)\)
\( \bar{u}_{cr} \) = critical depth-averaged velocity according to Shields, see Eq. (7.2.47)
\( h \) = water depth \((m)\)
\( \bar{u} \) = depth-averaged velocity

Equation (7.3.46) only requires \( \bar{u}, h, d_{50}, d_{90} \) and \( \nu \) as input data and can be used to get a first estimate of the suspended load transport.

When the bed load transport and the suspended load transport are known, the total load transport of bed material can be determined by summation \((q_t = q_b + q_s)\).

The ratio of the suspended and total load transport can be expressed as:

\[
\frac{q_s}{q_t} = \frac{q_s}{q_b + q_s} = \frac{1}{q_b/q_s + 1} \tag{7.3.47}
\]

7.73
For reasons of simplicity the bed load transport is here defined as:

\[ q_b = a \cdot c_s \cdot \bar{u}_a \]  

(7.3.48)

in which:
- \( c_s \) = reference concentration
- \( \bar{u}_a \) = effective transport velocity of bed-load particles
- \( a \) = reference level (= bed-load layer thickness)

Substitution of Eq. (7.3.43) and (7.3.48) in Eq. (7.3.47) yields:

\[ \frac{q_s}{q_t} = \frac{1}{(F^{-1})(\bar{u}_s/\bar{u})(a/h)} + 1 \]  

(7.3.49)

The ratio \( \bar{u}_s/\bar{u} \) may be identified as the ratio of the average transport velocity of the bed load and suspended load particles, which varies from about 0.4 for large, steep bed forms in the lower regime to about 0.8 for flat bed conditions in the upper flow regime. Figure 7.3.16 shows the ratio of the suspended load and the total load as a function of the ratio of the bed-shear velocity and particle fall velocity for \( \bar{u}_s/\bar{u} = 0.6, \kappa = 0.4, \beta = 1 \) and \( a/h = 0.05 \). An empirical relationship given by Laursen (1958) and some data of Guy et al (1966) are also shown.

Figure 7.3.16  Ratio of suspended and total load transport
7.3.9 Stratification effects in high-concentration suspensions

1. Stratification and hyperconcentrations

A flow system with a vertical density gradient is called:
- stable when the density decreases upwards,
- unstable when the density increases upwards,
- neutral when the density is constant over the depth.

In sediment suspension above a movable bed the sediment concentrations decrease in upward direction from the bed. Consequently, the flow is stable. In uniform flow with an equilibrium concentration profile the solid particles at a certain level statistically remain at that level, because the turbulence forces produced by the shear action in the flow are continuously doing work as they try to mix the sediment upwards against the action of gravity. This reduces the turbulence energy available for mixing the fluid resulting in damping of turbulence.

The stability of the suspension can be quantified by the Richardson number or the Monin-Obukhov number. This latter parameter (herein used) is defined as the ratio of the energy per unit volume required at a particular level to keep the sediment in suspension and the available turbulence kinetic energy supplied by the shear action at the same level, as follows:

\[
\xi = \frac{(\rho_s - \rho)g c w_{s,m}}{\tau \frac{du}{dz}} = \frac{(\rho_s - \rho)g c w_{s,m}}{\rho \varepsilon_f (\frac{du}{dz})^2} = \frac{(s-1)g \kappa z c w_{s,m}}{u_*^3}
\]  

(7.3.50)

In which:
- \( c \) = volume concentration at height \( z \) above bed
- \( w_{s,m} \) = fall velocity in fluid-sediment mixture
- \( \varepsilon_f \) = fluid mixing coefficient in clear water (= \( \kappa u_\tau z \))
- \( \frac{du}{dz} \) = fluid velocity gradient at height \( z \) (= \( u_\tau / \kappa z \))
- \( \tau \) = shear stress at height \( z \) (= \( \rho \varepsilon_f \frac{du}{dz} \))
- \( \kappa \) = constant of Von Karman (= 0.4)

Using the \( \xi \)-parameter, the suspension is stable for \( \xi \geq 0 \). The \( \xi \)-parameter has been used extensively in studies of the velocity profile in the atmospheric boundary layer (Businger et al, 1971). The effect of the suspended sediment on the near-bed velocity profile in terms of the \( \xi \)-parameter was discussed by Soulsby and Wainwright (1987).

Stratification effects are most important in the high-concentration or hyperconcentration range.

Volume concentrations in the range of 0.2 to 0.6 (20% to 60 vol %) are herein called hyperconcentrations. For concentrations larger than 0.2 the hindered settling effect will have a dominating effect on the suspension characteristics, as shown by Winterwerp et al (1990) and by Chien-Zhaohui (1986). The sediment load will move more or less as a neutrally buoyant load with the same behaviour as a pure liquid (pseudo one-phase flow). The flow resistance is only slightly larger than that of a clear water flow in the turbulent rough regime. Hyperconcentrations in the range of 500 to 1500 kg/m³ were observed at velocities of 6 to 8 m/s in the middle reach of the Yellow river in China (Zhaoyin, 1990). Because of the high viscosity, the flow behaviour was almost laminar. In the turbulent regime the velocity profiles were observed to be logarithmic.

Hyperconcentrations in the Yellow river consist of a range of size classes. About 40% of the material is smaller than 50 \( \mu \)m.
Based on flume experiments (Zhaoyin, 1990), the maximum hyperconcentration of uniform particles was found to vary from about 1000 kg/m\(^3\) for fine particles (d < 150 \(\mu\)m) to about 200 kg/m\(^3\) for coarse particles (d = 5000 \(\mu\)m). Higher values up to 1250 kg/m\(^3\) were obtained when graded sediments were suspended.

2. Velocity profile and flow resistance

Based on the Boussinesq hypothesis, the shear stress at height z in a fluid-sediment mixture flow is defined, as:

\[
\tau_z = \rho_m (\varepsilon_{f,m} + \nu_m) \frac{du}{dz}
\]  

(7.3.51)

in which:
\(\tau_z\) = fluid shear stress at height z above bed
\(\varepsilon_{f,m}\) = mixing coefficient in fluid-sediment mixture
\(\nu_m\) = kinematic viscosity coefficient in fluid-sediment mixture (= \(\eta_m/\rho_m\))
\(u\) = fluid velocity at height z above bed
\(\rho_m\) = \(\rho(1+(s-1)c)\) = density of mixture at height z
\(c\) = sediment concentration at height z
\(\eta_m\) = dynamic viscosity coefficient

The shear-stress at height z and the bed-shear stress can also be expressed as:

\[
\tau_z = \bar{\rho}_{m,1} g (h-z) I
\]

(7.3.52)

\[
\tau_b = \bar{\rho}_m g h I
\]

(7.3.53)

in which:
\(I\) = energy gradient
\(h\) = flow depth
\(\bar{\rho}_{m,1}\) = \(\rho(1+(s-1)c)\) = mean density of layer above height z to surface
\(\bar{\rho}_m\) = \(\rho(1+(s-1)c)\) = mean density over depth
\(\bar{c}\) = mean concentration of layer above height z to surface
\(c\) = mean concentration over depth
\(s\) = specific density (= \(\rho_s/\rho\))
\(\rho_s\) = sediment density
\(\rho\) = fluid density

Equations (7.3.52) and (7.3.53) can be expressed as:

\[
\tau_z = \frac{\bar{\rho}_{m,1}}{\bar{\rho}_m} \left(1 - \frac{z}{h}\right) \tau_b = \frac{\bar{\rho}_{m,1}}{\bar{\rho}_m} \left(1 - \frac{z}{h}\right) u_*^2
\]

(7.3.54)

The velocity gradient can be obtained from Eqs. (7.3.51) and (7.3.54):

\[
\frac{du}{dz} = \frac{(1 + (s-1)\bar{c})}{(1 + (s-1)c)} \left(\varepsilon_{f,m} + \nu_m\right) \frac{u_*^2}{(1 + (s-1)c) (\varepsilon_{f,m} + \nu_m)}
\]

(7.3.55)
For a clear fluid Eq. (7.3.55) reduces to:

\[
\frac{du}{dz} = \frac{(1 - z/h) u_*^2}{\varepsilon_f + \nu}
\]  \hspace{1cm} (7.3.56)

In the fully turbulent logarithmic layer in the near-bed region the viscosity can be neglected and the fluid mixing coefficient can be expressed as: \( \varepsilon_f = \kappa u_* z(1-z)/h \), yielding:

\[
\frac{du}{dz} = \frac{u_*}{Kz} - \frac{u_*}{l_m}
\]  \hspace{1cm} (7.3.57)

in which \( l_m = \kappa z \) = mixing length.

In the viscous sublayer adjacent to the (plane)bed the velocity gradient is given by:

\[
\frac{du}{dz} = \frac{u_*^2}{\nu}
\]  \hspace{1cm} (7.3.58)

In a fluid-sediment mixture the turbulence structure and hence the velocity profile and the flow resistance are modified. Experimental work was done by Vanoni (1946), Einstein-Chien (1955), Ippe (1971), Coleman (1980), Wang (1981) and Winterwerp et al (1990). Most experiments were done in the concentration range below \( \bar{c} \leq 0.1 \). Wang and Winterwerp et al did experiments with concentrations upto \( \bar{c} = 0.4 \).

Analysis of measured velocity profiles in sediment suspensions shows a decrease of the near-bed velocities and an increase of the near-surface velocities, as shown in Figure 7.3.17 (Lau, 1982 and Woo et al, 1988). Clearly, an increase of the velocity gradient in the outer region \( (z/h > 0.05) \) can be observed.

The effect of the suspended sediment on the flow resistance is not very clear. A decrease of the flow resistance (Itakura-Kishi, 1980) as well as an increase of the flow resistance (Winterwerp et al, 1990) has been reported.

Defining the flow resistance as,

\[
\tau_b = 1/8 \ \rho_m \ f_m \ \bar{u}^2
\]  \hspace{1cm} (7.3.59)

in which:
\( \tau_b \) = bed-shear stress
\( f_m \) = friction factor of fluid-sediment flow
\( \rho_m \) = mixture density = \( \rho(1 + (s-1) \ \bar{c}) \)
\( \bar{c} \) = depth-averaged concentration
\( \rho \) = fluid density
Figure 7.3.17 Measured velocity profiles in high-concentration flows of Einstein-Chien (1955)

Figure 7.3.18 Friction factor
it follows that both the mixture density and the friction factor are important. The mixture
density increases for increasing concentration resulting in an increase of flow resistance. The
behaviour of the friction factor is presented schematically in Figure 7.3.18, showing:
• no change of friction factor in fully rough regime,
• small decrease of friction factor entering the transition regime from the rough regime
  (decreasing Re-number) at the same relative roughness (k_r/D),
• increase of friction factor for decreasing Re-number in smooth regime,
• small decrease of friction factor by transition from smooth to laminar regime,
• increase of friction factor for decreasing Re-number in laminar regime.

Analysis of the high-concentration mixture flows of Einstein-Chien (1955) and Winterwerp
et al (1990) shows u*-values in the range of 0.1 to 0.2 m/s, k_r-values in the range of 0.001
to 0.005 m (see also Section 6.2.3). The viscosity of the fluid-sediment mixture in the
near-bed region with concentrations of about 0.4 will be about 10^{-5} m^2/s. Based on these values,
the Re_*/u_* k_r/\nu_m number will be in the range of 10 to 100, which means flow in the
transition regime (Re_*/u_* = 5 to 70) or in the rough regime (Re_*/u_* > 70), depending on the
hydraulic conditions.

The stratification phenomena affecting the velocity profile and the flow resistance can be
described as:

• damping of turbulence because turbulence energy is consumed in keeping the particles in
  suspension (transfer of momentum from fluid to particles). This effect which is related to
  the concentration gradient (maximum in near-bed region) can be interpreted as a (slight)
  decrease of the Reynolds number. This will not influence the friction factor in the rough
  regime; the friction factor in the transition regime may be slightly reduced, whereas it will
  be increased in the smooth regime.

• damping of turbulence by increased viscous dissipation. This effect is directly related to
  the value of the concentration. In high-concentration suspension (\bar{c} = 0.2 to 0.4) the
effective viscosity increases by a factor 5 to 10 resulting in an increase of the viscous
sublayer thickness. In the smooth regime this effect can be interpreted as an increase of the
effective roughness (z_o = 3.3 \nu_m/u_*) and hence as an increase of the flow resistance.

• reduction of mean flow energy by transfer of momentum from the fluid to the particles (in
  the longitudinal direction) to keep the solid mass in motion. This effect is related to the
  concentration value results in an increase of the flow resistance (increase of f-factor).

The overall effect probably is a slight decrease of the flow resistance at low concentrations
(\bar{c} < 0.1) near the transition regime (Re_*/u_* = 50 to 100), as observed by many resarchers
(Vanoni, 1946) and an increase of the flow resistance at high concentrations (\bar{c} > 0.1) near
the smooth regime (Re_*/u_* = 5 to 50) as observed by Winterwerp et al (1990).

A very informative discussion about the effect of suspended sediment on the flow resistance
has been given by Chien and Zhaohui (1986), based on Chinese research related to
hyperconcentrations.

In the fully rough regime almost no influence was found of hyperconcentrations on flow
resistance. A slight decrease of flow resistance was found when hyperconcentrated flow
entered the transition regime (Re = 2000 to 4000) from smooth to laminar flow due to an
increase of the concentration and hence viscosity resulting in a sudden damping of turbulence
(Chien-Zhaohui, 1986; Zenghai, 1988). In the fully laminar regime (Re > 2000) the friction
factor increased with (hyper)concentration due to increase of viscosity and hence decrease of
Re-number. These observed phenomena can be explained from the behaviour of the friction
factor (see Fig. 7.3.18).
The turbulence damping effect was clearly observed by Xingkui and Ning (1989). Based on
turbulence measurements obtained with a pressure probe, they found a considerable reduction
(factor 2) of the near-bed turbulence intensity from $u_{ms} \approx 2u_*$ to $1u_*$ for a volume
concentration increasing from 0 to 8%.

Many researchers have tried to model the effect of the suspended sediment on the velocity
profile.

Roughly, four types of approaches can be found in the literature:

- modification of the mixing length scale ($l_m$) or the fluid mixing coefficient ($\varepsilon_{f,m} = \varepsilon_{f,m} l_m / \kappa_m$) by reducing the $\kappa_m$-value of the fluid-sediment mixture; $\kappa_m$ is either constant over the
depth or varies with $z$, Einstein-Chien (1955), Lppen (1971), Yalin-Finlayson (1972), Wang

$$ l_m = \kappa_m z \quad \text{or} \quad \varepsilon_{f,m} = \kappa_m u_* z \quad (7.3.60) $$

- modification of the mixing length scale ($l_m$) by introducing the Monin-Obukhov length scale
($L_M^0$), Itakura-Kishi (1980), McLean (1991a,b),

$$ l_m = \frac{\kappa z}{1 + \gamma z / L_M^0} \quad \text{and} \quad \kappa = \text{constant} \quad (7.3.61) $$

- modification of the mixing length scale ($l_m$) by introducing the Coles wake function ($\alpha = \text{wake strength parameter}$), Coleman (1981), Valiani (1988)

$$ l_m = \frac{\kappa}{z^{-1} + (\alpha \pi / h) \sin(\pi z / h)} \quad \text{and} \quad \kappa = \text{constant} \quad (7.3.62) $$

- modification of the mixing coefficient ($\varepsilon_{f,m}$) by using a higher order turbulence model
($k$-Epsilon model with equations for the turbulent energy and its dissipation rate),

\[ \kappa_m = \text{Von Karman coefficient in fluid-sediment mixture}, \quad \kappa = \text{Von Karman coefficient in clear fluid}; \gamma = \text{coefficient (5 to 10),} \quad \alpha = \text{wake strength parameter (0.1 to 1)}. \]

In most methods the $l_m$-value or the $\varepsilon_{f,m}$-value is reduced by the suspended sediment resulting
in an increase of the velocity gradient ($du/dz = u_*/l_m$).

Einstein and Chien (1955) found that the $\kappa_m$-value of the fluid-sediment mixture is related
to the ratio of the energy required to keep the sediments in suspension and the total available
energy. This can be expressed as:

$$ \kappa_m = f \left( \frac{(s-1)c s \overline{w}}{\overline{u} I} \right) \quad (7.3.63) $$
in which:
\[ \bar{c} = \text{depth-averaged volume concentration} \]
\[ \bar{u} = \text{depth averaged velocity} \]
\[ w_s = \text{fall velocity of suspended sediment} \]
\[ s = \text{specific density } (\rho_s/\rho) \]
\[ l = \text{energy gradient} \]

Equation (7.3.63) shown in Fig. 7.3.19, yields values from 0.4 to 0.2 for increasing concentrations.

IPPCN (1971) considered both the effect of turbulence damping by viscous dissipation and mixing against gravitation forces. He found:

\[ \kappa_m = \frac{\kappa(1 + (s-1) \bar{c})}{1 + 2.5 \ c_b} \]  \hspace{1cm} (7.3.64)

in which:
\[ c_b = \text{bed-load concentration (volume).} \]

Equation (7.3.64) yields values from 0.4 to 0.2 for increasing \( c_b \)-values up to 0.4.

Based on energy considerations and analysis of velocity profiles measured in the Yellow river, Wang (1981) found:

\[ \kappa_m^{-1} = \kappa^{-1} + \frac{1.14(s-1)g(\bar{w}_{s,m} - \bar{u} l) \bar{c}}{u_* l} \]  \hspace{1cm} (7.3.65)

in which:
\[ \bar{w}_{s,m} = \text{fall velocity of suspended sediment in a fluid-sediment mixture (based on } \bar{c}). \]

Wang explicitly took into account the transfer of potential energy into kinetic energy associated with the longitudinal flow of the solid mass. This is expressed by the term \( \bar{w}_{s,m} - \bar{u} l \), yielding

- a decrease of \( \kappa_m \) for increasing concentrations, if \( \bar{w}_{s,m} - \bar{u} l > 0 \)
- an increase of \( \kappa_m \) for increasing concentrations, if \( \bar{w}_{s,m} - \bar{u} l < 0 \)

At very high concentrations (hyperconcentrations) the fall velocity is greatly reduced by the hindered settling effect. This may lead to \( \bar{w}_{s,m} - \bar{u} l < 0 \), implying transport of solids without energy supplied by bottom-related turbulence. The energy is now supplied by the loss of potential energy of the solid mass during transportation. This stage was termed auto-suspension by Bagnold (1962).

Wang found \( \kappa_m \)-values between 0.4 and 0.1 for volume concentrations up to 0.4.

Winterwerp et al (1990) observed that the measured velocity profiles had a logarithmic behaviour in the outer region \( (z/h > 0.1) \). The velocity profiles were fitted by (see Fig. 7.3.28):

\[ \frac{u}{u_*} = \frac{1}{\kappa_m} \ln \left( \frac{u_z}{u_m} \right) + B \]  \hspace{1cm} (7.3.66)
Figure 7.3.19 $\kappa_m$-values according to Einstein-Chien (1955)

Figure 7.3.20 Measured velocity and concentration profiles, Winterwerp et al (1990)
The slope of the velocity profiles (Fig. 7.3.20) first decreases with increasing concentrations up to \( \bar{c} = 0.2 \) indicating an increase of turbulence damping (decrease of \( \kappa_m \)-value). A further increase of the concentration (\( \bar{c} = 0.2 \) to 0.4) results in a steepening of the slope of the velocity profile up to the clear water value again (increase of \( \kappa_m \)-value). Figure 7.3.21 shows the behaviour of the \( \kappa_m \)-values as a function of \( \bar{c} \). A shift of the logarithmic profiles with increasing concentrations was also observed indicating an increase of the flow resistance. This is caused by an increase of the thickness of the viscous sublayer related to an increase of the effective viscosity and by additional friction related momentum transfer from the fluid to the solid particles (particles are accelerated by the fluid and retarded by grain-grain collisions and grain bed collisions).

Van Rijn used a logarithmic velocity distribution for the transitional flow regime to compute the velocity distribution of Test 57 (mean concentration \( \bar{c} = 0.11 \)) and Test 83 (\( \bar{c} = 0.44 \)), experiments of Winterwerp et al (1990) and Run S-15 (\( \bar{c} = 0.030 \)), experiments of Einstein-Chien (1955). The velocity distribution is given by (Van Rijn, 1990):

\[
\bar{u}_z = \frac{\bar{u}}{1 + \ln(h/z_o)} \cdot \ln(z/z_o) \tag{7.3.67}
\]

where:
- \( \bar{u} \) = depth-averaged velocity
- \( h \) = water depth
- \( z_o = 0.033 k_s + 0.11 \frac{v_m}{u_*} \) = zero-velocity level
- \( k_s = 3 \theta d_{50} \) = effective grain roughness, Eq. (6.2.12b)
- \( u_* = u_s^2/(g d_{50}) \) = mobility parameter
- \( v_m \) = kinematic viscosity coefficient of fluid-sediment mixture near bed \( (\approx 10^{-5} \text{ m}^2/\text{s}) \)
- \( u_s = (g h z_o)^{1/3} \) = bed shear velocity

Measured and computed velocity distributions are shown in Fig. 7.3.22. Reasonably good agreement can be observed. The maximum relative difference is about 25%.

Based on this, it is concluded that the velocity profiles in high-concentration flows can be described reasonably well by generally-accepted expressions for turbulent flow.
Winterwerp 1990
Test 57
\( \bar{u} = 1.57 \text{ m/s} \)
\( h = 0.098 \text{ m} \)
\( l = 0.0102 \)
\( d_{50} = 120 \mu\text{m} \)
\( d_{90} = 180 \mu\text{m} \)
\( \bar{c} = 0.11 \)

Winterwerp 1990
Test 83
\( \bar{u} = 1.87 \text{ m/s} \)
\( h = 0.0803 \text{ m} \)
\( l = 0.0291 \)
\( d_{50} = 120 \mu\text{m} \)
\( d_{90} = 180 \mu\text{m} \)
\( \bar{c} = 0.44 \)

Einstein Chien 1955
Run S-15
\( \bar{u} = 2.2 \text{ m/s} \)
\( h = 0.124 \text{ m} \)
\( l = 0.0168 \)
\( d_{50} = 280 \mu\text{m} \)
\( d_{90} = 380 \mu\text{m} \)
\( \bar{c} = 0.03 \)

Figure 7.3.22 Measured and computed velocity profiles in high-concentration flows
Itakura-Kishi (1980) and McLean (1991a,b) assumed a constant $\kappa$-value equal to the clear fluid value ($\kappa_m = \kappa = 0.4$). The effect of the suspended sediment on the velocity profile was modeled by introduction of the Monin-Obukhov length scale ($L_{MO}$), as follows:

$$L_{MO} = \frac{u_*^3}{(s-1) g \kappa \bar{c} w_s}$$  (7.3.68)

in which:
- $\bar{c}$ = depth-averaged concentration
- $w_s$ = fall velocity in clear fluid
- $\kappa$ = Von Karman coefficient in clear fluid ($= 0.4$)

The velocity gradient is given by:

$$\frac{dU}{dz} = \frac{n_s}{\kappa z} \left( 1 + \gamma \frac{z}{L_{MO}} \right)$$  (7.3.69)

Based on analysis of measured velocity profiles in suspensions ($\bar{c} < 0.05$), the $\gamma$-coefficient was found to be $\gamma = 7$.

The velocity profiles are expressed as:

$$\frac{u}{u_*} = 5.5 + \frac{1}{\kappa} \left[ 7 \left( \frac{z}{L_{MO}} \right) + \ln \left( \frac{u_* z}{v_m} \right) \right] \quad \text{for } \frac{u_* k_s}{v_m} < 5$$  (7.3.70)

$$\frac{u}{u_*} = 8.5 + \frac{1}{\kappa} \left[ 7 \left( \frac{z}{L_{MO}} \right) + \ln \left( \frac{z}{k_s} \right) \right] \quad \text{for } \frac{u_* k_s}{v_m} > 70$$  (7.3.71)

in which:
- $v_m$ = viscosity coefficient of mixture = $v (1 + 2.5 c_b)$
- $c_b$ = bed-load concentration

The depth-averaged velocity can be obtained by integration over the depth:

$$\frac{\bar{u}}{u_*} = 3 + \frac{7h}{2\kappa L} + \frac{1}{\kappa} \ln \left( \frac{u_* h}{v_m} \right) \quad \text{for } \frac{u_* k_s}{v_m} < 5$$  (7.3.72)

$$\frac{\bar{u}}{u_*} = 6 + \frac{7h}{2\kappa L} + \frac{1}{\kappa} \ln \left( \frac{h}{k_s} \right) \quad \text{for } \frac{u_* k_s}{v_m} > 70$$  (7.3.73)

The extra term $(7h/2\kappa L)$ in Eqs. (7.3.72) and (7.3.73) results in an increase of the mean velocity in a sediment-laden flow with respect to a clear flow at the same $u_*$-value. This means a decrease of the friction factor. The method is valid for mixtures, with depth-averaged concentrations up to 0.05.

To apply Eqs. (7.3.72) and (7.3.73), the depth-averaged concentration $\bar{c}$ and the bed-load concentration $c_b$ must be known. Generally, the $\bar{c}$-value is not known because the velocity profile will influence the concentration profile. This effect was also studied by Itakura-Kishi (1980), see next Section.
An iterative solution method is required to compute both the velocity and concentration profile.

McLean (1991a, b) also used the Monin-Obukhov concept based on a local value of the concentration in stead of the depth-averaged concentration. The effect of the suspended sediment on the viscosity was also taken into account.

The fluid and sediment mixing coefficients in a mixture were modelled as:

$$\varepsilon_{f,m} = \frac{\varepsilon_f}{1 + \gamma \xi} \quad (7.3.74)$$

$$\varepsilon_{s,m} = \frac{\varepsilon_f}{\sigma + \beta \xi} \quad (7.3.75)$$

in which:

- $\varepsilon_f$ = fluid mixing coefficient (clear water)
- $\gamma, \beta, \sigma$ = coefficients
- $\xi$ = stratification parameter based on the Monin-Obukhov concept

A numerical iterative method is required to compute the velocity and concentration profile.

Coleman (1981) analysed his own data (maximum concentration $\bar{c} = 0.005$) and re-examined the data from earlier experiments. He found that the change in $\kappa$-value as observed by others was caused by the incorrect application of the logarithmic velocity distribution in the outer flow region where the logarithmic profile is not really valid. By applying the logarithmic profile to the near-bed region (where it is valid), Coleman found that the presence of the suspended sediments ($\bar{c} < 0.005$) had no effect on the $\kappa$-value (thus, $\kappa_m - \kappa = 0.4$). The velocity profiles in the outer region were found to be affected by the wake-effect. The velocity distribution valid for the whole depth is given by:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln(u_*z/v) + B + \frac{2\alpha}{\kappa} \sin^2(\pi z/2h) \quad (7.3.76)$$

in which:

- $\alpha$ = wake strength parameter
- $B$ = coefficient depending on hydraulic conditions
- $\kappa$ = constant of Von Karman (= 0.4)
- $h$ = water depth

The velocity gradient is given by:

$$\frac{du}{dz} = u_* \left( \frac{1}{z} + \frac{\pi \alpha}{h} \sin(\pi z/h) \right) \quad (7.3.77)$$

The effect of the suspended sediment on the fluid viscosity was taken into account. The B-coefficient represents the effect of bottom roughness and the effect of the solid particles on the flow. Close to the bottom where the sine function is insignificant, the wake function does not affect the velocity profile. Equation (7.3.76) was fitted to measured velocity profiles. Coleman found that $\kappa$ was not affected by the suspended sediment and equal to its clear water value. The only effect of the suspended sediment is to increase the wake strength parameter ($\alpha$). The $\alpha$-value varied between $\alpha = 0.19$ for clear water and $\alpha = 0.86$ for a suspension
with $\bar{c} = 0.005$. Valiani (1988) reanalysed the data of Coleman using the same velocity profile, Eq. (7.3.76). Valiani used an (objective) analytical-numerical fitting method, whereas Coleman used a graphical fitting method. Valiani found that the $\kappa_m$-value decreased for increasing concentrations ($\kappa_m = 0.4$ to 0.2). The $B$-coefficient was also found to be related to the suspended sediment concentration. The wake strength parameter $\alpha$ was only poorly affected by the sediment concentrations. The results of Valiani (using the same data set) are different from those of Coleman. Valiani stressed the importance of accurate measurements in the near-bed region.

Umeyama and Gerritsen (1992) modified the mixing length scale, as follows:

$$l_m = \kappa z(1 - z/h)^\alpha$$  \hspace{1cm} (7.3.78)$$

in which:

- $\kappa$ = Von Karman constant in clear water (0.4)
- $\alpha = 1/2 + 1/2 \beta (c/c_a) = \text{coefficient } (c = 0 \text{ yields } \alpha = 1/2)$
- $c = \text{concentration}$
- $c_a = \text{reference concentration}$
- $\beta = \text{coefficient}$

The velocity gradient is given by:

$$\frac{du}{dz} = \frac{u_*}{\kappa z} \left(1 - \frac{z}{h}\right)^{1/2 - \alpha} \left(1 + (s - 1)c\right)^{-1/2}$$  \hspace{1cm} (7.3.79)$$

The $\beta$-coefficient was determined by fitting with measured velocity profiles and found to increase with the concentration. A general relationship of the $\beta$-coefficient was not given. A similar method was proposed to compute the concentration profiles.

3. Concentration profiles and transport rates

In the upper regime with high or hyperconcentrations the effects of the suspended sediments on the density, viscosity, on the (hindered) fall velocity and on the turbulence structure (damping) cannot be neglected.

Assuming that the hindered settling effect can be represented as $w_{c_m} = (1-c)c_w$, and that damping factor $\phi$ is constant over the depth (see Eq. (7.3.16)), an analytic solution of the diffusion equation can be obtained for a parabolic mixing coefficient, as follows:

$$\sum_{n=1}^{4} \frac{1}{n(1-c)^n} - \sum_{n=1}^{4} \frac{1}{n(1-c_a)^n} + \ln \left[\frac{c(1-c)}{c_a(1-c)}\right] = \ln \left[\frac{a(h-z)}{z(h-a)}\right] \left[\frac{w_s/(\beta \phi \kappa u_*)}{\beta \phi \kappa u_*}\right]$$  \hspace{1cm} (7.3.80)$$

in which:

- $c = \text{concentration at height } z$
- $c_a = \text{reference concentration}$
- $a = \text{reference level}$
- $h = \text{water depth}$
- $Z = w_s/(\beta \phi \kappa u_*) = \text{ suspension number}$
- $\phi = \text{turbulence damping factor (constant over depth)}$
- $\beta = \text{ratio of sediment and fluid mixing (constant over depth)}$
- $\kappa = \text{Von Karman constant (= 0.4)}$
- $u_* = \text{bed-shear velocity}$
- $w_s = \text{fall velocity of suspended sediment in clear water}$
A numerical solution method is required when the turbulence damping effect varies with the concentration or concentration gradient (φ not constant). An iterative solution method is required when the damping effect is related to the velocity gradient; both the concentration profile and the velocity profile must be computed simultaneously.


Einstein and Chien (1955) used the following set of equations to compute the concentration profile:

$$c \, w_{s,\text{in}} + \varepsilon_s \frac{dc}{dz} = 0$$

$$w_{s,\text{m}} = w_s (1-c)^\gamma$$

$$\varepsilon_s = \frac{\tau}{\rho (1+(s-1)\bar{c})} (du/dz)$$

The velocity gradient and the depth-averaged concentration (\bar{c}) must be known. Hence, an iterative solution method is required.

Ippen (1971) used a modified velocity gradient to compute the mixing coefficient distribution. The velocity gradient was modeled as:

$$\frac{du}{dz} = \frac{u_s}{\kappa_m z} \left( \frac{z/h - \varphi}{z/h - \varphi \ln(z/h)} \right) \quad (7.3.81)$$

The \(\varphi\)-parameter is an empirical parameter which was found to be dependent on the \(\kappa_m\)-value (according to Eq. (7.3.64)) and the Z-parameter.

Good agreement between computed and measured (data Einstein-Chien, 1955) concentration profiles was obtained.

Itakura and Kishi (1980) introduced the Monin-Obukhov length scale and proposed:

$$\frac{c}{c_a} = \left[ \frac{h - z}{h - a} \right]^{1+\phi} \left( \frac{a}{z} \right)^{w_s/(\kappa u_s)} \quad (7.3.82)$$

in which:

- \(c_a\) = reference concentration at height \(z=a\) above bed
- \(\phi\) = correction factor = \(7 \, h/L_{MO}\)
- \(L_{MO}\) = Monin-Obukhov length scale, Eq. (7.3.68)
- \(Z\) = suspension number = \(w_s/(\kappa u_s)\)
- \(\kappa\) = Von Karman constant in clear water (=0.4)
- \(w_s\) = fall velocity in clear water
- \(h\) = water depth

Iterative computations are required because \(L_{MO}\) depends on the mean concentration \(\bar{c}\).

7.88
McLean (1991a, b) also proposed a (numerical) method based on the Monin-Obukhov concept. 

DeVantier and Larock (1983) studied high-concentration effects by introducing a higher-order turbulence model (K-Epsilon model) to model the effect of the suspended sediment on the mixing coefficient. This sophisticated numerical approach offers the most promising representation of the physical phenomena involved.

Woo et al (1988) modeled the effect of the sediment concentrations on the fall velocity (hindered settling), on the density and on the viscosity. The velocity gradient was determined from measured velocity profiles (curve fitting). The computed concentration profiles showed good agreement with the measured profiles of Einstein-Chien (1955). The method of Woo et al (1988) is not predictive because the velocity profiles must be known to determine the velocity gradient.

Winterwerp et al (1990) found that the concentration profiles in suspensions with mean concentrations in the range of $\bar{C} = 0.2$ to 0.4 can be represented by the Rouse profile using $\kappa = 0.4$ and $w_{s,m} = w_s (1-c)^4$, as shown in Figure 7.3.20. The Rouse concentration profile was solved by iteration as $w_{s,m}$ varies with concentration. The turbulence damping effect as expressed by the modified $\kappa_m$-value was not taken into account.

Van Rijn (1984b) assumed that:

$$\kappa_m = \phi \kappa \quad \text{and} \quad \varepsilon_s = \beta \phi \varepsilon_f \quad (7.3.83)$$

in which $\phi$ = damping factor dependent on concentration and $\beta$ = ratio of sediment and fluid mixing, see Eq. (7.3.16).

The fluid mixing coefficient ($\varepsilon_f$) in clear water was modeled by a parabolic-constant distribution according to Equation (7.3.15).

Using $w_{s,m} = (1-c)^5 w_s$, Equation (7.3.10) was solved numerically. The $\phi$-factor was determined by data fitting using the experimental results of Einstein-Chien (1955), Barton-Lin (1955) and Vanoni-Brooks (1957), yielding:

$$\phi = 1 + \left(\frac{c}{c_0}\right)^{0.8} - 2\left(\frac{c}{c_0}\right)^{0.4} \quad \text{for} \quad c > 0.001 \text{ or } c > 2.5 \text{ kg/m}^3 \quad (7.3.84)$$

in which $c_0$ = maximum volume concentration ($= 0.65$).

Equation (7.3.84) is shown in Figure 7.3.23. Values reported by Yalin-Finlayson (1972) are also reported. Computed and measured (Run S-15, Einstein-Chien, 1955) concentration profiles are shown in Fig. 7.3.24. The reference concentration was assumed to be equal to the concentration measured in the lowest sampling point ($c_s = 625,000$ ppm at $a = 0.005$ m).

The Rouse concentration profile with $\phi = 1$ (no damping effect) yields concentrations which are much too large.

The proposed method is not very suitable for practical use because it requires numerical computations. Therefore, a simplified method based on Eq. (7.3.21) in combination with a modified suspension number $Z'$ was introduced by Van Rijn (1984b). The modified $Z'$-value is defined as:

$$Z' = Z + \Psi \quad (7.3.85)$$
**Figure 7.3.23** ϕ-factor according to Van Rijn (1984b)

**Figure 7.3.24** Measured and computed concentration profile for Run S-15 of Einstein-Chien (1955)

**Figure 7.3.25** Measured and computed sand transport rates at velocities up to 3 m/s
in which:
\[ Z = \frac{w_s}{\beta ku_*} \]
\[ \psi = \text{stratification correction parameter} \]
\[ w_s = \text{fall velocity suspended sediment transport in clear water} \]
\[ K = \text{Von Karman constant (}=0.4) \]
\[ \beta = \text{ratio of sediment and fluid mixing, Eq. (7.3.17)} \]

The \( \psi \) parameter represents the turbulence damping effect and the hindered settling effect. The \( \psi \) parameter was determined by a trial and error method which implied the numerical computation of the concentration profiles for a range of hydraulic conditions and the determination of the \( \psi \) value giving the same concentration profile as that according to the numerical computation. Based on analysis of the available \( \psi \) values, the following relationship was obtained (for \( c > 0.001 \) or \( c > 2.5 \text{ kg/m}^3 \)):

\[
\psi = 2.5 \left( \frac{w_s}{u_*} \right)^{0.8} \left( \frac{c_s}{c_0} \right)^{0.4} \text{ for } 0.01 \leq \frac{w_s}{u_*} \leq 1 \text{ and } 0.01 \leq a/h \leq 0.1 \quad (7.3.86)
\]

in which:
\[ c_s = \text{reference volume concentration, use Eq. (7.3.31)} \]
\[ c_0 = \text{maximum volume concentration (}= 0.65) \]

Figure 7.3.24 shows an example of the concentration profile according to the simplified method for Run S-15 of the Einstein-Chien data. The simplified method is valid for depth-averaged concentrations up to \( \bar{c} = 0.1 \).

Voogt et al (1991) have shown that the method of Van Rijn can be used to compute the sand transport rates for velocities up to 3 m/s, as shown in Figure 7.3.25. The methods of Ackers-White (1973) and Engelund-Hansen (1967) also yield good results. The Van Rijn method without the stratification correction \( (\psi = 0) \) is also shown in Figure 7.3.25. The suspended load transport without the \( \psi \) correction is a factor 1.8 higher for \( \bar{u} = 1 \text{ m/s} \) and a factor 2.8 higher for \( \bar{u} = 2.5 \text{ m/s} \).

### 7.4 Total load transport

#### 7.4.1 Prediction methods

Herein, the following methods are described:

- Einstein (1950)
- Bagnold (1966)
- Engelund-Hansen (1967)
- Bijker (1971)
- Ackers-White (1973, 1990)
- Yang (1973)
- Van Rijn (1984b)

The methods of Engelund-Hansen, Ackers-White and Yang do not make a distinction between bed load and suspended-load transport, but directly give the total load transport rate. Computed transport rates according to the methods of Engelund-Hansen and Van Rijn for specific cases are given in Appendix A.

The total load transport of bed material particles can be obtained by summation of the bed load and suspended load transport, as follows:

\[ q_{t,c} = q_{b,c} + q_{s,c} \]  \hspace{1cm} (7.4.1)

A numerical program-TRANSPOR of the method of Van Rijn is available, see Appendix A.

2. *Engelund-Hansen (1967)*

The method of Engelund-Hansen is based on a energy-balance concept. The work (per unit time and width) required to elevate the sediment load over a height equal to the bed form height \( \Delta \) is:

\[ w_r = (\rho_s - \rho) g q_t \Delta \]  \hspace{1cm} (7.4.2)

The work (per unit time and width) done by the fluid on moving the particles over a length equal to the bed form length \( \lambda \) is

\[ W_a = \alpha_1 (\tau_b' - \tau_{b,cr}) u_* \lambda \]  \hspace{1cm} (7.4.3)

The energy balance \( W_r = W_a \) yields:

\[ q_t = \alpha_1 \left( \frac{\tau_b' - \tau_{b,cr}}{(\rho_s - \rho) g d} \right) u_* d \frac{\lambda}{\Delta} \frac{1}{f} \]  \hspace{1cm} (7.4.4)

in which:
- \( q_{t,c} \) = volumetric total load transport (m\(^3\)/s)
- \( \tau_b' \) = effective bed-shear stress (N/m\(^2\))
- \( \tau_{b,cr} \) = critical bed-shear stress according to Shields (N/m\(^2\))
- \( d \) = particle diameter (m)
- \( u_* \) = overall bed-shear velocity (m/s)
- \( f \) = friction coefficient = 2g/C\(^2\)
- \( C \) = Chézy-coefficient
- \( \lambda, \Delta \) = bed-form length and height (m)

Based on data analysis, it was found that \( \lambda f/\Delta \) is approximately constant, giving:

\[ q_t = \frac{\alpha_2}{f} (\theta' - \theta_{cr}) u_* d \]  \hspace{1cm} (7.4.5)

The dimensionless transport rate \( \phi_t = q_t/((s - 1)^{0.5} g^{0.5} d^{1.5}) \) is introduced, yielding:

\[ \phi_t = \frac{\alpha_2}{f} (\theta' - \theta_{cr}) \theta^{0.5} \]  \hspace{1cm} (7.4.6)

Using Eq. (6.2.26) and assuming \( \theta_{cr} = 0.06 \), it follows that \( \theta' - \theta_{cr} = 0.4 \theta^2 \) for the lower regime. Thus:

7.92
\[ \phi_t = \frac{\alpha_3 \theta^{2.5}}{f} \]  \hfill (7.4.7)

The \( \alpha_3 \)-coefficient was determined by data fitting (approx. 100 flume data), giving \( \alpha_3 = 0.1 \). Thus

\[ \phi_t = \frac{0.1 \theta^{2.5}}{f} \]  \hfill (7.4.8)

in which:

\[ \phi_t = \frac{q_{t,c}}{(s - 1)^{0.5} g^{0.5} d_{50}^{1.5}} \]

\[ \theta = \frac{u^2}{(s - 1) g d_{50}} = \frac{\tau_b}{(\rho_s - \rho) g d_{50}} = \frac{h l}{(s - 1) d_{50}} \]

\[ f = \frac{2g}{C^2} \]

Equation (7.4.8) does not account for the critical bed-shear stress and is, therefore, not accurate close to initiation of motion. Computed values are given in Appendix A.

Equation (7.4.8) can be rearranged to:

\[ q_{t,c} = \frac{0.05 \bar{u}^5}{(s - 1)^2 g^{0.5} d_{50} C^3} \]  \hfill (7.4.9)

in which:

\( q_{t,c} \) = volumetric current-related total load transport (m\(^3\)/s)
\( u \) = depth-averaged velocity (m/s)
\( C \) = Chézy-coefficient (m\(^{1/2}\)/s)
\( s \) = specific density (= \( \rho_f / \rho \))
\( d_{50} \) = median particle size of bed material (m)

3. Yang (1973)

Yang (1973) assumed that the sediment transport is related to the unit stream power, defined as: \( \bar{u} l \). The total sediment concentration \( (c_t) \), defined as the ratio of the sediment and fluid discharge per unit width, \( (c_t = q_t/q) \), was expressed as:

\[ \log(c_t) = \alpha_1 + \alpha_2 \log \left( \frac{\bar{u} l - \bar{u}_s l}{w_s} \right) \]  \hfill (7.4.10)

Analysis of flume and field data resulted in:

\[ \alpha_1 = 5.435 - 0.286 \log(w_s d_{50}/v) - 0.457 \log(u_s/w_s) \]  \hfill (7.4.11)

\[ \alpha_2 = 1.799 - 0.409 \log(w_s d_{50}/v) - 0.314 \log(u_s/w_s) \]  \hfill (7.4.12)
in which:

\( c_t \) = total load concentration in parts per million by weight (ppm)
\( u \) = depth-averaged velocity (m/s)
\( u_e \) = depth-averaged velocity at initiation of motion (m/s)
\( I \) = energy gradient
\( d_{50} \) = median particle diameter of bed material (m)
\( w_s \) = fall velocity (based on \( d_{50} \) of bed material) (m/s)
\( u_* \) = bed-shear velocity (m/s)

\[
\bar{u}_e = \left( \frac{2.5}{\log \left( \frac{u_* d_{50}}{v} \right) - 0.06} + 0.66 \right) w_s \quad \text{for} \quad 1.2 < \frac{u_* d_{50}}{v} < 70
\]

\[
\bar{u}_e = 2.05 \ w_s \quad \frac{u_* d_{50}}{v} \geq 70
\]

The total load transport rate (in kg/sm) is given by:

\[
q_{k,c} = 10^{-3} \ c_t \ \bar{u} \ h
\]  

in which:

\( q_{k,c} \) = total load transport rate (kg/sm)
\( h \) = water depth (m)

4. Ackers-White (1973)

Based on analysis of 925 sets of flume and field data, the following empirical formula was proposed:

\[
q_{k,c} = K \ \bar{u} \ d_{35} \left( \frac{u}{u_*} \right)^n \left( \frac{Y - Y_e}{Y_e} \right)^m
\]  

in which:

\( q_{k,c} \) = total load transport (m³/s)
\( \bar{u} \) = depth-averaged velocity (m/s)
\( u_* \) = bed-shear velocity (m/s)
\( Y \) = particle mobility parameter (-)
\( Y_e \) = critical particle mobility parameter (-)
\( n, m, K \) = coefficients (-)
\( v \) = kinematic viscosity coefficient (m²/s)
\( s \) = specific density (-)
\( d_{35} \) = representative diameter of bed material (m)

\[
Y = \left( \frac{u_*^n}{((s - 1)g d_{35})^{0.5}} \right) \left( \frac{\bar{u}}{5.66 \log(10h/d_{35})} \right)^{-n}
\]

\[
D_* = d_{35} \left( \frac{(s - 1)g}{v^2} \right)^{1/3}
\]
\[ K = 10^{-3.33 + 2.86 \log(D_s) - (\log D_s)^2} \]

for \( 1 < D_s < 60 \)

\[ n = 1 - 0.56 \log(D_s) \]

for \( 1 < D_s < 60 \)

\[ m = \frac{9.66}{D_s} + 1.34 \]

for \( 1 < D_s < 60 \)

\[ Y_{cr} = \frac{0.23}{D_s^{0.5}} + 0.14 \]

for \( 1 < D_s < 60 \)

\[ K = 0.025, \ n = 0, \ m = 1.5, \ Y_{cr} = 0.17 \]

for \( D_s \geq 60 \)

The \( K \) and \( m \) coefficients were later (HR Wallingford, 1990) revised:

\[ K = 10^{-3.46 + 2.79 \log(D_s) - 0.98 (\log D_s)} \]

for \( 1 < D_s < 60 \)

\[ K = 0.025 \]

for \( D_s \geq 60 \)

\[ m = \frac{6.83}{D_s} + 1.67 \]

for \( 1 < D_s < 60 \)

\[ m = 1.78 \]

for \( D_s \geq 60 \)

The form of the \( Y \)-parameter ensures that the fine sediment transport depends upon the total bed shear stress and the coarse transport rate upon the effective grain shear stress.

Revision of the original coefficients was necessary because the original formula predicted transport rates which were considerably too large for relatively fine sediments \( (d_{so} < 200 \mu m) \) and relatively coarse sediments.

**7.4.2 Comparison of methods**

Computed transport rates according to the methods of Engelund-Hansen (1967) and Van Rijn (1984b) are shown in Appendix A.

Van Rijn (1984b) used 486 sets of river data to verify the methods of Engelund-Hansen (1967), Ackers-White (1973), Yang (1973) and Van Rijn (1984). Bed material sizes were in the range of 100 to 400 \( \mu m \). Flow velocities were in the range of 0.4 to 2.4 m/s. The results have been expressed in terms of of a discrepancy ratio \( (r) \) defined as the ratio of the predicted and measured transport rate. Table 7.9 shows the percentage of \( r \)-values of all data falling in the range of \( 0.5 \leq r \leq 2 \). The method of Van Rijn yields the best results for field data with 76\% of the predicted transport rates within a factor 2 of the measured values. The method of Yang yields excellent results for flume data and small-scale river data, but very poor results for large-scale river data (depth > 1 m). This cannot be attributed to the quality of the field data because the other other methods produce reasonable results for these data sets. Therefore, the method of Yang must have serious systematic errors for large depths. On the average, the predicted values were much too small.

Voogt et al (1989) carried out large-scale flume experiments with bed material of 200 \( \mu m \) and velocities in the range of 1 to 3 m/s. Comparison of predicted and measured transport rates showed good results for all three methods. Voogt et al also compared predicted rates with 120 sets of measured estuary data. Bed material sizes were in the range of 200 to 300 \( \mu m \). Flow velocities were in the range of 1 to 2 m/s. The results are given in the Table 7.9. The method
of Van Rijn yields the best results with about 80% of the predicted transport rates within a factor 2 of measured values. The results of the other two methods are less good, yielding a considerable overprediction, particularly the method of Ackers-White.

<table>
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<th>Method</th>
<th>297 Flumedata</th>
<th>486 River data</th>
<th>120 Estuary data</th>
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<td>64%</td>
</tr>
<tr>
<td>Ackers-White</td>
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<td>68%</td>
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<td>39%</td>
</tr>
<tr>
<td>Van Rijn</td>
<td>(1984)</td>
<td>76%</td>
<td>76%</td>
</tr>
</tbody>
</table>

Table 7.9 Percentage of predicted values within factor 2 of measured values

White et al (1973) examined various transport formulae using about 1000 flume data and 260 field data. Froude numbers greater than 0.8 were excluded. The results of the formulae of Ackers-White (1973), Engelund-Hansen (1967), Einstein (1950) and Bagnold (1966) are given in Table 7.10.

<table>
<thead>
<tr>
<th>Method</th>
<th>Flume and field data (1260)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ackers-White</td>
<td>(1973)</td>
</tr>
<tr>
<td>Engelund-Hansen</td>
<td>(1967)</td>
</tr>
<tr>
<td>Einstein</td>
<td>(1950)</td>
</tr>
<tr>
<td>Bagnold</td>
<td>(1966)</td>
</tr>
<tr>
<td></td>
<td>68%</td>
</tr>
<tr>
<td></td>
<td>63%</td>
</tr>
<tr>
<td></td>
<td>46%</td>
</tr>
<tr>
<td></td>
<td>22%</td>
</tr>
</tbody>
</table>

Table 7.10 Percentage of predicted values within factor 2 of measured values

The method of Ackers-White yields the best results with 68% of the predicted transport rates within a factor 2 of measures values. The method of Bagnold yields poor results with a score of 22%.

Yang and Molinas (1982) compared the Yang-formula with 1093 laboratory data and 166 river data; yielding a value of 95% of the predicted transport rates within a factor 2 of measured transport rates. Large-scale river data were not considered.

Usually, the available sediment transport formulae require as input data: mean velocity, water depth, energy slope and particle characteristics. Van Rijn (1984c) tried to predict the transport rate in combination with a roughness predictor using as input data: the mean velocity, the water depth and the particle characteristics. The methods of Engelund-Hansen (1967) and Ackers-White (1973) were used in a similar way. The predicted transport rates were compared with measured values from USA rivers (266 data sets). The results are presented in Table 7.11. The relatively poor results of the methods of Engelund-Hansen and Ackers-White stem from the relatively strong influence of the predicted bed roughness on the transport rate.

<table>
<thead>
<tr>
<th>Method</th>
<th>Field data (266)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van Rijn</td>
<td>75%</td>
</tr>
<tr>
<td>Engelund-Hansen</td>
<td>44%</td>
</tr>
<tr>
<td>Ackers-White</td>
<td>44%</td>
</tr>
</tbody>
</table>

Table 7.11 Percentage of predicted values within a factor 2 of measured values
Finally, it is noted that it is very difficult to predict the sediment transport rate with an accuracy smaller than a factor 2 because of errors in the measured transport rates. Figure 7.4.1 shows mean concentrations ($C = q_f/q$) for two size ranges as measured by different researchers. Deviations of more than a factor 2 can be observed for "similar" conditions.

**Figure 7.4.1** Mean sediment concentration measured by different researchers under "similar" conditions

---

$d_{50} = \{100 - 110\ \mu m\}$

- $h \geq 0.2\ m$
- $T_e \geq 22^\circ\ C$
- $\sigma_s \geq 1.3$
- $x$ Laursen ($b=0.91\ m$)
- $\bullet$ Oxford ($b=1.22\ m$)

$d_{50} = \{180 - 190\ \mu m\}$

- $h \geq 0.25\ m$
- $T_e \geq 20^\circ\ C$
- $\sigma_s \geq 1.3$
- $x$ Guy et al ($b=2.44\ m$)
- $\bullet$ Barton-Lin ($b=1.22\ m$)
7.4.3 Examples and problems

1. Sand concentration measurements were carried out in the Mississippi river at station 1250, 20 April 1961 by Scott and Stephens (1966). The results are presented in Table 7.9. The measured suspended load transport was 2.4 kg/sm. The bed-load transport was not measured.

The hydraulic parameters are: water depth $h = 12.05$ m, energy gradient $I = 0.000076$, mean velocity $\bar{u} = 1.55$ m/s. The bed material characteristics are: $d_{50} = 350$ μm, $d_{90} = 400$ μm, $d_{90} = 1500$ μm, $\sigma_s = 2.5$. Other data are: $\rho_s = 2650$ kg/m$^3$, $\rho = 1000$ kg/m$^3$, $\nu = 10^{-5}$ m$^2$/s.

a. Use the Rouse concentration profile and reference concentration of Van Rijn and Smith-McLean to compute the sediment concentrations at $z = 0.55, 1.59, 3.63, 6.62, 9.39$ and $10.95$ m above the bed?

b. Compute the suspended load transport according to Bagnold and Van Rijn?

c. Compute the total load transport according to Bagnold, Engelund-Hansen, Ackers-White, Yang and Van Rijn?

Solution:

**a. Concentration profile**

Bed-shear stress : $\tau_b = \rho gh I = 9$ N/m$^2$

Bed-shear velocity : $u_* = 0.095$ m/s

Chézy-coefficient : $C = \bar{u}(hI)^{-0.5} = 51.2$ m$^{1/2}$/s

Effective bed roughness : $k_s = 12 \text{ h} \times 10^{-0.1} = 0.21$ m

Particle size parameter : $D_s = 10.1$

Critical bed-shear stress : $\tau_{b,cr} = 0.23$ N/m$^2$ (Fig. 4.1.5)

**Van Rijn**

Grain Chézy-coefficient : $C' = 18 \log(12h/3d_{90}) = 81$ m$^{1/2}$/s

Grain bed-shear stress : $\tau'_b = \rho g(\bar{u}/C')^2 = 3.6$ N/m$^2$

T-parameter : $T = (\tau' - \tau_{b,cr})/\tau_{b,cr} = 14.6$

Reference concentration Van Rijn : $c_s = 0.015 \frac{d_{50}}{k_s} \frac{T^{1.5}}{D_s^{0.3}} = 0.0008$

(reference level $a = k_s = 0.21$ m)

$= 0.0008 \rho_s = 2.1$ kg/m$^3$

Suspended sediment size : $d_s = (1 + 0.011(\sigma_s 1)(T 25))d_{50} = 330$ μm

Fall velocity : $w_s = 0.046$ m/s (Fig. 3.2.6)
Suspension number: \[ Z' = Z + \psi \]

\((\kappa = 0.4, c_o = 0.65)\)

\[ \beta = 1 + 2(w_s/u_s)^2 = 1.47 \]

\[ Z = w_s/(\beta \kappa u_s) = 0.82 \]

\[ \psi = 2.5(w_s/u_s)^{0.8} (c_s/c_o)^{0.4} = 0.1 \]

\[ Z' = 0.82 + 0.1 = 0.92 \]

Concentration profile: \[ c = c_s \left( \frac{h-z}{z} \frac{a}{h-a} \right)^{Z'}, \text{ see Table 7.12} \]

**Smith-McLean**

Reference level: \[ a = 3 d_{90} + 26.3 (\theta' - \theta_{cr}) d_{50} \]

\[ \theta' = \tau_b/((\rho_s - \rho) g d_{50}) \]

\[ = 3.6/(1650 9.81 0.0004) = 0.56 \]

\[ \theta_{cr} = \tau_{b,cr}/((\rho_s - \rho) g d_{50}) = 0.036 \]

\[ a = 0.01 \text{ m} \]

Reference concentration, Eq. (7.3.30): \[ c_s = 0.004 \ c_o \left( \frac{S}{1 + 0.004S} \right) \]

\[ c_o = 0.6 \]

\[ S = (\tau'_b - \tau_{b,cr})/\tau_{b,cr} = 14.6 \]

\[ c_s = 0.033 \]

\[ = 0.033 \rho_s = 88 \text{ kg/m}^3 \]

Suspension number: \[ Z = w_s/(\kappa u_s) = 1.2 \]

Concentration profile: \[ c = c_s \left( \frac{h-z}{z} \frac{a}{h-a} \right)^{Z}, \text{ see Table 7.12} \]

<table>
<thead>
<tr>
<th>Height above bed (z) (m)</th>
<th>Concentrations (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>0.41</td>
</tr>
<tr>
<td>1.59</td>
<td>0.24</td>
</tr>
<tr>
<td>3.63</td>
<td>0.14</td>
</tr>
<tr>
<td>6.62</td>
<td>0.10</td>
</tr>
<tr>
<td>9.39</td>
<td>0.08</td>
</tr>
<tr>
<td>10.95</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Table 7.12** Measured and computed concentrations for Mississippi river
Comparison of measured and computed concentrations shows good agreement in the lower layer (z < 4 m), but less good agreement in the upper layer where the computed values are much too small. This is caused by the gradation of the bed material. The finer fractions will be suspended in the upper layers and the coarser fractions in the near-bed region. This cannot be represented by using one representative fall velocity.

b. **Suspended load transport (see Table 7.13)**

**Bagnold**

\[ \tau_b = 9 \text{ N/m}^2 \]

\[ \bar{u} = 1.55 \text{ m/s} \]

\[ w_s = 0.055 \text{ m/s} \quad (d_{s0} = 400 \mu \text{m, Fig. 3.2.6}) \]

\[ e_b = 0.1, \quad e_a = 0.02 \]

\[ q_s = \frac{e_a(1 - e_b) \tau_b \bar{u}}{(\rho_s - \rho) g (w_s/\bar{u} - 1)} = 4.4 \times 10^{-4} \text{ m}^2/\text{s} = 1.2 \text{ kg/sm} \]

**Van Rijn**

\[ a = k_s = 0.21 \text{ m} \]

\[ c_a = 2.1 \text{ kg/m}^3 \]

\[ Z' = 0.92 \]

\[ F = \frac{(a/h)Z' - (a/h)^{1.2}}{(1 - a/h)Z' (1.2 - Z')} = 0.058 \]

\[ q_s = c_a \bar{u} h F = 2.3 \text{ kg/sm} \]

c. **Total load transport**

**Bagnold**

\[ q_s = \frac{e_b \tau_b \bar{u}}{(\rho_s - \rho) g \tan\phi} = 1.44 \times 10^{-4} \text{ m}^2/\text{s} = 0.4 \text{ kg/sm} \]

\[ q_t = q_b + q_s = 1.2 + 0.4 = 1.6 \text{ kg/sm} \]

**Van Rijn**

\[ q_b = 0.1((s-1)g)^{0.5} d_{s0}^{1.5} D_*^{-0.3} T^{1.5} = 9 \times 10^{-2} \text{ m}^2/\text{s} = 0.24 \text{ kg/sm} \]

\[ q_t = q_b + q_s = 0.2 + 2.3 = 2.5 \text{ kg/sm} \]

**Engelund-Hansen**

\[ C = 51.2 \text{ m}^{3/4}/\text{s} \]

\[ q_t = \frac{0.05 \bar{u}^5}{(s-1)^2 g^{0.5} d_{s0} C^3} = 9.8 \times 10^{-4} \text{ m}^2/\text{s} = 2.6 \text{ kg/sm} \]

**Yang**

\[ u_* d_{s0}/\nu = (0.095 0.0004)/10^{-6} = 38 \]

\[ w_s d_{s0}/\nu = (0.055 0.0004)/10^{-6} = 22 \]

\[ u_*/w_s = 0.095/0.055 = 1.73 \]

\[ \bar{u}_{cr} = 0.126 \text{ m/s} \]

7.100
\[ \alpha_1 = 4.94, \alpha_2 = 1.18 \]

\[ \log(c_t) = \alpha_1 + \alpha_2 \log \left( \frac{(\bar{u} - \bar{u}_c) I}{w_s} \right) \]

\[ c_t = 58 \text{ ppm} \]

\[ q_b = 10^{-3} c_t \bar{u} h = 1.1 \text{ kg/sm} \]

**Ackers-White**

\[ D_* = d_{35}/((s - 1) g/\nu^2)^{1/3} = 8.9 \]

\[ K = 0.0202 \]

\[ m = 2.44 \]

\[ n = 0.468 \]

\[ Y_{cr} = 0.217 \]

\[ Y = 0.892 \]

\[ q_b = K \bar{u} d_{35} \left( \frac{\bar{u}}{u_*} \right)^n \left( \frac{Y - Y_{cr}}{Y_{cr}} \right)^m = 6.5 \times 10^{-4} \text{ m}^2/\text{s} \]

\[ = 1.7 \text{ kg/sm} \]

<table>
<thead>
<tr>
<th>Method</th>
<th>Computed</th>
<th>Measured suspended load transport (kg/sm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bed load transport (kg/sm)</td>
<td>Suspended load transport (kg/sm)</td>
</tr>
<tr>
<td>Bagnold</td>
<td>0.4</td>
<td>1.2</td>
</tr>
<tr>
<td>Engelund-Hansen</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Yang</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ackers-White</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Van Rijn</td>
<td>0.2</td>
<td>2.3</td>
</tr>
</tbody>
</table>

|                        | 2.4 |

**Table 7.13 Measured and computed transport rates for Mississippi river**

2. A wide river has a water depth of \( h = 3 \text{ m} \), energy gradient \( I = 2.1 \times 10^4 \), depth-averaged flow velocity \( \bar{u} = 1.5 \text{ m/s} \). The bed material characteristics are \( d_{35} = 250 \mu m \), \( d_{50} = 300 \mu m \), \( d_{90} = 600 \mu m \), \( \alpha_s = 1.8 \). The water temperature is \( T_e = 20^\circ C \), \( \nu = 1.1 \times 10^{-6} \text{ m}^2/\text{s} \). Other data are \( \rho_s = 2650 \text{ kg/m}^3 \), \( \rho = 1000 \text{ kg/m}^3 \).

a. Use the Rouse concentration profile and reference concentration of Van Rijn and McLean to compute sediment concentrations at \( z = 0.03, 0.1, 0.5, 1, 2 \) and \( 3 \text{ m above bed} \)?

b. Compute suspended load transport according to Bagnold and Van Rijn?
c. Compute total load transport according to Bagnold, Engelund-Hansen, Ackers-White, Yang and Van Rijn?

Solution:

Computed concentrations are given in Table 7.14.

<table>
<thead>
<tr>
<th>Height above bed</th>
<th>Concentration (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Van Rijn</td>
</tr>
<tr>
<td>z (m)</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>15.4</td>
</tr>
<tr>
<td>0.1</td>
<td>3.8</td>
</tr>
<tr>
<td>0.5</td>
<td>0.53</td>
</tr>
<tr>
<td>1</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>0.04</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table 7.14 Computed concentrations*

Suspended load transport:
- Bagnold: \( q_b = 1.04 \text{ kg/sm} \)
- Van Rijn: \( q_v = 1.72 \text{ kg/sm} \)

Total load transport:
- Bagnold: \( q_t = 1.3 \text{ kg/sm} \)
- Engelund-Hansen: \( q_t = 1.7 \text{ kg/sm} \)
- Ackers-White: \( q_t = 2.1 \text{ kg/sm} \)
- Yang: \( q_t = 0.9 \text{ kg/sm} \)
- Van Rijn: \( q_t = 2.0 \text{ kg/sm} \)

3. A wide river has a water depth of \( h = 3 \text{ m} \), mean flow velocity of \( \bar{u} = 1.2 \text{ m/s} \), energy gradient \( I = 2.10^4 \), bed material characteristics \( d_{50} = 120 \text{ µm} \), \( d_{90} = 150 \text{ µm} \), \( d_{\infty} = 300 \text{ µm} \), \( \sigma_s = 1.5 \). Other data are \( T_e = 20^\circ \text{C} \), \( \nu = 1 \times 10^{-6} \text{ m²/s} \), \( \rho_s = 2650 \text{ kg/m}^3 \), \( \rho = 1000 \text{ kg/m}^3 \).

a. Compute representative size of suspended sediment?
b. Compute reference concentration according to Van Rijn?
c. Compute sediment concentration (Rouse profile) at \( z = 0.1, 0.3, 0.5, 1 \text{ and } 2 \text{ m above the bed?} \)
d. Compute suspended load transport according to Bagnold and Van Rijn?
e. Compute ratio of suspended load transport according to Bagnold, Van Rijn and Laursen?
f. Compute total load transport according to Engelund-Hansen, Ackers-White, Bagnold and Van Rijn?

Solution:

a. \( d_s = 140 \text{ µm} \)

b. \( c_s = 2.2 \text{ kg/m}^3 \text{ at } a = 0.068 \text{ m} \)
c. \( c_{0.1} = 1.8 \text{ kg/m}^3 \)
\( c_{0.3} = 1.0 \text{ kg/m}^3 \)
\( c_{0.5} = 0.75 \text{ kg/m}^3 \)
\( c_1 = 0.47 \text{ kg/m}^3 \)
\( c_2 = 0.24 \text{ kg/m}^3 \)

d. Bagnold
\( q_s = 1.7 \text{ kg/m}^3 \)
Van Rijn
\( q_s = 1.6 \text{ kg/m}^3 \)

e. Bagnold
\( q_s/q_l = 0.90 \)
Van Rijn
\( q_s/q_l = 0.97 \)
Laursen
\( q_s/q_l = 0.92 \)

f. Engelund-Hansen
\( q_t = 2.2 \text{ kg/sm} \)
Ackers-White
\( q_t = 0.8 \text{ kg/sm} \)
Bagnold
\( q_t = 1.9 \text{ kg/sm} \)
Van Rijn
\( q_t = 1.7 \text{ kg/sm} \)

4. A wide river has a depth of \( h = 3 \) m; the energy gradient is \( I = 5 \times 10^{-4} \). The bed material characteristics are \( d_{s0} = 150 \mu \text{m}, d_{s0} = 300 \mu \text{m} \). The fall velocity is \( w_s = 0.017 \text{ m/s} \). The bed is plane. The depth-averaged concentration is \( \overline{c} = 5 \text{ kg/m}^3 \). Other data are: \( v = 1 \times 10^{-6} \text{ m}^2/\text{s}, \rho_s = 2650 \text{ kg/m}^3, \rho = 1000 \text{ kg/m}^3 \).

What is the depth-averaged velocity with and without stratification according to Itakura and Kishi?
What are the velocities at \( z = 0.05, 0.1, 0.5, 1, 2 \) and \( 3 \) m above the bed?

Solution:

Assume near-bed viscosity
\( \nu_m = 10 \nu = 10^{-5} \text{ m}^2/\text{s} \)

Bed-shear velocity
\( u_* = (gh)^{0.5} = 0.121 \text{ m/s} \)

Mobility parameter
\( \theta = u_*^2/((s-1)g d_{s0}) = 6.1 \)

Effective bed roughness,
\( k_{s,c} = 3 \theta d_{s0} = 0.0055 \text{ m} \)

Take \( k_s = 0.01 \text{ m} \)

Reynolds number
\( u_* k_s/\nu_m = 121 \rightarrow \text{rough regime} \)

Monin-Obukhov length scale
\( \overline{c}_{\text{volume}} = 5/\rho_s = 5/2650 = 0.0019 \)
\( L = (u_*)^3/(s-1)g \kappa \overline{c} w_s = 8.5 \text{ m} \)

Depth-averaged velocity
\( \overline{u} = u_*[6 + 7h/(2KL) + 2.5 \ln(h/k_s)] \)
\( = 23.3 u_* = 2.83 \text{ m/s} \)

Depth-averaged velocity without stratification
\( \overline{u} = u_*[6 + 2.5 \ln(h/k_s)] \)
\( = 20.3 u_* = 2.45 \text{ m/s} \)
Velocity profile: \[ u_z = u \cdot [8.5 + 2.5(7z/8.5 + \ln(z/k_s))] \]

- \[ u_{0.05} = 1.53 \text{ m/s} \]
- \[ u_{0.1} = 1.75 \text{ m/s} \]
- \[ u_{0.5} = 2.34 \text{ m/s} \]
- \[ u_1 = 2.67 \text{ m/s} \]
- \[ u_2 = 3.13 \text{ m/s} \]
- \[ u_3 = 3.50 \text{ m/s} \]

5. A wide river has a discharge of \( q = 10 \text{ m}^2/\text{s} \). The energy gradient is \( I = 5 \times 10^4 \). The total sediment discharge is \( q_s = 60 \text{ kg/sm} \). The bed material characteristics are \( d_{50} = 150 \mu\text{m} \), \( d_{90} = 300 \mu\text{m} \). The fall velocity is \( w_s = 0.017 \text{ m/s} \). The bed roughness is \( k_s = 0.01 \text{ m} \). Other data: \( v = 1 \times 10^{-6} \text{ m}^2/\text{s} \), \( \rho_s = 2650 \text{ kg/m}^3 \), \( \rho = 1000 \text{ kg/m}^3 \).

What is the mean (volume) sediment concentration?
What is the flow depth according to the method of Itakura and Kishi (assume rough flow)?

Solution:
\[ c_{\text{mean}} = 0.0023 \]
\[ h = 3.3 \text{ m} \text{ (by trial and error)} \]
8.5 References


REFERENCES (continued)


REFERENCES (continued)


REFERENCES (continued)


REFERENCES (continued)


REFERENCES (continued)


REFERENCES (continued)


8 BED MATERIAL SUSPENSION AND TRANSPORT IN WAVES

8.1 Introduction

Wave motion over an erodible sand bed can generate a sediment suspension with relatively large sediment concentrations in the near-bed region in the case of non-breaking waves, as shown by the laboratory experiments of Nakato et al. (1977) and by Bosman (1982) for the ripple regime and by Horikawa et al. (1982), Staub et al. (1984) and Ribberink and Al Salem (1991, 1992) for the plane bed (sheet flow) regime.

Field measurements showing similar results were performed by Huntley and Hanes (1987), Antsyferov and Kosyan (1990), Greenwood et al. (1991), Beach and Sternberg (1991), Van Rijn and Kroon (1992) and by others.

As an example time-averaged concentrations in and near the surf zone of Kamchia (Black Sea) during swell conditions which were measured by Antsyferov and Kosyan (1990) using poles equipped with traps, are shown in Figure 8.1.1. Plunging breaking started at pole 9; the breaking zone was situated between poles 6 and 9. The key role of the breaking waves on the sediment concentration field in the coastal zone can be clearly observed from Figure 8.1.1. The concentrations are maximum near the plunging point (pole 9) and decrease sharply on both sides of the plunging area. The bed material was coarsest in the most energetic area (850 μm) and finer inside the surf zone (650 μm) and outside the surf zone (450 μm).

![Figure 8.1.1 Sediment concentration field in surf zone near Kamchia, Black Sea (Antsyferov and Kosyan, 1990)](image)

Wave-induced transport processes are related to the velocities generated by high and low-frequency wave phenomena. Net onshore transport is dominant in non-breaking wave conditions, whereas net offshore transport is dominant in breaking wave conditions.

In this chapter the various wave-related transport processes (transport of particles by the oscillating fluid motions) are discussed. Measured wave-induced concentrations and transport rates over rippled and plane beds are analyzed. Mathematical models to compute instantaneous and time-averaged concentration profiles and transport rates are discussed.
Transport processes in combined wave and current conditions with the steady current as the dominant transporting agent are discussed in Chapter 9.

8.2 Identification of transport processes

8.2.1 Non-breaking waves

Cross-shore transport processes in (non-breaking) shoaling waves are strongly related to the instantaneous near-bed velocities and concentrations in the high and low frequency ranges.

Instantaneous velocities and concentrations in shoaling waves in field conditions just seaward of the breaking zone have been measured by Huntley and Hancs (1987), Wright et al. (1991), Osborne and Greenwood (1992) and others.

To demonstrate the relative importance of different transport processes, the instantaneous velocity and concentration are decomposed in three components, as follows:

\[
U = u + \bar{U} + \hat{U}_L
\]

\[
V = v + \bar{V} + \hat{V}_L
\]

\[
C = c + \bar{C} + \hat{C}_L
\]

(8.2.1)

in which:

- \(U, V\) = instantaneous cross-shore and longshore velocity at height \(z\) above bed
- \(C\) = instantaneous concentration at height \(z\) above bed
- \(u, v\) = time-averaged velocity
- \(c\) = time-averaged concentration
- \(\bar{U}, \bar{V}\) = high-frequency velocity oscillations, short waves, \((T < 12 \text{ s})\)
- \(\bar{U}_L, \bar{V}_L\) = low-frequency velocity oscillations, long waves \((12 < T < 200 \text{ s})\)
- \(\hat{C}, \hat{C}_L\) = high-frequency concentration oscillation
- \(\hat{C}_L\) = low-frequency concentration oscillation

High-frequency velocity oscillations are caused by turbulent eddies and by wind waves.

Low-frequency velocity oscillations are caused by macroscale turbulence and by long waves (bound long waves, free long waves).

High-frequency turbulence components show a random and chaotic nature; low-frequency turbulence components show a more regular pattern. Separation of turbulence-induced and wave-induced oscillations in the frequency domain (by filtering techniques) is hardly possible, because of the overlap in frequencies of both effects.

Figure 8.2.1 shows a schematic representation of instantaneous wave-induced velocities and concentrations in the near-bed region under irregular (non-breaking) shoaling wind waves.

The following phenomena can be observed:
- asymmetric high-frequency velocity oscillation (\(\bar{U}_L\)) with peak forward (onshore) orbital velocities larger than the peak backward (offshore) orbital velocities (\(\bar{U}_{\text{s, on}} > \bar{U}_{\text{s, off}}\)),
- nearly symmetric low-frequency velocity oscillations (\(\bar{U}_L\)) generated by bound long waves \((12 < T < 200 \text{ s})\) which are related to groups of alternating low and high waves; forward-directed velocities during passage of low waves and backward-directed velocities during passage of high waves,
- low concentrations under low waves and high concentrations under high waves.
Figure 8.2.1 Schematic representation of instantaneous velocities and concentrations under irregular shoaling waves

The net cross-shore transport at a particular level (near-bed region) can be obtained by averaging over time (indicated by an overbar), as follows:

\[
q_{\text{net}} = \frac{1}{\Delta t} \int U C \, dt - \overline{UC}
\]  
(8.2.2)

Substituting Eq (8.2.1) in Eq. (8.2.2) yields:

\[
\overline{UC} = u_c + \overline{U_s C_s} + \overline{U_s C_L} + \overline{U_L C_s} + \overline{U_L C_L}
\]  
(8.2.3)

The \( \overline{UC} \) fluxes are termed the wave-related fluxes and are discussed in this Chapter. The \( u_c \) flux is termed the current-related flux (see Chapter 9); the following steady and quasi steady currents may occur:
- tide induced currents,
- coriolis-induced currents,
- wind induced currents,
- density-induced currents,
- currents due to non-breaking waves,
- longshore-currents due to breaking waves,
- cross-shore return currents due to breaking waves and storm surge (undertow),
- cross-shore rip currents.

The transport processes under shoaling waves (seaward of breaking zone) associated with the various terms of Eq. (8.2.3) can be described as follows:
- the generation of a quasi-steady weak current in forward (onshore) direction near the bed results in a net forward-directed transport (\( u_c \)) in case of a plane bed or a flat
rippled bed (see also Figure 8.2.2); strong vortex motions over a steep rippled bed may induce a net backward (offshore) steady current and transport,

- the generation of high-frequency asymmetric wind waves ($\bar{U}_w$) with relatively large forward peak velocities under the wave crests and relatively small backward peak velocities under the wave troughs results in a net forward transport ($\bar{U}_s\bar{C}_s$) in case of a plane bed or a flat rippled bed (Fig. 8.2.3); strong vortex motions over steep ripples may give a net backward transport (Fig. 8.2.3),

- the generation of low-frequency bound-long waves related to a mean water surface increase under low-amplitude wind waves and a mean water surface decrease under high-amplitude wind waves results in a net backward transport ($\bar{U}_w\bar{C}_s$) because the concentrations ($\bar{C}_s$) are relatively large under high waves and relatively small under low waves ($U_L$ and $C_L$ out of phase), see Fig. 8.2.4,

- macro-scale (low-frequency) turbulence may generate a net transport in forward or backward direction depending on the hydrodynamic conditions,

- the flux terms ($\langle \bar{U}_s, \bar{C}_s \rangle$ and $\langle \bar{U}_w, \bar{C}_s \rangle$) related to interaction of high and low frequency components will be approximately zero because $U_s$ and $C_s$ are uncorrelated.

![Wave propagation diagram](image)

**Figure 8.2.2** Net velocity profile due to non-linear effects in shoaling waves

![Transport processes diagram](image)

**Figure 8.2.3** Transport processes due to (high frequency) asymmetric wave motion
The net wave-related transport process can also be analyzed by assuming that the instantaneous transport rate is related to the third power of the instantaneous velocity (see Section 8.3.3), as follows:

\[ q_s \approx U \, U^2 \]  
\[ q_{w,\text{net}} \approx \overline{U \, U^2} \]  

The instantaneous velocity is defined as:

\[ U = u + \overline{U_s} + \overline{U_L} \]  

Substitution of Equation (8.2.6) in Equation (8.2.5) and assuming \( u < \overline{U_{s,\text{max}}} \) and \( \overline{U_{L,\text{max}}} < \overline{U_{s,\text{max}}} \), the terms with \( u^3 \), \( u^2 \) and \( \overline{U_L^2} \) can be neglected, yielding:

\[ q_{w,\text{net}} = \overline{U_s \, U_s^2} + 3u \, \overline{U_s^2} + 3 \overline{U_L \, U_s^2} + 6u \, \overline{U_s \, U_L} \]  

in which:

\( \overline{U_s \, U_s^2} \) = net transport due to wave asymmetry (in wave propagation direction)

\( 3u \, \overline{U_s^2} \) = net current-related transport due to wave action (\( \overline{U_s^2} \) = stirring parameter) in the direction of the mean current

\( 3 \overline{U_L \, U_s^2} \) = net transport rate due to interaction of long and short waves; against wave propagation direction in case of group-bound long waves (negative sign)

\( 6u \, \overline{U_s \, U_L} \) = net transport rate due to interaction of mean current, short and long waves; approximately zero because \( \overline{U_s} \) and \( \overline{U_L} \) are uncorrelated.

The magnitude and direction of the low-frequency transport processes are not very well known despite many field research efforts. Huntley and Hanes (1987) observed a weak offshore-directed transport near the sea bed. Wright et al. (1991) reported that low-frequency effects caused measurable but not dominant cross-shore fluxes; the low-frequency fluxes were just as often directed seaward as shoreward.
velocity at 2.2 cm above mean bed level (T < 5 sec) high frequency

velocity at 2.2 cm above mean bed level (T > 5 sec) low frequency

velocity at 2.2 cm above mean bed level (T < 5 sec) high frequency

velocity at 2.2 cm above mean bed level (T > 5 sec) low frequency

Figure 8.2.5 High-frequency and low-frequency oscillations of cross-shore velocity and sand concentration at z = 0.022 m above the bed, Test T14.30.90, Havinga (1992)
Figure 8.2.6 Vertical distribution of mean concentration, cross-shore velocity and transport rates, Test T14.30.90, Havinga (1992)
An illustrative example of high-frequency and low-frequency oscillations of velocities and concentrations in cross-shore direction is given in Fig. 8.2.5 for an experiment carried out by Havinga (1992) in a wave-current basin with a rippled sand bed of 100 μm. The low-frequency velocity is offshore-directed under groups of high waves (see t = 1222 s, 1270 s and 1290 s), which is related to the presence of bound-long waves. The low-frequency concentrations are found to be maximum under high waves.

Figure 8.2.6 shows the vertical distribution of the mean sand concentration, mean cross-shore velocity and the cross-shore transport rates for the same test. The high-frequency transport component \( \langle U_v \rangle \) is directed onshore, whereas the low-frequency transport component \( \langle U_l \rangle \) is directed offshore (bound-long wave effect). Both effects are of the same order of magnitude.

Another transport process of importance is related to gravity-forces acting on bed-load particles on a sloping bed (see Sections 7.2.6 and 8.5.4). This will result in an increase of the transport rate during downsloping flow and a decrease of the transport rate during upsloping flow. The gravity effect on the suspended load transport is negligible small, which can be derived from Equation (7.2.49) given by Bagnold (1966); the mean velocity \( \bar{u} \) for unidirectional flow should be replaced by the net velocity \( \Delta u \) for oscillating flow, yielding \( \Delta u \geq \tan \beta \).

### 8.2.2 Breaking waves

Transport processes in breaking waves (surf zone) are caused by the following mechanisms:

- net backward (offshore) transport due to the generation of a net return flow (undertow, see Figure 8.2.7) in the near-bed region in spilling and plunging breaking waves.
- net forward (onshore) transport by asymmetric wave motion (\( \bar{u}_w \)) in weakly spilling breaking waves.
- longshore and offshore-directed transport due to the generation of large-scale circulation cells with longshore currents and offshore rip currents (see Fig. 8.2.8).
- gravity-induced transport (bed load) in downsloping direction.

Low-frequency transport processes \( \langle U_v U_l \rangle \) under breaking waves in the surf zone are different from those under shoaling waves seaward of the breaking zone. In the surf zone the wave group structure is much less pronounced; low-frequency effects are more related to edge waves and free long waves (reflected waves) than to bound long waves (wave groups). Onshore as well as offshore-directed low-frequency transport processes were observed by Osborne and Greenwood (1992).

![Figure 8.2.7 Undertow current in breaking waves](image)

8.8
Figure 8.2.8 Longshore and offshore-directed rip currents in and near the surf zone

Kroon and Van Rijn (1993) found relatively large high-frequency velocity oscillations (upto 2 m/s) and low-frequency velocity oscillations (upto 0.5 m/s) in the surf zone near Egmond, The Netherlands. The high- and low-frequency sediment flux components at approximately \( z = 0.05 \) m above the bed were however quite small, see Fig. 8.2.9. The current-related component (uc) was found to be dominant.

Transport processes related to longshore and offshore-directed steady currents in the surf zone can be described in terms of the time-averaged concentrations and velocities (UC ~ uc). These latter processes are less complicated to describe and have been studied by many researchers (see Chapter 9).

Figure 8.2.9 Transport vectors for the surf zone of Egmond, The Netherlands, Kroon and Van Rijn (1993)
8.2.3 Vector presentation of fluxes

Figures 8.2.9 and 8.2.10 show a vector presentation of the various transport components. The overall net transport direction is given by vector addition of the time-averaged values of the measured instantaneous variables. The transport vectors can be decomposed in components along the current and wave directions. The instantaneous sediment fluxes ($F_c$, $F_w$) in current and wave direction ($\alpha \neq \beta$) can be derived from the (measured) instantaneous fluxes in $x$ and $y$ directions, as follows:

$$F_c = -\left(\frac{\sin \beta}{\sin(\alpha - \beta)}\right) UC + \left(\frac{\cos \beta}{\sin(\alpha - \beta)}\right) VC$$ (8.2.8)

$$F_w = +\left(\frac{\sin \alpha}{\sin(\alpha - \beta)}\right) UC - \left(\frac{\cos \alpha}{\sin(\alpha - \beta)}\right) VC$$ (8.2.9)

in which:

- $F_c = U_c C$ = instantaneous flux in current direction
- $F_w = U_w C$ = instantaneous flux in wave direction
- $U_c$ = instantaneous velocity in current direction
- $U_w$ = instantaneous velocity in wave direction
- $U$ = instantaneous velocity in $x$ direction
- $V$ = instantaneous velocity in $y$ direction
- $C$ = instantaneous concentration
- $\alpha$ = angle of current direction with positive $x$-axis (Fig. 8.2.10)
- $\beta$ = angle of wave direction with positive $x$-axis (Fig. 8.2.10)
The $F_e$ and $F_w$-fluxes can be decomposed in mean and oscillating components. Equations (8.2.8) and (8.2.9) do not give accurate results for $\alpha = \beta$.

8.3 Analysis of measured concentration profiles and transport rates

8.3.1 Instantaneous concentrations

In section 8.2 it has been shown that instantaneous concentrations are especially important with respect to the wave-related transport processes. Instantaneous concentrations generated by non-breaking waves in the ripple regime in laboratory conditions have been measured by Nakato et al. (1977) and by Bosman (1982). Instantaneous concentrations in the plane bed regime (sheet flow) in laboratory conditions have been measured by Horikawa et al. (1982), by Staub et al. (1984) and by Ribberink and Al-Salem (1991, 1992).

Instantaneous sediment concentrations in field conditions have been measured by Huntley and Hanes (1987), Wright et al. (1991), Osborne and Greenwood (1992).

Sediment concentrations measured close to the bed ($z < 0.05$ m), especially in field conditions, should be considered with great care. Black and Rosenberg (1991) using an underwater camera observed a 'rooster-tail' plume of sand downstream of a small optical sensor placed at 0.03 m above the bed level. After flow reversal much of this material was swept back to the sensor!

Ripple regime

Figure 8.3.1 shows ensemble mean values of instantaneous concentrations within a wave cycle in a tunnel measured by Bosman (1982). A sinusoidal oscillatory motion with a period of $T = 1$ sec and a velocity amplitude of $\bar{U}_s = 0.3$ m/s was generated over a sand bed ($d_{90} = 200$ $\mu$m, $d_{90} = 240$ $\mu$m). The bed was covered with almost perfectly two dimensional ripples (length = 0.055 m, height = 0.01 m). An optical instrument was used to measure the concentration values above the ripple crest and trough. The exact measuring locations are shown in Figure 8.3.1. The ripple crest measurements show ensemble mean concentrations and standard deviations based on 100 periods. About 70% of all measurements are within the standard deviation curves. As regards the trough measurements, only ensemble mean values are shown. The following phenomena can be observed above the crest:

- the (random) scatter is quite large (roughly $\pm 50\%$),
- two large concentration peaks just after flow reversal and probably generated by leeside eddy-velocities,
- two smaller concentration peaks at the moment of maximum flow, probably generated by stoss-side velocities,
- asymmetric concentration distribution (water motion is symmetric).

The phenomena above the trough are:

- the (random) scatter is also quite large (not shown),
- two larger concentration peaks after flow reversal and probably generated by leeside eddy-velocities (time lag is larger compared with concentration measurements at the crest),
- two smaller concentration peaks after maximum flow and probably generated by stoss-side velocities,
- asymmetric concentration distribution,
- the peaks above the trough are smaller than those above the crest due to dispersion and settling of sediment particles.
Figure 8.3.1  Ensemble mean values and standard deviations of instantaneous concentrations over a wave cycle, $d_{50} = 200 \mu m$, Bosman (1982)
According to Bosman, the scatter is mainly caused by (slight) local ripple changes resulting in small differences in the local velocities and hence concentrations. Based on this, it seems very difficult to relate the local instantaneous sediment concentration to a local instantaneous fluid velocity. The measurements of Nakato et al. (1977) with periods in the range of 1 to 3 s show similar results as those of Bosman.

Figure 8.3.2 shows sediment concentrations in shoaling waves in field conditions measured by Greenwood et al. (1991). The peaky behaviour of the concentrations is similar to that observed by Bosman.

Sheet flow regime

Instantaneous concentrations generated by non-breaking waves in the sheet flow regime have been measured by Horikawa et al. (1982) in oscillatory flow over a sand bed ($d_{50} = 200 \ \mu m$) in a wave tunnel using an electro-resistance concentration meter. Results are shown in Figure 8.3.3. Large concentration gradients can be observed in a layer of 0.003 to 0.005 m above and below the initial bed surface level. The latter refers to the bed level before the experiment. The maximum concentrations are generated at the moment of maximum velocities ($\theta = 90^\circ$). The concentration is minimum at the moment of minimum velocities ($\theta = 0^\circ$).

The thickness of the concentration layer was about 0.02 m. At this latter level the time-averaged concentration was about 0.5 kg/m$^3$. The velocities have been determined by applying a camera technique focusing on the interior particles to eliminate the wall effect. Analysis of the instantaneous sand transport rates show that most of the transport occurs below the initial bed level ($\theta = 30^\circ, 60^\circ, 90^\circ$).

Ribberink and Al Salem (1992) also measured instantaneous sand concentrations ($d_{50} = 210 \ \mu m$) in the sheet flow layer. Figure 8.3.4 shows instantaneous sand concentrations at elevations $z = 0.5, 0.7, 0.9$ and $1.8$ cm above the initial surface level under regular
asymmetric wave motion. A large peak concentration can be observed just after the maximum forward velocity of 1.6 m/s. The time lag of the concentration increases with the height above the bed. The relatively small backward velocity of 0.6 m/s does not generate a separate peak concentration. This phase of the wave cycle is dominated by the phenomena of the previous half cycle (settling processes).

Figure 8.3.3 Instantaneous concentrations, velocities and fluxes in sheet flow regime, \(d_{50} = 200 \mu m\), Horikawa et al. (1982)
Figure 8.3.5 shows instantaneous sand concentrations \( d_{50} = 210 \mu m \) inside the sheet flow layer from \( z = -6 \) mm to \( z = 3 \) mm with respect to the initial bed surface. Regular sinusoidal wave motion was generated during this test. Large concentration gradients over a small vertical distance of about 0.01 m can be observed. The concentration varies from \( c = 200 \) kg/m\(^3\) to 1000 kg/m\(^3\) over a layer thickness of 0.01 m. A remarkable phenomenon is the existence of a thin layer at \( z = 0 \) with an almost constant concentration from 500 to 600 kg/m\(^3\). This was also observed by Horikawa et al. (see Figure 8.3.3). The minimum concentrations are approximately in phase with the peak velocity for \( z < 0 \) and the maximum concentrations are in phase with the peak velocity for \( z > 0 \) m.

![Image](image)

**Figure 8.3.4** Instantaneous concentrations in sheet flow regime, \( d_{50} = 210 \mu m \). Irregular, asymmetric wave motion, \( \bar{U}_c = 1.1 \) m/s, \( \bar{U}_r = 0.6 \) m/s, \( T = 6.5 \) s, Ribberink and Al-Salem (1992)

**Figure 8.3.5** Instantaneous concentrations in sheet flow regime, \( d_{50} = 210 \mu m \). Regular sinusoidal wave motion, \( \bar{U} = 1.6 \) m/s, \( T = 7.2 \) s, Ribberink and Al-Salem (1992)

### 8.3.2 Time-averaged concentrations

Time-averaged concentrations (averaged over many waves) are relevant for the current-related sediment transport. Time averaging is necessary to eliminate the large random scatter of the instantaneous concentrations.

Time-averaged concentrations have been measured by various researchers. Some of the experimental results are presented here focusing on the following cases:

- concentrations by non-breaking waves over a rippled bed,
- concentrations by breaking waves,
- concentrations in the sheet flow layer.
1. Concentrations by non-breaking waves over a rippled bed

In case of oscillatory flow over a rippled bed the instantaneous sediment concentrations are varying in time and space, as shown in Figure 8.3.1. To eliminate these variations, it is necessary to average the concentrations over time (many waves) and over space (many bed forms). In laboratory conditions this can be done quite simply by moving the measuring instruments forward and backward over a certain distance during the sampling period. This method has been used by Bosman (1982), Nieuwjaar-Van der Kaaij (1987) and Nap-Van Kampen (1988) for experiments in a wave flume.

Figure 8.3.6 shows time-averaged and bed-averaged concentrations measured by Nieuwjaar-Van der Kaaij (1987) using a pump sampler in a wave flume with a water depth of 0.5 m and a sand bed of $d_{50} = 200 \ \mu m$. Irregular waves were generated ($H_s =$ significant wave height). The sand bed was covered with ripples in all experiments ($\Delta_r =$ ripple height).

The following phenomena can be observed:
- concentrations increase for increasing wave height $H_s$,
- negligible small concentrations for $z/h > 0.3$,
- constant concentration gradient at each wave height $H_s$,
- steeper concentration profiles for increasing wave heights,
- ripple height increases slightly for increasing wave height.

![Figure 8.3.6 Time-averaged concentrations in non-breaking waves in a flume with a rippled bed. Nieuwjaar-Van der Kaaij (1987)](image)

Figure 8.3.7 shows time-averaged concentrations measured by Van Rijn (1987) using a pump sampler in a large-scale wave flume with a water depth of 2 m and a sand bed with $d_{50} = 210 \ \mu m$. Irregular waves were generated. The bed was covered with pronounced ripples with a height of about 0.02 m for $H_s \leq 0.69$ m; smooth ripples with a height of 0.001 m were observed after the test with $H_s > 1.1$ m. Bed averaging of the concentrations was not applied.
The following phenomena can be observed:
- increasing concentrations for increasing wave height up to $H_s = 0.69$ m and decreasing concentrations for $H_s = 1.1$ m in the near-bed layer,
- two layer concentration profiles with large gradients in the near-bed layer of $z/h < 0.05$ (3 ripple heights) and small gradients for $z/h > 0.05$; finer size fractions are dispersed in the upper layers,
- decrease of ripple height from $\Delta_r = 0.02$ m to 0.001 m for wave height increasing from $H_s = 0.69$ m to $H_s = 1.1$ m.

The experimental results indicate a simultaneous decrease of the near-bed concentrations and the ripple height when the wave height increases from $H_s = 0.69$ m to $H_s = 1.1$ m. This can be observed more clearly in Figure 8.3.8 showing the concentrations at three elevations ($z = 0.025$ h, 0.05 h and 0.1 h) as a function of the mobility parameter $\Psi - \bar{U}^2 / [(s-1)gd_{50}]$ in which $\bar{U}$ is amplitude of near-bed orbital velocity. The ripple height is also plotted as a function of $\Psi$. Figure 8.3.8 clearly shows that the concentrations are largest for $\Psi \approx 150$ in the presence of ripples with a height of 0.02 m. For $\Psi > 150$ there is a gradual transition from the ripple regime (height $\approx 0.02$ m) with relatively large concentrations to the smooth bed regime (smooth flat ripples of 0.001 m) with relatively small concentrations. It is most likely that the ripple-generated eddies, which are most effective in the entrainment of particles from the bed, are gradually disappearing for $\Psi = 150$ resulting in a gradual decrease of the concentrations, as shown in Figure 8.3.8.

Figures 8.3.9 and 8.3.10 show time and bed-averaged concentrations measured by Bosman (1982) and by Nap-Van Kampen (1988) using a pump sampler in a wave flume with a water depth of 0.5 m and a sand bed of $d_{50} \approx 100$ $\mu$m. Irregular waves were generated. Occasional (spilling) breaking waves were observed during the tests of Bosman. The bed was covered with ripples for wave heights $H_s \leq 0.18$ m. Bosman did not report the ripple heights.
The following phenomena can be observed:

- increasing concentrations for increasing wave heights,
- slightly flatter concentration profiles for increasing wave heights,
- constant concentration gradients at each wave height (for \( h_s \leq 0.18 \) m),
- two layer concentration profiles for \( h_s = 0.2 \) m and \( 0.23 \) m with constant concentration gradient in near-bed region \( (z/h < 0.15) \) and increasing concentration gradients for \( z/h > 0.15 \); finer size fractions in the upper layers,
- decreasing ripple height for increasing wave height (upto \( h_s = 0.18 \) m).
These results show slightly flatter concentration profiles for a decreasing ripple height, which is related to a somewhat less intensive mixing in the near-bed layer. The concentrations in the region \( z/h > 0.15 \) are relatively large for wave heights of \( H_s = 0.2 \) and \( 0.23 \) m in the experiments of Bosman. This may be caused by the effect of occasional breaking of the waves.

2. Concentrations by breaking waves over a flat bed

The effect of breaking waves can be clearly observed in Figure 8.3.9, presenting the experimental results of Bosman (1982) measured (pump samplers) in a wave flume at a water depth of 0.3 m and a sand bed of \( d_{50} = 100 \) \( \mu m \). Irregular waves were generated. According to Bosman (1982), the bed was flat at locations near breaking waves.

Two phenomena can be observed:
- the near-bed concentrations are approximately constant (\( \approx 10 \) kg/m\(^3\)) for increasing wave heights,
- the concentrations at higher levels show a large increase for wave heights increasing from \( H_s = 0.12 \) m (non-breaking waves) to \( H_s = 0.19 \) m (plunging breaking waves).

Figure 8.3.11 shows time-averaged concentrations measured by Kana (1979) in surf zones near USA-coasts. Spilling and plunging breaking waves did occur. The bed material was \( d_{50} \approx 200 \) \( \mu m \). The types of bed forms were not reported.

![Graph showing concentration profiles](image)

**Figure 8.3.10** Time-averaged concentrations in (non)breaking waves in a flume with rippled bed
Figure 8.3.11 Time-averaged concentrations in breaking waves, USA-coast, Kana (1979)

Figure 8.3.12 Time-averaged concentrations in breaking waves in large-scale flume with a flat bed, Steetzel (1987)

The following phenomena can be observed:
- concentration profiles have a similar shape in spilling and plunging breaking waves,
- concentrations are a factor 5 larger in case of plunging breaking waves.

Figure 8.3.12 shows concentration profiles in spilling and plunging breaking waves measured by Steetzel (1987) using a pump sampler in a large flume with a water depth of 0.9 m and a sand bed of 210 μm. Irregular waves were generated. The bed was flat. The concentration profiles are similar to those of Kana (1979), both in shape and magnitude.

Figure 8.3.13 shows concentration profiles measured by Kroon using a pump sampler in the surf zone and in the swash zone near the Dutch coast of Egmond (Van Rijn and Kroon, 1992). The bed consisting of sand (d₅₀ ≈ 300 μm) was covered with small-scale undulations (≈ 0.01 m). Spilling and plunging breaking waves did occur. Weak currents were present.
The following phenomena can be observed:
- a clear increase of the concentrations with relative wave height \((H_s/h)\),
- near-bed concentrations in the range of 0.2 to 1.0 kg/m\(^3\) for relative wave height of 0.4 to 0.6 (spilling breakers),
- near-bed concentrations in the range of 1 to 10 kg/m\(^3\) for relative wave height in the range of 0.6 to 1.1 (plunging breakers in swash zone),
- more uniform concentration profiles for increasing relative wave heights.

Similar results were observed by Antsyferov and Kosyan (1990), who measured concentration profiles in the surf zone by using sediment traps (see Figure 8.1.1).

![Graph showing concentration profiles](image)

**Figure 8.3.13** Time-averaged concentrations in breaking waves, Dutch coast near Egmond, *Van Rijn and Kroon (1992)*

Figure 8.3.14 shows concentrations at \(z = 0.01\) m derived from concentration profiles measured in the inner surf zone (water depths of 0.5 to 1.5 m) of the Dutch coast near Egmond (Van Rijn and Kroon, 1992). The scatter is relatively large because the accuracy of the vertical position of the measurement points close to the bed (large concentration gradients) is not very high. The field data show reasonable agreement with the tunnel data of Ribberink and Al-Salem (1991, 1992), see Figure 8.3.14. Computed results based on the methods of Bijkjer (1971) and Van Rijn (1989) are also shown, see Section 8.4.6. The method of Bijkjer yields an almost constant concentration.

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8.21
The ratio of the concentrations measured at \( z = 0.01 \) m and \( z = 0.1 \) m above the bed was found to decrease from about 12 in non-breaking waves to about 2 for strong plunging breaking waves.

Similar results were observed by Antsyferov and Kosyan (1990), who measured concentration profiles in the surfzone using sediment traps (Fig. 8.1.1).

![Graph](image)

**Figure 8.3.14 Time-averaged concentrations at 0.01 m above the bed**

Based on the available measurements, the mean concentration ranges for the surf zone with bed material in the range of 200 to 300 \( \mu \)m and water depths of 1 to 2 m are given in Table 8.1

<table>
<thead>
<tr>
<th>Sediment concentrations</th>
<th>Spilling breakers</th>
<th>Plunging breakers</th>
</tr>
</thead>
<tbody>
<tr>
<td>near-bed concentrations (( z \approx 0.05 ) m)</td>
<td>0.2 - 1.0 ( \text{kg/m}^3 )</td>
<td>1 - 10 ( \text{kg/m}^3 )</td>
</tr>
<tr>
<td>near surface concentrations</td>
<td>0.02 - 0.2 ( \text{kg/m}^3 )</td>
<td>0.5 - 3 ( \text{kg/m}^3 )</td>
</tr>
</tbody>
</table>

**Table 8.1 Sediment concentrations in breaking waves**

The data indicate that the breaker type is a dominating factor in wave-related sediment suspensions. Spilling breakers are less effective than plunging breakers, which is probably caused by the relatively small-scale eddies generated by spilling breakers. Furthermore, these small eddies are confined to the near-water surface region and do not extend below the trough level (Miller, 1976). Plunging breakers show strong jets penetrating to the seabed resulting in the generation of large sediment concentrations.
3. Concentrations in sheet flow layer

Observations have shown that bed forms are washed out when the mobility parameter $\psi$ is larger than about 200 to 250. In that case a thin (\(\approx 0.02\) m) layer of moving sediment particles with high concentrations close to the bed is generated. This is called the sheet flow layer. Time-averaged concentrations in the sheet flow layer have only been measured in wave tunnel experiments (Horikawa et al., 1982; Staub et al., 1984, Ribberink and Al-Salem, 1991, 1992). Field data are not available. Horikawa et al. (1982) used an electro-resistance probe to measure the concentrations over a sand bed of 200 $\mu$m. Staub et al. (1984) and Ribberink and Al-Salem (1991, 1992) used a pump sampling instrument. A regular sinusoidal oscillatory motion was generated by Horikawa et al. and by Staub et al., whereas irregular asymmetric oscillatory flow (Jonswap spectrum) was generated by Ribberink and Al-Salem.

Figure 8.3.15 shows concentration profiles measured by Horikawa et al. (1982) and by Staub et al. (1984) in sinusoidal oscillatory flow.

Figures 8.3.16, 8.3.17 and 8.3.18 show concentration profiles measured by Ribberink and Al-Salem (1991, 1992) in regular and irregular waves. Near-bed concentrations are given in Table 8.2 and are shown in Figures 8.3.15 and 8.3.19.

<table>
<thead>
<tr>
<th>Test(^1)</th>
<th>$\dot{u}_{\text{sig}}$ (m/s)</th>
<th>$T_p$ (s)</th>
<th>$\Delta_{\text{f}}$ (m)</th>
<th>$\Delta_{\text{r}}$ (m)</th>
<th>Concentration at $z = 0.01$ m (kg/m$^3$)</th>
<th>Concentration at $z = 0.1$ m (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>crest</td>
<td>trough</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td>1.2</td>
<td>0.73</td>
<td>6.5</td>
<td>plane</td>
<td>4</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.52</td>
<td>6.5</td>
<td>0.0035</td>
<td>0.087</td>
<td>0.7</td>
</tr>
<tr>
<td>3</td>
<td>1.08</td>
<td>0.7</td>
<td>6.5</td>
<td>plane</td>
<td>1.3</td>
<td>0.010</td>
</tr>
<tr>
<td>4</td>
<td>1.17</td>
<td>0.73</td>
<td>9.1</td>
<td>plane</td>
<td>4.5</td>
<td>0.030</td>
</tr>
<tr>
<td>5</td>
<td>0.79</td>
<td>0.49</td>
<td>9.1</td>
<td>0.003</td>
<td>0.084</td>
<td>0.7</td>
</tr>
<tr>
<td>6</td>
<td>1.11</td>
<td>0.7</td>
<td>9.1</td>
<td>plane</td>
<td>2.0</td>
<td>0.02</td>
</tr>
<tr>
<td>7R</td>
<td>0.95</td>
<td>0.5</td>
<td>6.5</td>
<td>plane</td>
<td>3.0</td>
<td>0.03</td>
</tr>
<tr>
<td>8R</td>
<td>1.31</td>
<td>0.7</td>
<td>6.5</td>
<td>plane</td>
<td>4.5</td>
<td>0.04</td>
</tr>
<tr>
<td>9R</td>
<td>1.72</td>
<td>0.86</td>
<td>6.5</td>
<td>plane</td>
<td>20</td>
<td>0.10</td>
</tr>
<tr>
<td>10R</td>
<td>0.96</td>
<td>0.53</td>
<td>9.1</td>
<td>plane</td>
<td>3</td>
<td>0.03</td>
</tr>
<tr>
<td>11R</td>
<td>1.25</td>
<td>0.69</td>
<td>9.1</td>
<td>plane</td>
<td>6</td>
<td>0.04</td>
</tr>
<tr>
<td>12R</td>
<td>1.75</td>
<td>0.99</td>
<td>9.1</td>
<td>plane</td>
<td>35</td>
<td>0.50</td>
</tr>
<tr>
<td>13R</td>
<td>1.20</td>
<td>0.90</td>
<td>6.5</td>
<td>plane</td>
<td>3.0</td>
<td>0.02</td>
</tr>
<tr>
<td>14R</td>
<td>1.19</td>
<td>0.92</td>
<td>9.1</td>
<td>plane</td>
<td>4.5</td>
<td>0.05</td>
</tr>
<tr>
<td>15R</td>
<td>0.98</td>
<td>0.54</td>
<td>5.0</td>
<td>plane</td>
<td>2.0</td>
<td>0.02</td>
</tr>
<tr>
<td>16R</td>
<td>1.03</td>
<td>0.63</td>
<td>12.0</td>
<td>plane</td>
<td>2.0</td>
<td>0.02</td>
</tr>
<tr>
<td>17</td>
<td>0.44</td>
<td>0.28</td>
<td>6.5</td>
<td>0.0143</td>
<td>0.11</td>
<td>4.0</td>
</tr>
<tr>
<td>18</td>
<td>0.64</td>
<td>0.37</td>
<td>6.5</td>
<td>0.009</td>
<td>0.09</td>
<td>2.0</td>
</tr>
<tr>
<td>19</td>
<td>0.66</td>
<td>0.39</td>
<td>5.0</td>
<td>0.0112</td>
<td>0.0112</td>
<td>4.0</td>
</tr>
<tr>
<td>20</td>
<td>0.60</td>
<td>0.38</td>
<td>9.1</td>
<td>0.0071</td>
<td>0.0071</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\(^1\) R = Regular

**Table 8.2** Measured concentrations near sand bed ($d_{50} = 210$ $\mu$m, $d_{90} = 290$ $\mu$m) for regular and irregular asymmetric waves, Ribberink and Al-Salem (1991, 1992)
Figure 8.3.15 Time-averaged concentrations in sheet flow layer. A. Horikawa et al. (1982), B. Staub et al. (1984)

Figure 8.3.16 Time-averaged concentrations - influence of wave irregularity, Ribberink and Al-Salem (1992)

Figure 8.3.17 Time-averaged concentrations - influence of velocity, Ribberink and Al-Salem (1992)
**Figure 8.3.18** Time-averaged concentrations - influence of wave period, Ribberink and Al-Salem (1992)

**Figure 8.3.19** Measured concentrations at 0.01 m and 0.1 m above the bed, Ribberink and Al-Salem (1992)
When regular waves are generated, the concentration profile is more uniform, but the near-bed concentrations are about the same as those for irregular waves (see Figure 8.3.16).

The sediment concentrations increase with increasing velocities \((u_{rms})\) in the plane bed regime; the shape of the concentrations is almost constant. In the ripple regime the concentrations decrease with increasing velocities because the ripples are gradually washed out (see Figure 8.3.17). According to Ribberink and Al-Salem (1992), the relative concentrations in the plane bed regime can be represented as: \(c/c_s = (a/z)^2\), with \(c_s\) = reference concentration at \(z = a = 0.01\) m above the bed. The concentrations decrease by a factor 100 over a height of 0.1 m near the bed.

The sediment concentrations decrease with increasing wave period in the ripple regime; a clear influence of the wave period in the plane bed regime cannot be detected (see Figure 8.3.18).

The concentrations at \(z = 0.01\) m and 0.1 m above the bed as measured by Ribberink and Al-Salem are shown in Figure 8.3.19. The concentrations at \(z = 0.01\) m are close for low peak velocities when the ripple height and steepness (and hence the vortex motions) are maximum. For increasing peak velocities the ripples are gradually washed out (flattened) and the concentration decreases to a minimum value at the beginning of the sheet flow regime. After that a strong increase of the concentration can be observed for increasing peak velocities. The concentration at \(z = 0.1\) m above the bed shows a continuous increasing trend.

A clear influence of the wave period cannot be detected.

Sheet flow tests with calcareous sediments (coral sand) were executed at Delft Hydraulics (1992). The near-bed concentrations were found to be slightly higher (10% to 30%) compared to quartz sand under similar conditions.

Based on all available data, the mean concentration ranges at levels of 0.01, 0.02 and 0.1 m above the initial bed (measured before the tests) are given in Table 8.3.

<table>
<thead>
<tr>
<th>Height above the bed (m)</th>
<th>Horikawa et al., 1982</th>
<th>Staub et al., 1984</th>
<th>Ribberink and Al-Salem</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>200 μm ( \bar{u} = 1.08) m/s (T = 4) s</td>
<td>190 μm ( \bar{u} = 1.3) m/s (T = 9.1) s</td>
<td>380 μm ( \bar{u} = 1.9) m/s (T = 6.8) s</td>
</tr>
<tr>
<td>0.10</td>
<td>( \bar{u}_{1/2} = 0.8) - 1.7 m/s (T_p = 5-12) s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 μm ( \bar{u} = 1.68) m/s (T = 6.8) s</td>
<td>10 - 25</td>
<td>50</td>
<td>10 - 30</td>
</tr>
<tr>
<td>200 μm ( \bar{u} = 1.9) m/s (T = 6.8) s</td>
<td>5 - 10</td>
<td>20</td>
<td>5 - 15</td>
</tr>
<tr>
<td>200 μm ( \bar{u} = 2.1) m/s (T = 6.8) s</td>
<td>1 - 10</td>
<td>20</td>
<td>0.5 - 20</td>
</tr>
<tr>
<td>200 μm ( \bar{u} = 2.1) m/s (T = 6.8) s</td>
<td>0.005 - 0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8.3 Sediment concentrations in sheet flow regime

4. Summary

Summarizing all available concentration data for non-breaking and breaking waves in water depths of 0.5 to 2 m and bed material sizes from 100 to 300 μm, the following characteristics can be observed:

- Two-layer concentration profile with rather large concentration gradients in the near-bed layer (≈ 3 ripple heights) and smaller concentration gradients in the upper layer, in case of non-breaking waves over a rippled bed,
• strong influence of the ripples giving relatively large concentrations in the near-bed layer due to eddy-generated motions,
• increasing concentrations in the upper layers by spilling breaking waves,
• strongly increasing concentrations in the near-bed layer and in the upper layer by plunging breaking waves (factor 5 to 10 larger than in spilling breaking waves).

8.3.3 Sediment load and transport rate

1. Experimental data related to high-frequency wave motion

Sediment transport rates under short waves (high frequency) have been investigated by Abou-Seida (1965), Kalkanis (1964), Vincent (1957), Manohar (1955), Sleath (1978), Sato Horikawa (1986), Sawamoto Yamashita (1987), Ribberink and Al-Salem (1991, 1992), Havinga (1992) and others. Most of the experiments of Abou-Seida, Kalkanis, Vincent and Sleath were carried out with flat (oscillating) beds. Transport rates were generally small. Manohar performed his experiments in a still water tank by oscillating the bed. The sand particles were trapped by trays installed in the bed. Manohar measured transport rates over flat and rippled beds using sand material of 280 μm and 1000 μm. An asymmetric type of motion was generated. Although ripples were present in some experiments, the transport mode was dominantly bed load with rolling grains, as described by Manohar: 'particles rolled to the crest and over the crest from the adjacent trough'. Suspension was only observed after the disappearance of the ripples. In all cases Manohar observed a net transport rate in the direction of the largest peak velocity.

Sato and Horikawa performed experiments in a wave tunnel with sand material of 180 μm in the ripple regime. Asymmetric regular oscillatory flows were generated. Sand traps were installed at both ends of the test section. The net sand transport rates were determined from bed profile recordings in (many) longitudinal sections. Measurements were carried out after steady state asymmetric ripples had been developed along the sand bed. Sato and Horikawa report that suspended sand clouds are formed above the steeper flank of a ripple when the flow direction is forward. The cloud is then thrown up over the ripple crest when the flow direction is changed and the sand is transported backward. In nearly all tests the net transport rate was in the backward direction, while the largest peak velocity was in the forward direction. Havinga (1992) found transport rates in forward (onshore) direction for irregular (asymmetric) wave motion over relatively flat ripples.

Sheet flow experiments in sinusoidal oscillatory flow have been carried out by Horikawa et al. (1982) in a wave tunnel with sand of 200 μm. The time-averaged (half wave period values) transport rates are reported in Table 8.4.

<table>
<thead>
<tr>
<th>Peak orbital velocity $U_0$ (m/s)</th>
<th>Wave period T (s)</th>
<th>Transport rate in half period $q_{w, \text{half}}$ (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.27</td>
<td>3.6</td>
<td>416 $10^6$</td>
</tr>
<tr>
<td>1.08</td>
<td>2.6</td>
<td>261 $10^6$</td>
</tr>
<tr>
<td>1.08</td>
<td>3.4</td>
<td>208 $10^6$</td>
</tr>
<tr>
<td>0.87</td>
<td>5.4</td>
<td>182 $10^6$</td>
</tr>
<tr>
<td>0.76</td>
<td>6.0</td>
<td>104 $10^6$</td>
</tr>
</tbody>
</table>

Table 8.4 Half period transport rates for sinusoidal waves, $d_{50} = 200 \mu m$, Horikawa et al. (1982)
Sawamoto and Yamashita (1987) performed similar tests with sediment sizes of 200, 700 and 1800 \( \mu \)m.

Ribberink and Al-Salem (1992) measured time-averaged concentrations and net transport rates in the ripple and sheet flow regime in a wave tunnel with a sand bed \((d_{10} = 150 \mu \text{m}, d_{50} = 210 \mu \text{m}, d_{90} = 320 \mu \text{m})\). The experimental procedure was similar to that of Sato and Horikawa (1986). Irregular and regular asymmetric oscillatory flow (Jonsnapspectrum, 2nd order Stokes) was generated in the tunnel. A weak mean current of about 0.05 m/s against the wave propagation direction was observed in the near-bed layer \((z < 0.05 \text{ m})\) under asymmetric wave motion. The measured net transport rates were however in the direction of the largest peak velocity (see Table 8.5 and 8.6).

Ribberink and Al-Salem found that the net transport rate is related to the third power of velocity. The net transport is relatively high for wave periods \(T \geq 9.1 \text{ s}\) and relatively low (50\% lower) for \(T \leq 6.5 \text{ s}\). They also found that 90\% of the sediment transport took place in a layer with a thickness of 0.01 m adjacent to the bed (see Fig. 8.3.20). Above this layer there was a small net suspended load transport in offshore (backward) direction, which was caused by phase differences between the peak velocities and peak concentrations. This latter effect will become increasingly important for finer sediments \((d_{50} < 150 \mu \text{m})\) and smaller periods \((T < 5 \text{ s})\) because the suspended load will increase.

Ribberink and Chen (1993) did similar tests with fine sediment material \((d_{50} = 130 \mu \text{m})\) and found net transport rates against the wave propagation direction (backward direction).

![Figure 8.3.20](Image)

**Figure 8.3.20** Vertical distribution of net transport rate, \(d_{50} = 200 \mu \text{m}\. Asymmetric wave motion

The most important conclusions which can be drawn from the results of Ribberink and Al-Salem (1991, 1992) are:

1. The vertical distribution of the transport rate under asymmetric wave motion over a 200 \( \mu \text{m}\)-bed shows a dominant onshore transport in the lowest layer \((z < 0.01 \text{ m})\) and offshore transport in higher layers \((z \geq 0.01 \text{ m})\), see Fig. 8.3.20.

8.28
2. The net depth-integrated transport rate under asymmetric wave motion over a 200 \( \mu m \) bed is onshore.

3. The net transport rate increases weakly with increasing wave period.

4. Simple transport formulae (transport proportional to power of velocity) give good results for \( d_{50} = 200 \mu m \).

Chen (1992) used harmonic component analysis to study the wave-related transport process. The instantaneous velocity and concentration are defined as:

\[
U = u + u' + \hat{U}_1 \cos(\omega t - \alpha_1) + \hat{U}_2 \cos(2\omega t - \alpha_2) + \ldots \quad (8.3.1)
\]

\[
C = c + c' + \hat{C}_1 \cos(\omega t - \beta_1) + \hat{C}_2 \cos(2\omega t - \beta_2) + \ldots \quad (8.3.2)
\]

The net sediment flux at height \( z \) above the bed can now be described as:

\[
\overline{UC} = uc + uc'/c' + \frac{1}{2} \hat{U}_1 \hat{C}_1 \cos(\beta_1 - \alpha_1) + \frac{1}{2} \hat{U}_2 \hat{C}_2 \cos(\beta_2 - \alpha_2) + \ldots \quad (8.3.3)
\]

in which:
- \( \hat{U}_n \) = amplitude of harmonic velocity component,
- \( \hat{C}_n \) = amplitude of harmonic concentration component,
- \( u \) = mean velocity,
- \( c \) = mean concentration,
- \( u' \) = random velocity fluctuation,
- \( c' \) = random concentration fluctuation,
- \( \alpha_n \) = phase angle,
- \( \beta_n \) = phase angle.

The uc-term represents the flux component due to the mean current. The other terms represent the flux component due to the oscillatory flow. The harmonic-random interaction terms have been neglected. The depth-integrated values can be obtained by integration over the depth.

<table>
<thead>
<tr>
<th>Test</th>
<th>RMS-velocity ( U_{rms} ) (m/s)</th>
<th>Peak period ( T_p ) (s)</th>
<th>Peak velocities ( \hat{U}_{ik} ) (m/s)</th>
<th>Bed forms</th>
<th>Net transport rate ( q_{w, net} ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.48</td>
<td>6.5</td>
<td>1.20 0.73</td>
<td>plane</td>
<td>12.30 ( 10^4 )</td>
</tr>
<tr>
<td>B2</td>
<td>0.32</td>
<td>6.5</td>
<td>0.80 0.52</td>
<td>plane</td>
<td>3.30 ( 10^4 )</td>
</tr>
<tr>
<td>B3</td>
<td>0.43</td>
<td>6.5</td>
<td>1.08 0.70</td>
<td>plane</td>
<td>8.10 ( 10^4 )</td>
</tr>
<tr>
<td>B4</td>
<td>0.48</td>
<td>9.1</td>
<td>1.17 0.73</td>
<td>plane</td>
<td>14.60 ( 10^4 )</td>
</tr>
<tr>
<td>B5</td>
<td>0.33</td>
<td>9.1</td>
<td>0.79 0.49</td>
<td>plane</td>
<td>4.10 ( 10^4 )</td>
</tr>
<tr>
<td>B6</td>
<td>0.44</td>
<td>9.1</td>
<td>1.11 0.70</td>
<td>plane</td>
<td>12.00 ( 10^4 )</td>
</tr>
<tr>
<td>B17</td>
<td>0.20</td>
<td>6.5</td>
<td>0.44 0.28</td>
<td>plane</td>
<td>0.26 ( 10^4 )</td>
</tr>
<tr>
<td>B18</td>
<td>0.25</td>
<td>6.5</td>
<td>0.64 0.37</td>
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<td>2.00 ( 10^4 )</td>
</tr>
<tr>
<td>B19</td>
<td>0.25</td>
<td>5.0</td>
<td>0.66 0.39</td>
<td>plane</td>
<td>0.38 ( 10^4 )</td>
</tr>
<tr>
<td>B20</td>
<td>0.25</td>
<td>9.1</td>
<td>0.60 0.48</td>
<td>plane</td>
<td>2.80 ( 10^4 )</td>
</tr>
</tbody>
</table>

Table 8.5 Net transport rates for irregular asymmetric wave motion, \( d_{50} = 210 \mu m \), Ribberink and Al-Salem (1992)
<table>
<thead>
<tr>
<th>Test</th>
<th>RMS-velocity $U_{ms}$ (m/s)</th>
<th>Peak period $T_p$ (s)</th>
<th>Peak velocities $\bar{U}_{stk}$ (m/s)</th>
<th>Bed forms</th>
<th>Net transport rate $q_{m, net}$ (m$^2$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B7</td>
<td>0.46</td>
<td>6.5</td>
<td>0.95</td>
<td>0.50</td>
<td>plane</td>
</tr>
<tr>
<td>B8</td>
<td>0.70</td>
<td>6.5</td>
<td>1.31</td>
<td>0.70</td>
<td>plane</td>
</tr>
<tr>
<td>B9</td>
<td>0.92</td>
<td>6.5</td>
<td>1.72</td>
<td>0.86</td>
<td>plane</td>
</tr>
<tr>
<td>B10</td>
<td>0.54</td>
<td>9.1</td>
<td>0.96</td>
<td>0.53</td>
<td>plane</td>
</tr>
<tr>
<td>B11</td>
<td>0.70</td>
<td>9.1</td>
<td>1.25</td>
<td>0.69</td>
<td>plane</td>
</tr>
<tr>
<td>B12</td>
<td>0.97</td>
<td>9.1</td>
<td>1.75</td>
<td>0.99</td>
<td>plane</td>
</tr>
<tr>
<td>B13</td>
<td>0.70</td>
<td>6.5</td>
<td>1.20</td>
<td>0.90</td>
<td>plane</td>
</tr>
<tr>
<td>B14</td>
<td>0.71</td>
<td>9.1</td>
<td>1.19</td>
<td>0.92</td>
<td>plane</td>
</tr>
<tr>
<td>B15</td>
<td>0.51</td>
<td>5.0</td>
<td>0.98</td>
<td>0.54</td>
<td>plane</td>
</tr>
<tr>
<td>B16</td>
<td>0.56</td>
<td>12.0</td>
<td>1.03</td>
<td>0.63</td>
<td>plane</td>
</tr>
</tbody>
</table>

Table 8.6 Net transport rates for regular asymmetric wave motion, $d_{50} = 210$ μm, Ribberink and Al-Salem (1992)

Chen measured instantaneous velocities (regular waves with a weak current) and concentrations above a rippled sand bed in a flume. Analysis of the data shows that the contributions of the harmonic components with a frequency higher than 2ω together with the contribution of the random component are less than 5% of the other contributions. The first order harmonic flux component near the bed was found to be relatively small because the phase angle difference $\beta_1 - \alpha_1$ was about 90°. The second order harmonic flux component near the bed was significant because $\beta_2 - \alpha_2$ was about 180°.

The current-related flux term $(u_c)$ was found to be of essential importance even for small values of the ratio $u_r/\bar{U}_1$ with $u_r = \text{mean velocity near the bed}$ and $\bar{U}_1 = \text{peak orbital velocity of first harmonic component near the bed}$. The depth-integrated current-related flux term was about 80% of the total flux for $u_r/\bar{U}_1 = 0.2$ (Test D) and about 55% for $u_r/\bar{U}_1 = 0.1$ (Test E).

2. Experimental data related to low-frequency wave motion

Sediment transport related to low-frequency oscillations (bound-long waves) was studied by Havinga (1992) in a laboratory wave-current basin. He measured instantaneous longshore and cross-shore velocities and instantaneous concentrations at various elevations above the bed under irregular non-breaking waves combined with a longshore current, see also Figs. 8.2.5 and 8.2.6. In most tests the waves were normal to the shore ($\phi = 90°$). Test with $\phi = 60°$ and $\phi = 120°$ were also carried out. The experimental data are presented in Table 8.7.

The dimensionless transport rate is shown as a function of $\Psi_{on}$ in Figure 8.3.21. The low-frequency transport increases with the peak orbital velocity and the mean longshore current. The low-frequency transport component was found to be related to bound-long waves.

Van de Meene (1992) measured instantaneous velocities and concentrations in shoaling waves combined with tidal currents on (shoreface connected) ridges in the North Sea (water depths 14 to 18 m). The near-bed velocity recordings clearly showed the presence of groups of low and high waves. The low-frequency velocities ($T > 20$ s) were as large as 0.1 m/s, but they were not correlated to the wave groups as may be expected for bound-long waves. Probably, the free long waves were dominant. The low-frequency sediment flux near the bed was found to be negligible.
Kroon and Van Rijn (1993) analyzed instantaneous velocities and concentrations measured near the bed in the swash and surf zone of the Dutch coast near Egmond, see also Fig. 8.2.9. The low-frequency velocity components in cross-shore direction were as large as 0.5 m/s, but they were not correlated to the wave groups. The cross-shore low-frequency sediment flux component near the bed was found to be negligible.

<table>
<thead>
<tr>
<th>Test</th>
<th>Sign. wave height</th>
<th>Peak orbital velocities near bed</th>
<th>Mean longshore current velocity</th>
<th>Depth-integrated low-frequency cross-shore transport rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_s$ (m)</td>
<td>$U_{oo}$ (m/s)</td>
<td>$U_{oe}$ (m/s)</td>
<td>$u$ (m/s)</td>
</tr>
<tr>
<td>T</td>
<td>7.10.90</td>
<td>0.068</td>
<td>0.153</td>
<td>0.133</td>
</tr>
<tr>
<td>T</td>
<td>7.20.90</td>
<td>0.076</td>
<td>0.155</td>
<td>0.136</td>
</tr>
<tr>
<td>T</td>
<td>7.30.90</td>
<td>0.076</td>
<td>0.181</td>
<td>0.168</td>
</tr>
<tr>
<td>T</td>
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</table>

Table 8.7 High-frequency and low-frequency depth-integrated transport rates in cross-shore direction (- offshore), $d_{50} = 100 \mu m$, Havinga (1992)

![Figure 8.3.21 Low frequency transport](image)

3. Analysis of transport processes

Herein, the sediment loads and the transport rates are discussed. The time-averaged sediment load ($L$ in kg/m²) is defined as the integration over the depth of the time averaged concentrations. An exponential curve was used to estimate the load between the lowest measuring point and the bed.
Figure 8.3.22 shows the Load $L_t$ as a function of the peak forward orbital velocity based on the tunnel data of Ribberink and Al-Salem (1991, 1992) and the Egmond surf zone data of Kroon and Van Rijn (1993).

The plotted data can be represented by:

$$\frac{L_t}{\rho_s d_{50}} = 0.0006 (\psi_{\text{crest}} \psi_{cr})^{1.3}$$  \hfill (8.3.4)

in which:

- $L_t$ = sediment load (kg/m$^2$)
- $\rho_s$ = sediment density (kg/m$^3$)
- $d_{50}$ = median particle diameter of bed material (m)
- $\bar{U}_{\text{sig,crest}}$ = near-bed significant peak orbital velocity under wave crest (m/s)
- $\bar{U}_{cr}$ = near-bed peak orbital velocity at initiation of motion (m/s)
- $s = \rho_s/\rho$ = relative density (-)

$$\psi_{\text{crest}} = \frac{(\bar{U}_{\text{sig,crest}})^2}{(s-1) g d_{50}}$$ = mobility parameter (-)

$$\psi_{cr} = \frac{(\bar{U}_{cr})^2}{(s-1) g d_{50}}$$ = critical mobility parameter (-)

A sediment load of 1.6 kg/m$^2$ corresponds to a moving sediment layer with a thickness of about 1 mm, which will occur at peak velocitics larger than about 1.2 m/s. The maximum (moving) layer thickness observed by Ribberink and Al-Salem was about 5 mm ($L_t = 8$ kg/m$^2$) at a peak velocity of 1.7 m/s.

Figure 8.3.22 Time-averaged sediment load
The net transport rate (magnitude and direction) under short waves is strongly related to the type of bed forms generated. In regular asymmetric wave motion (swell waves) relatively steep ripples with $\Delta_r/\lambda_r$ in the range of 0.2 to 0.1 are generated, whereas in irregular asymmetric wave motion (wind waves) relatively flat ripples with $\Delta_r/\lambda_r$ in the range of 0.1 to 0.01 or plane-bed sheetflow conditions are generated. The net transport rate and direction strongly depends on the flow structure (vortices or no vortices) near the ripples, as shown schematically in Figure 8.3.23.

**Figure 8.3.23 Schematic presentation of transport processes in ripple regime**

The underlying mechanism of wave-related transport against the wave propagation direction is the existence of phase differences between the peak concentrations and peak velocities which are related to the generation of vortices on the lee-sides of steep ripples. When the wave crest passes (forward motion) a steep ripple, a strong lee-side vortex is generated (see Figure 8.2.3), which erodes a relatively large amount of sand (high concentration in vortex). When the wave trough passes (backward motion), the high-concentration eddy is lifted and carried against the wave direction. During this movement, it will partly disintegrate and most of the sand particles will settle to the bed. Simultaneously, another less strong lee-side vortex ($\bar{U}_{\text{trough}} < \bar{U}_{\text{crest}}$) with a lower sand concentration is generated.

Thus, near the bed there is a two-phase process of relatively high concentration vortices transported backward by relatively small velocities and relatively low concentration vortices transported forward by relatively high velocities resulting in a net transport against the wave direction (Nielsen, 1988; Sato and Horikawa, 1986). A similar process with phase differences between the peak velocities and the peak concentrations may occur in sheet flow transport over a very fine sand bed (50 to 150 $\mu$m), when the suspended load transport is dominant.

The experimental results of Sato and Horikawa (1986) for regular asymmetric wave motion over a sand bed of 180 $\mu$m and those of Ribberink and Al-Salem (1991, 1992) for irregular and regular asymmetric wave motion over a sand bed of 210 $\mu$m are summarized in Fig. 8.3.24.

Sato and Horikawa observed an increasing net transport rate against the wave propagation direction up to $\bar{U}_{\text{crest}} = 0.6$ m/s, when steep ripples were present. The net transport rates decreased considerably when the ripples were gradually washed out (flattened) for $\bar{U}_{\text{crest}} > 0.6$ m/s, preventing the generation of lee-side vortices.

Ribberink and Al-Salem observed a net transport rate in the wave direction increasing with the significant peak orbital velocity under the wave crest $\bar{U}_{\text{sig, crest}}$ for irregular asymmetric waves. For regular asymmetric waves under sheet flow conditions they measured net transport rates which were a factor 2 larger than those for irregular waves (see Fig. 8.3.24). The net transport for regular wave motion under sheet flow conditions was in the wave propagation direction (largest peak velocity).
Comparing the data of Sato-Horikawa and Ribberink-Al Salem for regular waves, it seems that net transport direction is changed for $\hat{U}_{\text{crest}}$ of approximately 0.7 to 0.8 m/s when the ripples are fully washed out (flattened).

The experimental results of Sato and Horikawa (1986) for regular asymmetric wave motion and those of Ribberink and Al-Salem (1991, 1992) can be represented by simple formulae, as presented in Section 8.5.3.

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**Figure 8.3.24** Measured net transport rates in regular and irregular asymmetric wave motion over a sand bed of 200 $\mu$m
8.4 Computation of instantaneous and time-averaged concentration profiles

8.4.1 Introduction

In the Literature various models are proposed to compute the sediment concentration profiles. Most models are based on the time-averaged convection-diffusion equation (Nielsen, 1979; Bosman-Stetzel, 1986; Dally, 1980; Skafel-Krishnappan, 1984), as follows:

\[ w_{s,m} c + \varepsilon_{s,w} \frac{dc}{dz} = 0 \]  

(8.4.1)

in which:
- \( c \) = sediment concentration
- \( w_{s,m} \) = particle fall velocity in fluid-sediment mixture
- \( \varepsilon_{s,w} \) = sediment diffusion or mixing coefficient related to the wave motion
- \( z \) = vertical coordinate

Some models are based on the time-dependent convection-diffusion equation, as follows:

\[ \frac{\partial C}{\partial t} - w_{s,m} \frac{\partial C}{\partial z} - \frac{\partial}{\partial z} \left( \varepsilon_{s,w} \frac{\partial C}{\partial z} \right) = 0 \]  

(8.4.2)

These latter models are only valid for plane (sheet flow) bed conditions and are developed to compute the variation of the instantaneous concentrations in the thin (= 0.02 m) wave boundary layer above the bed. Analytic solutions of the convection-diffusion equation are given by Nielsen (1979) assuming that the mixing or eddy viscosity coefficient is constant in space and time.

Numerical solution requires specification of the fall velocity \( (w_{s,m}) \), the sediment mixing coefficient \( (\varepsilon_{s,w}) \) and boundary conditions at the bed and at the upper boundary. Usually, a separate oscillatory flow model is applied to compute the mixing coefficients and the bed-shear stresses, see also Fig. 2.4.4 and see Section 2.3.4. This latter parameter is then applied to generate the instantaneous bed concentration (boundary condition), see section 8.4.6. Based on this, the concentration profiles can be computed as a function of time and also the time-averaged values by averaging over the wave period. Bakker (1974) applied a mixing length concept to determine the sediment mixing coefficient. The expressions of Einstein were applied to specify the bed concentration. Recently, the theories of Bagnold (1954) related to particle interactions in case of high concentrations were incorporated in the model of Bakker. The influence of the sediment particles on the fall velocity and turbulence was taken into account (Bakker and Van Kesteren, 1986). Fredse et al. (1985) apply a numerical model based on the eddy viscosity concept. The eddy viscosity coefficients are related to the instantaneous bed-shear stresses and the boundary-layer thickness. The bed-boundary condition for the sediment was represented by a concentration function, which specifies the bed concentration at a height of two particle diameters above the bed as a function of the instantaneous bed-shear stress. At the water surface the vertical sediment flux was set at zero. The influence of the sediment particles on the fall velocity and eddy viscosity coefficient was not taken into account. Ribberink and Al-Salem (1991) also presented a numerical model based on the mixing length concept.

Hagatum and Eidsvik (1986) presented a numerical model based on a two-equation turbulence model to represent the eddy viscosity coefficients. It is questionable whether such a
sophisticated improvement is necessary considering the experimental problems related to
the measurement of the concentrations in the sheet flow layer (unreliable verification
data).
Given the complexity of all phenomena (eddy viscosity, hindered settling, turbulence
damping) within this thin (≈ 0.02 m) high-concentration sheet flow layer, the predicting
ability of these time-dependent models is rather questionable. Verification is hardly
possible due to measurement problems in the near-bed region with extremely large vertical
concentration gradients in a thin layer (≈ 0.02 m).

8.4.2 Time-averaged convection-diffusion equation

Usually, the classical convection-diffusion equation is applied to compute the equilibrium
concentration profile in steady flow. This equation reads as:

$$w_{s,m} c + \varepsilon_s \frac{dc}{dz} = 0 \quad (8.4.3)$$

in which:

- $w_{s,m}$ = particle fall velocity of suspended sediment in a fluid-sediment mixture =
  \[(1-c)^2 w_s\]
- $w_s$ = particle fall velocity of suspended sediment in clear fluid
- $\varepsilon_s$ = sediment mixing coefficient
- $c$ = time-averaged concentration at height $z$ above the bed

Here, it is assumed that Equation (8.4.3) is also valid for wave-related mixing. To verify
this, Equation (8.4.3) is expressed as:

$$\frac{d}{dz} (\ln c) = - \frac{w_{s,m}}{\varepsilon_s} \quad (8.4.4)$$

The diffusion concept is valid when the slopes of measured concentration profiles
represented in a plot of log $c$ versus $z$ are inversely proportional to the fall velocity in
the case of constant wave conditions. To investigate this, some experimental results
presented by Roelvink (1985) are analyzed. Roelvink measured concentration profiles of different
size fractions above a rigid bottom consisting of artificial ripples. A limited amount of
sediment particles was fed to the (oscillatory) flow. Figure 8.4.1 shows measured
concentration profiles for various size fractions in case of a constant wave height. Since
the wave height is constant, the mixing coefficients will be approximately constant
assuming that this parameter is not or only weakly correlated to the sediment particle size.
As can be observed, there is a tendency for the slopes of the concentration profiles to be
inversely proportional to the particle fall velocity, which supports the applicability of the
diffusion concept for wave-induced mixing. The sediment mixing coefficient in oscillatory
flow should be interpreted as an apparent mixing coefficient, because diffusive as well as
convective processes are involved. Similar results have been obtained by Van der Graaff
(1988). Nielsen (1984) questions the applicability of the diffusion equation for wave-
induced mixing. He suggests that the process of particles trapped in a vortex and carried
away by the vortex is a convection process rather than a diffusion process. All sand
particle sizes, that are small enough to be trapped in an eddy will travel with the eddy
until it dissolves and their time-averaged concentrations will become distributed in very
much the same way, irrespective of settling velocity. Deigaard (1991) presented a similar
eddy trapping mechanism for sediment suspension in waves and found an apparent
turbulent diffusion coefficient increasing with settling velocity.
Figure 8.4.1 Vertical distribution of concentration in waves according to Roelvink (1985)
8.4.3 Particle size of suspended sediment

Observations in flume and field conditions have shown that the suspended sediment particles are not uniformly distributed over the depth in case of a non-uniform bed material composition. The coarser particles are suspended in the near-bed region, while the finer particles are suspended in the upper layers. Some information can be obtained from the measurements of Van Rijn (1987) in a large-scale wave flume (d_{50} = 210 \mu m, \sigma_s = 1.8). Sediment concentration profiles were determined by using a pump sampler to obtain water-sediment samples. The suspended sediment samples of each profile were put together to get a sufficiently large sample for analysis. Fall velocities of these samples were measured in a settling tube with a length of 2 m. The fall velocities were converted to particle sizes. The ratio of the median suspended sediment size (d_m) and the median bed material size (d_{50}) was found to be in the range of d_m/d_{50} = 0.6 to 1.0 for shoaling and spilling breaking waves.

8.4.4 Sediment mixing coefficient for non-breaking waves

Measurements in wave flumes show the presence of suspended sediment particles from the bed up to the water depth (Bosman, 1982; Van Rijn, 1987). The largest concentrations are found close to the bed where the diffusivity is rather large due to ripple-generated eddies. Further away from the bed the sediment concentrations decrease rapidly because the eddies dissolve rather rapidly travelling upwards.

Various researchers have tried to model the suspension processes by introducing an effective wave-related sediment mixing coefficient. Most expressions are empirical or semi-empirical.

1. Homma and Horikawa (1962)

\[ \varepsilon_{s,w} = \alpha \frac{H^3k}{T} \frac{\sinh^3(kz)}{\sinh^3(kh)} \]  

(8.4.5)

in which:
- H = wave height
- T = wave period
- L = wave length
- h = water depth
- z = vertical coordinate relative to the bed
- \alpha = constant (\approx 10^2)
- k = wave number (2\pi/L)

2. Bijker (1967, 1971)

\[ \varepsilon_{s,w} = \kappa u_{s,w} z \left( 1 - \frac{z}{h} \right) \]  

(8.4.6)

in which:
- \kappa = Von Karmen constant
- u_{s,w} = (\tau_{bw}/\rho)^{0.5} = time-averaged bed-shear velocity related to waves
- \tau_{bw} = 1/4 \rho f_w (\hat{U}_b)^2 = time-averaged bed-shear stress
- f_w = friction factor
Originally, the Bijker method was proposed for combined current and wave conditions. This method can also be applied for waves alone and is not dependent on calibration coefficients.

3. **Lundgren (1972)**

\[
\varepsilon_{s,w} = \frac{0.4 \bar{u}_{*,w} z}{1 + 1.34 (0.5f_w)^{0.25} (z/\delta_w) \exp(z/\delta_w)} \tag{8.4.7}
\]

in which:
- \(\bar{u}_{*,w} = (\tau_{b,w}/\rho)^{0.5}\) = peak value of wave-related bed-shear velocity
- \(f_w\) = friction factor
- \(\delta_w\) = maximum thickness of wave boundary layer \((\delta_{w,min} = \Delta_t)\)

The \(\delta_w\) parameter follows from:

\[
\left(\frac{2.74 \delta_w}{k_{s,w}}\right) \log\left(\frac{2.74 \delta_w}{k_{s,w}}\right) = 0.0282 \left(\frac{\hat{A}_5}{k_{s,w}}\right) \tag{8.4.8}
\]

in which:
- \(k_{s,w}\) = wave-related bed roughness
- \(\hat{A}_5\) = peak-value of orbital excursion at bed

4. **Bhattacharya (1971)**

\[
\varepsilon_{s,w} = \varepsilon_s \left(\frac{z}{a}\right) \tag{8.4.9}
\]

\[
\varepsilon_s = F \left(\frac{ha}{T}, \frac{H}{h}\right)
\]

in which:
- \(\varepsilon_s\) = sediment mixing coefficient at small height \((a)\) above bed

5. **Swart (1976)**

\[
\varepsilon_{s,w} = \left(\frac{w_s}{b_1}\right) z \tag{8.4.10}
\]

\[
b_1 = 1.05 \left(\frac{w_s}{\kappa u_{*,w}}\right)^{0.96} \left(\frac{a}{h}\right)^{0.013} w_s/(\kappa u_{*,w})
\]

in which:
- \(w_s\) = particle fall velocity
- \(\kappa\) = Von Karman constant

\[ \varepsilon_{s,w} = 0.00146 \left( \Delta_r + 0.4 \delta \right) g T \left( \frac{\hat{A}_\delta}{w_s} \right)^{0.32} \quad \text{for} \quad \frac{\hat{A}_\delta}{w_s} \geq 25 \quad (8.4.11) \]

\[ \varepsilon_{s,w} = 0.00035 \left( \Delta_r + 0.4 \delta \right) g T \left( \frac{\hat{A}_\delta}{w_s} \right)^{0.68} \quad \text{for} \quad \frac{\hat{A}_\delta}{w_s} < 25 \]

in which:
\[ \Delta_r = \text{ripple height} \]
\[ \delta = \text{maximum boundary layer thickness} \]
\[ T = \text{wave period} \]
\[ \omega = 2\pi/T \]

Equation (8.4.11) is valid for non-breaking waves.

7. Dally (1980)

\[ \varepsilon_{s,w} = \frac{1}{15} h \hat{u}_{*,w} \quad (8.4.12) \]


\[ \varepsilon_{s,w} = \beta \hat{A}_\delta \hat{u}_{*,w} \quad (8.4.13) \]

\[ \beta = 8.7 \left( \frac{\hat{u}_{*,w} d_{50}}{v} \right)^{-2.2} \]


\[ \varepsilon_{s,w} = \varepsilon_{s,0} + \varepsilon_{s,t} \quad (8.4.14) \]

\[ \varepsilon_{s,0} = \frac{\pi H^2}{2.82 T} \frac{\sinh^2(kz)}{\sinh^2(kh)} \quad (8.4.15) \]

\[ \varepsilon_{s,t} = \frac{b (\hat{U}_\delta - w_s)(z/\delta_w)}{1 + 0.06 (z/\delta_w) \exp(z/\delta_w)} \quad (8.4.16) \]

\[ b = \left( \frac{\rho}{\rho_s - \rho} \right) \left( \frac{v^2}{g} \right)^{1/2} \]

in which:
\[ \varepsilon_{s,0} = \text{orbital-related mixing coefficient} \]
\[ \varepsilon_{s,t} = \text{turbulence-related mixing coefficient} \]
\[ \delta_w = \text{maximum thickness of wave-boundary layer} \]
\[ \hat{U}_\delta = \text{peak value of orbital velocity near bed} \]
The apparent mixing coefficient distribution was computed from measured concentration profiles (Bosman, 1982) using Equation (8.4.3). Figure 8.4.2 shows the computed mixing coefficient distribution for non-breaking waves ($H_s/h = 0.4$ and $0.5$)

![Graphs showing concentration and mixing coefficient for different $H_s/h$ ratios](image)

**Figure 8.4.2** Sediment mixing coefficient in non-breaking waves, *Van Rijn (1989)*

Based on these results, the following relationship was proposed:

\[
\begin{align*}
z \leq \delta_s & \quad \varepsilon_{s,w} = \varepsilon_{s,w,\text{bed}} \\
0.5 \leq z & \quad \varepsilon_{s,w} = \varepsilon_{s,w,\text{max}} \\
\delta_s < z < 0.5h & \quad \varepsilon_{s,w} = \varepsilon_{s,w,\text{bed}} + \left( \varepsilon_{s,w,\text{max}} - \varepsilon_{s,w,\text{bed}} \right) \left( \frac{z - \delta_s}{0.5h} \right)
\end{align*}
\] (8.4.17a, 8.4.17b, 8.4.17c)

Equation (8.4.17) is also shown in Figure 8.4.2. Equation (8.4.17) is fully defined when the following three parameters are known:

- thickness of near-bed sediment mixing layer ($\delta_s$)
- mixing coefficient near the bed ($\varepsilon_{s,w,\text{bed}}$)
- mixing coefficient in the upper half of the depth ($\varepsilon_{s,w,\text{max}}$)

The thickness of the near-bed sediment mixing layer ($\delta_s$) was found to be equal to about three times the ripple height in the ripple regime ($\delta_s = 3 \Delta_r$). The sheet flow data of Ribberink and Al Saleem (1991, 1992) show that the moving sediment layer thickness is about three times the boundary layer thickness ($\delta_s = 3 \delta_w$). Thus,
\[ \delta_s = 3 \Delta_t \text{ in ripple regime} \]
\[ \delta_s = 3 \delta_w \text{ in sheet flow regime} \]  \hspace{2cm} (8.4.18)

in which:
\[ \Delta_t = \text{ripple height} \]
\[ \delta_w = \text{wave boundary layer thickness (Equation 2.3.8)} \]

Equation (8.4.18) yields values in the range of \( \delta_s = 0.03 \text{ to } 0.3 \text{ m} \).
Kroon and Van Rijn (1993), using the model of Van Rijn, found a value of \( \delta_s = 0.2 \text{ m} \) for concentrations in non-breaking wave conditions \( (H_s/h < 0.4) \) near the Dutch coast of Egmond.

The sediment mixing coefficient in the near-bed layer \( (\varepsilon_{s,w,\text{bed}}) \) was obtained by fitting an exponential concentration distribution (constant mixing coefficient) to measured concentrations in the near-bed layer. Based on analysis of the results, it was found that (Van Rijn, 1989):

\[ \varepsilon_{s,w,\text{bed}} = \alpha_p \bar{U}_b \delta_s \quad \text{for } z = \delta_s \]  \hspace{2cm} (8.4.19)

in which:
\[ \bar{U}_b = \text{peak orbital velocity at edge of wave boundary layer, based on significant wave height and peak wave period (m/s)} \]
\[ \delta_s = \text{thickness of sediment mixing layer (m)} \]
\[ \alpha_p = 0.004D_s = \text{coefficient (-)} \]
\[ D_s = d_{50} [s-1]g/v^2)^{1/3} = \text{particle size parameter (-)} \]

The \( D_s \)-parameter expresses the influence of the observed increased mixing effect of larger particles in the near-bed region. This can be explained by the influence of the eddy-induced centrifugal forces acting on the particles. The mixing process in the near-bed layer is dominated by ripple-related eddy motions resulting in a 'spiralling out' process and hence in an increased mixing effect. This latter effect has been studied in detail by Nielsen (1979). Nielsen shows that the radius of a sand particle orbit can be increased by a factor 2 after one eddy revolution. Deigaard (1991) has found that apparent mixing coefficients in oscillatory flow in the near-bed region increases with the particle fall velocity.

The sediment mixing coefficient in the upper layer \( (\varepsilon_{s,w,\text{max}}) \) was found to be (Van Rijn, 1989):

\[ \varepsilon_{s,w,\text{max}} = \alpha_m \frac{H_s h}{T_p} \quad \text{for } z = 0.5 \text{ h} \]  \hspace{2cm} (8.4.20)

in which:
\[ H_s = \text{significant wave height} \]
\[ T_p = \text{peak wave period} \]
\[ h = \text{water depth} \]
\[ \alpha_m = 0.035 = \text{coefficient} \]

The basic parameters of the vertical distribution of the sediment mixing coefficient are the \( \alpha_p \)-coefficient, the \( \alpha_m \)-coefficient and the near-bed sediment mixing layer thickness, \( \delta_s \). To get a better understanding of the influence of these three parameters on the computed sediment concentration profile, a sensitivity analysis has been carried out. Each parameter

8.42
has been varied over an uncertainty range of approximately 50%, keeping the other parameters constant. The results are shown in Figure 8.4.3A. As can be observed in Figure 8.4.3A, the influence of the $\alpha_b$-coefficient is rather large, because it directly affects the mixing coefficient in the near-bed region. A reduction of the $\alpha_b$-coefficient with 50% has a large effect yielding a decrease of the mid-depth concentration by a factor 5!

The influence of the $\alpha_m$-coefficient, which affects the mixing coefficient in the upper layers, is less pronounced (Figure 8.4.3C). The $\delta_s$-value has a relatively small effect on the concentration profile (Figure 8.4.3B).

![Graphs showing influence of sediment mixing coefficients on concentration profile](image)

**Figure 8.4.3** Influence of sediment mixing coefficient on concentration profile
11. **Comparison of mixing coefficient distribution**

The above given expressions were used to compute the wave-related sediment mixing coefficient distribution for an experiment of Bosman (1984). The method of Van Rijn was not used because this method was calibrated using the data of Bosman.

The basic data are:

- $H_s$ = significant wave height = 0.12 m
- $T_p$ = peak period of wave spectrum = 1.9 s
- $h$ = water depth = 0.3 m
- $L$ = wave length = 3.08 m
- $d_{s0}$ = median particle size of bed material = 0.00105 m
- $d_{s90}$ = particle size larger than 90% = 0.000130 m
- $d_e$ = representative particle size of suspended sediment = 0.00008 m
- $w_s$ = particle fall velocity of suspended sediment = 0.005 m/s
- $\Delta_r$ = ripple height = 0.02 m
- $k_{b,w}$ = wave-related bed roughness height = 0.06 m
- $T_e$ = water temperature = 20 °C

Linear wave theory has been used to compute the wave parameters resulting in:

- $A_b$ = peak value of orbital excursion just outside boundary layer = 0.092 m
- $U_b$ = peak value of orbital velocity just outside boundary layer = 0.305 m/s
- $f_w$ = friction factor = 0.3
- $u_{b,w}$ = time-averaged bed-shear velocity (half cycle) = 0.078 m/s
- $\hat{u}_{b,w}$ = peak value of bed-shear velocity = 0.11 m/s

The results are shown in Figure 8.4.4A. The 'measured' values represent the sediment mixing coefficients derived from the measured sediment concentration profile, using the convection-diffusion equation (Equation 8.4.3). The measured values indicate a constant sediment mixing coefficient in the near-bed region. Above this region the sediment mixing coefficient increases rather strongly up to the mid-depth level. In the upper half of the water depth the measured sediment mixing coefficient shows a (relatively) weak increase. None of the proposed relationships has a distribution similar to that of the measured values. Most expressions yield values which are much too large. The expression proposed by Nielsen produces a mixing coefficient of the right order of magnitude in the near-bed region. The expression of Lundgren, which only represents the turbulence-related mixing process in the boundary layer, yields values of the right order of magnitude in the near-bed region. Outside the boundary layer ($\delta_w \approx 0.015$ m) the sediment mixing coefficient of Lundgren decreases rapidly.

Figure 8.4.4B shows measured and computed concentration profiles for the same experiment. The computed concentrations are based on a numerical solution of the diffusion equation (Equation 8.4.3) applying the proposed expressions for the sediment mixing coefficient. The concentration measured in the lowest sampling point has been used as bed-boundary concentration ($C_b = 9500$ mg/l at $z = 0.02$ m) for the numerical computation of the concentration profile. The concentration profile based on the sediment mixing coefficient of Lundgren shows reasonable results in the near-bed region ($z < 0.2$ m). Outside the boundary layer the concentrations are much too small. Comparing all methods, the Nielsen method yields the most reasonable results. The methods of Homma-Horikawa, Bijker, Swart, Dally, Skafel-Krishnappan and Kosyan produce incorrect results.
Figure 8.4.4  Vertical distribution of sediment mixing coefficient and sediment concentration, Van Rijn (1989)
8.4.5 Sediment mixing coefficient in breaking waves

The sediment mixing process of breaking waves is strongly related to the type of breaking (spilling or plunging). Spilling breaking waves do occur for $H_s/h \leq 0.6$; plunging breaking waves do occur for $H_s/h > 0.6$ (see Section 2.3.6).

Plunging breaking waves ($H_s/h > 0.6$) show strong mixing properties. During breaking a jet is generated which strikes upon the forward slope of the wave generating a series of eddies. Each eddy moves forward and downward to the bed yielding a strong mixing process. The forward velocity of the eddies is smaller than the peak onshore orbital velocity near the bed and therefore the eddies move toward the back of the wave where they break up under escape of the entrained air bubbles.

Shihayama et al. (1986) performed a flume study of sediment concentrations and transport under plunging breaking waves. The importance of the jets and vortices penetrating to the bed and eroding bed material was clearly observed. The depth-integrated suspended load was found to be relatively large under the wave crest region and relatively low under the wave trough region.

Spilling breakers ($H_s/h \leq 0.6$ have less effective mixing properties because the turbulence production (eddies) is confined to the near-water surface region where a bore (roller) is generated. Measurements of Stive (1984) show that the turbulence intensity below the wave through level is not noticeably affected by the spilling breaking process (generation of surface roller).

Figure 8.4.5 shows the vertical distribution of the sediment mixing coefficient as derived from measured concentration profiles in spilling breaking wave conditions (Bosman, 1982). The mixing coefficients are considerably larger than those for non-breaking waves (see Figure 8.4.2), but the vertical distribution is quite similar to that of non-breaking waves.

Based on this similarity, Van Rijn (1989) proposed to use Equations (8.4.17), (8.4.19) and (8.4.20) for breaking waves as well.

Kroon and Van Rijn (1993) used the model of Van Rijn to analyze concentration profiles measured in the surf and swash zone near the Dutch coast of Egmond. They found that the mixing effect of spilling and plunging breaking waves can be simply represented by increasing the mixing layer thickness ($\delta_s$), see Equation (8.4.17). Kroon and Van Rijn found a value of $\delta_s = 0.2$ m for all available data. In dimensionless notation the $\delta_s$-parameter can be represented as:

$$\frac{\delta_s}{h} = 0.3 \left( \frac{H_s}{h} \right)^{0.5} \quad \text{for } 0.2 < \frac{H_s}{h} < 1.2$$  \hspace{1cm} (8.4.21)

with:

$\delta_{s,\text{minimum}} = 0.05$ m

$\delta_{s,\text{maximum}} = 0.20$ m

Deigaard et al (1986) presented a mathematical model to describe the vertical sediment concentration distribution in broken waves (spilling breakers). The hydraulic conditions near the front of a spilling breaker were assumed to be similar to that of a hydraulic jump. The production of turbulent kinetic energy in a hydraulic jump is concentrated in and near the surface roller. The turbulence energy is transported downwards in the fluid by convective and diffusive processes. Deigaard et al used a one-equation (turbulence) model for the kinetic energy ($k$). Graphical results were presented in terms of a dimensionless
turbulence energy parameter \( (k_*) \) as a function of dimensionless depth \( (z/h) \). The mixing coefficient can be represented as:

\[
\varepsilon_w = 0.07 \, g^{0.25} k_*^{0.5} \frac{H^{0.5} h^{1.25}}{T^{0.5}} \quad \text{for } z/h \geq 0.13
\]  

\( 8.4.22 \)

in which:

\( H = \) wave height
\( h = \) water depth
\( T = \) wave period
\( k_* = \) dimensionless turbulence energy parameter

The \( k_* \)-parameter varies from approx. 0.05 near the bed to approx. 0.6 near the water surface. It is noted that Equation (8.4.22) is based on the same parameters as Equation (8.4.20).

The results of Deigaard et al were used by Van Rijn to compute the mixing coefficient distribution for a flume test \( (H_s/h = 0.57, \text{ see Figure 8.4.5, Bosman 1982}) \) with spilling breakers. The computed mixing coefficient was found to be a factor of 5 too large in the surface region and a factor of 15 too large in the near-bed region.

---

**Figure 8.4.5** Sediment mixing coefficient in breaking waves
8.4.6 Reference concentration in near-bed region


The reference concentration \( c_a \) is defined at the level \( z = a = k_s \) with \( k_s \) = equivalent bed-roughness height. Bijker (1967) assumed that the bed load transport occurred in a layer with a thickness equal to the bed roughness \( (k_s) \). The concentration in this layer was assumed to be constant over the layer thickness. Using the bed-load transport formula of Frijlink (Equation 7.2.30), the reference concentration can be expressed as:

\[
\begin{align*}
    c_a &= \frac{b \rho_s d_{50}}{6.34 a} \exp \left( \frac{-0.27(\rho_s - \rho)g d_{50}}{\mu \tau_{b,w}} \right) \\
\end{align*}
\]

(8.4.23)

in which:

\[
\begin{align*}
    c_a &= \text{reference concentration (kg/m}^2\text{)} \\
    \tau_{b,w} &= \frac{1}{4} \rho f_w (\bar{U}_b)^2 \\
    f_w &= \exp \left[ -6 + 5.2 \left( \frac{\bar{U}_b}{k_s} \right)^{0.19} \right] \\
    \bar{U}_b &= \text{time-averaged (half cycle) bed-shear stress (N/m}^2\text{)} \\
    f_w &= \text{friction factor, } f_{w,max} = 0.3 (-) \\
    \bar{U}_b &= \text{near-bed peak orbital velocity (m/s)} \\
    \text{efficiency factor (-)} &= \text{near-bed peak orbital excursion (m)} \\
    C &= 18 \log (12h/k_s) \\
    C' &= 18 \log (12h/d_{50}) \\
    h &= \text{overall Chézy coefficient (m}^{0.5}/\text{s}) \\
    k_s &= \text{effective bed roughness height (m)} \\
    a &= \text{reference level (m)} \\
    d_{50}, d_{90} &= \text{particle diameters (m)} \\
    \rho, \rho_s &= \text{fluid sediment density (kg/m}^3\text{)} \\
    b &= \text{coefficient (b = 1 for non-breaking waves and b = 5 for breaking waves)}
\end{align*}
\]

![Figure 8.4.6 Measured and predicted near-bed concentrations, \( d_{50} = 210 \mu m \)](image-url)
Equation (8.4.23) with \( b = 1 \) for non-breaking waves was used to predict the near-bed concentration for the experimental results of Ribberink and Al- Salem (1991, 1992, see Table 8.2). The reference level was applied at \( a = k_s \) with \( k_s = 0.01 \) m for sheet flow conditions (see Section 6.3.1) and \( k_s = 3\Delta_r \) for rippled bed conditions. The bed-shear stress was computed using \( \bar{U}_{sig,crest} \) and \( \tilde{A}_{sig,crest} \) as characteristic wave parameters. Computed and measured concentrations are shown in Figures 8.3.14 and 8.4.6. The computed concentrations are nearly constant for \( \bar{U}_{crest} > 1 \) m/s, expressing the exponential behaviour of Equation (8.4.23). The computed values are too large (factor 2 to 5) for \( \bar{U}_{crest} < 1.2 \) m/s and too small (factor 2 to 3) for \( \bar{U}_{crest} > 1.7 \) m/s.


The reference concentration was obtained by fitting an exponential function to measured near-bed concentrations and extrapolating this function to the mean bed level \( z = 0 \). This approach yields a reference concentration which should be interpreted as a non-physical parameter because the validity of the exponential concentration close to the bed is highly questionable (see Figures 8.3.15 to 8.3.18).

Based on this approach, Nielsen found:

\[
c_s = 0.005 \rho_s \theta_r^3
\]

\[\text{in which:}
\]

\[
c_s \quad \text{= reference concentration (kg/m}^3\text{)}
\]

\[
\theta_r = \left(1 - \pi \frac{\Delta_r}{\lambda_r}\right)^{-2} \theta'
\]

\[\text{= particle mobility parameter (-)}
\]

\[
\theta' = \frac{0.5f_w' \left(\bar{U}_b\right)^2}{(s - 1)g d_{s0}}
\]

\[\text{= effective particle mobility parameter}
\]

\[
f_w' = \exp\left[-6 + 5.2 \left(\frac{\tilde{A}_b}{2.5 \, d_{s0}}\right)^{-0.19}\right]
\]

\[\text{= grain-related friction factor (-)}
\]

\[a \quad \text{= reference level (z = a = 0), (m)}
\]

\[\Delta_r \quad \text{= ripple height (m)}
\]

\[\lambda_r \quad \text{= ripple length (m)}
\]

3. Fredsøe et al. (1985)

Fredsøe et al. used Equation (7.3.26), which was proposed for steady flow conditions, to compute the reference concentration in oscillatory flow. The particle mobility parameter \( \theta' \) is based on the bed-shear stress for waves \( \tau_{b,w} \). The reference level is \( a = 2d_{s0} \).


For steady flow Equation (7.3.31) was proposed to compute the reference concentration at a level \( a = k_s \) or \( a = \frac{1}{2} \Delta_r \).

The proposed equation is modified for oscillatory flow, as follows:

\[
c_s = 0.015 \rho_s \frac{d_{s0}}{a} \frac{\tau_{a,1.5}}{D_{s,0.3}}
\]

\[\text{(8.4.25)}
\]
in which:

c_s = reference concentration (kg/m³)
a = reference level

\[ D_0 = d_{50}[(s-1)g/v^2]^{1/2} \] = particle size parameter (-)

\[ T_a = \frac{(\tau'_b, w - \tau_{b,cr})}{\tau_{b,cr}} \] = bed shear stress parameter (-), see Appendix A

\[ \tau_{b,cr} \] = critical bed-shear stress according to Shields (N/m²)

\[ \tau_{b,w} = \frac{1}{4} \rho f_w (\hat{U}_o) - \frac{\mu_{w, a}}{k_{s,w}} \tau_{b, cr} \] = effective wave-related bed-shear stress (N/m²)

\[ f_w = \exp \left[ -6 + 5.2 \left( \frac{\hat{A}_o}{k_{s,w}} \right)^{0.19} \right] \] = friction factor, \( f_{w, \text{max}} = 0.3 \) (-)

\[ \hat{U}_o \] = near-bed peak orbital velocity based on significant wave height and peak wave period (m/s)

\[ \hat{A}_o \] = near-bed peak orbital excursion based on significant wave height and peak wave period (m)

\[ \mu_{w, a} = 0.6/D_0 \] = efficiency factor, \( \mu_{w, \text{min}} = 0.06 \) (-)

\[ k_{s,w} \] = wave-related bed roughness (m)

\[ \Delta_r \] = ripple height (m)

The \( \mu_w \) factor was found by calibration using measured concentrations in the near-bed region (Van Rijn, 1989). Particle sizes were in the range of 100 to 360 μm. Peak orbital velocities were in the range of 0.2 to 1.2 m/s and peak periods form 2 to 6 s. Most data were in the ripple regime.

The efficiency factor (\( \mu_w \)) represents that part of the bed-shear stress that is available to entrain the sediment particles from the bed into the flow. When bed forms are present, the efficiency factor should be smaller than unity (\( \mu_w < 1 \)) because a part of the bed-shear stress is exerted by pressure forces (from drag) generating eddies in the leeside region of the bed forms. In unidirectional flow the entrainment process is governed by skin-friction forces at the upsloping part of the bed forms. In oscillatory flow over bed forms (ripples) it is not realistic to relate the sediment entrainment process to skin-friction forces alone, as proposed by Nielsen (1979) and Grant and Madsen (1982). Visual observation clearly shows the active role of the ripple-related eddies (from drag) which are moving forward and backward over the ripples. Kennedy and Locher (1972) also emphasize the strong effect of the ripples on the flow-bed interaction. According to their observations, the entrainment process is largely governed by the ripple spacing and the strength of the lee eddies near the ripple crests.

Predicted and measured near-bed concentrations for the experimental results of Ribberink and Al-Salem (1991, 1992, see Table 8.2), are shown in Figures 8.3.14 and 8.4.6. The bed-shear stress was computed using the \( \hat{U}_{crest} \) and \( \hat{A}_{crest} \) parameters as the characteristic wave parameters. The two curves express the influence of the wave period and the bed roughness. Good agreement between measured and predicted values can be observed. The predicted values are too small (factor 2) for high peak velocities.
8.4.7 Methods for computation of time-averaged concentration profiles

The following methods are described:

4. Fredsøe et al. (1985)
5. Van Rijn (1989)

1. **Bijker (1967, 1971)**

Originally, this method has been developed by Bijker to compute the concentration profiles in case of combined currents and waves. In case of waves alone the method can be simplified to:

\[
\frac{c}{c_a} = \left( \frac{a}{h - a} \right)^{w_s/\kappa} \left( \frac{1 - z}{z} \right)^{u_{*,w}}
\]

(8.4.26)

\[
c_a = \frac{b \rho_s d_{50}}{6.34 \cdot \mu} \exp \left( \frac{0.27 (\rho_s - \rho) g d_{50}}{\mu \tau_{b,w}} \right)
\]

in which:

- \(c_a\) = reference concentration (kg/m³)
- \(a\) = thickness of bed-load layer (m)
- \(k_s\) = effective roughness of bed (m)
- \(h\) = water depth (m)
- \(\tau_{b,w}\) = time-averaged overall bed-shear stress, see Equation (8.4.23), (N/m²)
- \(d_{50}\) = median particle diameter of bed material (m)
- \(b\) = empirical coefficient (h = 1 for non-breaking waves and h = 5 for breaking waves)
- \(w_s\) = particle fall velocity of suspended sediment (m/s)
- \(\kappa\) = constant of Von Karman (0.4)
- \(u_{*,w} = (\tau_{b,w}/\rho)^{0.5}\) = time-averaged bed-shear velocity (m/s)
- \(\mu\) = \((C/C')^{1.5}\) = efficiency factor (-)
- \(C = 18 \log(12h/k_s)\) = Overall Chézy coefficient (m⁰.⁵/s)
- \(C' = 18 \log(12h/d_{50})\) = Chézy-coefficient related to grains (m⁰.⁵/s)

2. **Skafel-Krishnappan (1984)**

This method reads, as follows:

\[
\frac{c}{c_a} = \exp \left( - \frac{w_s}{\hat{u}_{*,w}} \cdot \frac{1}{\beta_1} \frac{z - a}{\hat{A}_s} \right)
\]

(8.4.27)

\[
\beta_1 = 8.7 \left( \frac{\hat{u}_{*,w} d_{50}}{v} \right)^{-2.2}
\]
\[ c_a = \frac{q_b}{a \hat{U}_s} \]

\[ q_b = 12.5 \rho_s w_s d_{50} \left( \frac{\hat{t}_{b,w}'}{(\rho_s - \rho) g d_{50}} \right)^3 \]

\[ \hat{t}_{b,w}' = 0.5 \rho f_w' (\hat{U}_s)^2 \]

\[ \hat{u}_{s,w} = [0.5 f_w (\hat{U}_s)^2]^{0.5} \]

\[ f_w' = \exp \left[ -6 + 5.2 \left( \frac{\hat{A}_s}{d_{90}} \right)^{-0.19} \right] \]

\[ f_w = \exp \left[ -6 + 5.2 \left( \frac{\hat{A}_s}{\Delta_r} \right)^{-0.19} \right] \]

in which,

- \( q_b \) = time-averaged bed-load transport rate during a half period (kg/sm)
- \( \hat{t}_{b,w}' \) = maximum value of bed-shear stress related to skin-friction (N/m²)
- \( f_w' \) = friction factor related to grains (-)
- \( \beta_1 \) = coefficient (-)
- \( a = 3d_{50} \) = thickness of bed-load layer
- \( d_{50}, d_{90} \) = particle sizes of bed material (m)
- \( \Delta_r \) = ripple height (m)


\[ \frac{c}{c_a} = \exp \left[ -z/l_s \right] \]  \hspace{1cm} (8.4.28)

\[ c_a = 0.005 \rho_s \theta_3^3 \]

\[ L_s = \frac{0.075 \Delta_r \omega \hat{A}_s}{w_s} \hspace{0.5cm} \text{for} \hspace{0.5cm} \frac{\omega \hat{A}_s}{w_s} \leq 15 \]

\[ L_s = 1.4 \Delta_r \hspace{0.5cm} \text{for} \hspace{0.5cm} \frac{\omega \hat{A}_s}{w_s} > 15 \]

\[ \theta_r = \frac{\theta'}{(1 - \pi \Delta_r/\lambda_r)^2} \]

\[ \theta' = 0.5 f_w' (\hat{U}_s)^2 \]

\[ f_w' = \exp \left[ -6 + 5.2 \left( \frac{\hat{A}_s}{2.5 d_{50}} \right)^{-0.19} \right] \]

8.52
in which:
\( c_s \) = reference concentration at \( z = 0 \), (kg/m³)
\( \theta' \) = mobility parameter (-)
\( \Delta_r \) = ripple height (m)
\( \lambda_r \) = ripple length (m)
\( L_a \) = length scale (m)
\( \omega = 2\pi/T \) = angular frequency (-)
\( w_s \) = particle fall velocity (m/s)
\( \hat{A}_b \) = peak value of near-bed orbital excursion (m)
\( f_w' \) = grain-related friction coefficient (-)
\( Z \) = vertical coordinate (m)

4. Fredsøe et al. (1985)

An instantaneous approach is followed to obtain the velocities and sediment concentrations within the wave period. The method is valid for non-breaking and breaking waves over a plane bed.

**Flow**

Potential flow outside the wave boundary layer:

\[
U = \hat{U}_b \sin(\omega t) \tag{8.4.29}
\]

The orbital velocities inside the wave boundary layer are derived from the instantaneous momentum equation applying a logarithmic velocity distribution for a rough bed:

\[
\frac{\partial U}{\partial t} + \frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{1}{\rho} \frac{\partial \tau}{\partial z} = 0 \tag{8.4.30}
\]

\[
\frac{\partial P}{\partial x} = -\rho \frac{\partial U_b}{\partial t}
\]

\[
U = \left( u_*/\kappa \right) \ln(30z/k_b)
\]

Vertical integration over the wave boundary layer yields:

\[
u_*^2 = \frac{\tau_b}{\rho} = \int_{z_o}^{z_b} \frac{\partial}{\partial t} (U_b - U) dt \tag{8.4.31}
\]

The eddy viscosity or mixing coefficient is represented as:

\[
\varepsilon_t^2 = \varepsilon_b^2 + \varepsilon_i^2 \tag{8.4.32}
\]

\[
\varepsilon_b = \kappa u_* z(1-z/\delta) = \text{bottom-induced mixing coefficient}
\]

\[
\varepsilon_i = \ell k^{0.5} = \text{wave-breaking induced mixing coefficient}
\]
\[ \ell = \text{mixing length scale (Deigaard et al., 1986)} \]
\[ k = \text{turbulent kinetic energy derived from transport equation for the kinetic energy (k-equation)} \]
\[ \delta = \text{instantaneous wave boundary layer thickness} \]
\[ u_* = \text{instantaneous bed-shear velocity} \]
\[ U = \text{instantaneous velocity} \]
\[ U = \text{peak value of orbital velocity just outside boundary layer} \]
\[ P = \text{instantaneous fluid pressure} \]
\[ \tau = \text{instantaneous fluid shear stress} \]
\[ \rho = \text{fluid density} \]
\[ \kappa = \text{Von Karman constant (= 0.4)} \]
\[ k_s = \text{bed roughness (= 2.5 d_{so})} \]
\[ z = \text{vertical coordinate} \]

**Sediment**

The sediment concentrations \( (C) \) are derived from the instantaneous convection-diffusion equation applying a prescribed concentration at the bed:

\[
\frac{\partial C}{\partial t} - w_s \frac{\partial C}{\partial z} - \frac{\partial}{\partial z} \left( \varepsilon_s \frac{\partial C}{\partial z} \right) = 0 \quad (8.4.33)
\]

Boundary conditions:

\[
c_n = F \left( \frac{\tau_b}{(\rho_s - \rho g d_{so})} \right) \quad \text{at } z = 2 d_{so}
\]

\[
w_s C + \frac{\partial}{\partial z} \left( \varepsilon_s \frac{\partial C}{\partial z} \right) = 0 \quad \text{at } z = h
\]

in which:

- \( C \) = instantaneous sediment concentration
- \( w_s \) = particle fall velocity
- \( \varepsilon_s \) = sediment mixing coefficient (\( \varepsilon_s = \varepsilon_t \))
- \( \rho_s \) = sediment density

Hindering settling effects and turbulence damping effects due to the large concentrations and gradients close to the bed are not taken into account. Numerical methods are applied to solve the momentum and convection-diffusion equation.

**5. Van Rijn (1989)**

Applying Equation (8.4.17) and neglecting the hindered settling effects (thus taking a constant fall velocity), the concentration profile can be obtained by integration of the time-averaged convection-diffusion Equation (8.4.3), yielding:

\[
\frac{c}{c_s} = \exp \left( \frac{w_s (a - z)}{\varepsilon_{s,w,bed}} \right) \quad \text{for } a \leq z \leq \delta_s \quad (8.4.34a)
\]

8.54
\[
\frac{c}{c_a} = \alpha \left( \frac{h}{h + \gamma(z - \delta_s)} \right)^\beta \quad \text{for} \quad \delta_s \leq z \leq 0.5h \quad (8.4.34b)
\]
\[
\frac{c}{c_a} = \alpha \eta \exp\left( \frac{w_s (0.5h - z)}{\varepsilon_{s,w,max}} \right) \quad \text{for} \quad 0.5h \leq z \leq h \quad (8.4.34c)
\]

with:
\[
\gamma = \left( \frac{h}{0.5h - \delta_s} \right) \left( \frac{\varepsilon_{s,w,max} - \varepsilon_{s,w,bed}}{\varepsilon_{s,w,bed}} \right)
\]
\[
\alpha = \exp\left( \frac{w_s (z - \delta_s)}{\varepsilon_{s,w,bed}} \right)
\]
\[
\beta = \frac{w_s h}{\gamma \varepsilon_{s,w,bed}}
\]
\[
\eta = \left( \frac{h}{h + \gamma(0.5h - \delta_s)} \right)^\beta
\]
\[
\varepsilon_{s,w,bed} = 0.004 D_s \hat{U}_d \delta_s
\]
\[
\varepsilon_{s,w,max} = 0.035 \frac{H_s h}{T_p}
\]
\[
\delta_s = 0.3 h \left( \frac{H_s}{h} \right)^{0.5} \quad \text{with} \quad \delta_{s,min} = 0.05 \text{ m} \quad \text{and} \quad \delta_{s,max} = 0.2 \text{ m}
\]
in which:
- \( c_a \) = reference concentration (Equation 8.4.25) at level \( z = a \) (kg/m³)
- \( h \) = water depth (m)
- \( \delta_s \) = thickness of near-bed mixing layer
- \( \varepsilon_{s,w,bed} \) = sediment mixing coefficient near the bed (m²/s)
- \( \varepsilon_{s,w,max} \) = sediment mixing coefficient in upper half of the depth (m²/s)
- \( w_s \) = particle fall velocity of suspended sediment (m/s)
- \( z \) = vertical coordinate (m)
- \( D_s \) = dimensionless particle parameter (-)
- \( \hat{U}_d \) = peak value of near-bed orbital velocity based on significant wave height (m/s)
- \( H_s \) = significant wave height (m)
- \( T_p \) = peak wave period (s)
- \( \alpha, \beta, \gamma, \eta \) = coefficients
The hindered settling effect, which means a concentration dependent fall velocity \( w_{s,m} = w_s (1-c)^5 \) has been neglected to obtain Equation (8.4.34). It is not clear whether this is justified. Therefore, the sediment concentration profile was computed numerically taken the hindered settling effect into account. Three cases were considered: \( c_s = 2.500, 5.000 \) and \( 10.000 \) mg/l. The results are presented in Figure 8.4.7, showing an increase of the concentration due to the hindered settling effect. Considering all uncertainties involved, the hindered settling effect can be neglected for concentrations smaller than \( 10.000 \) mg/l (10 kg/m²). Measurements of Ribberink and Al-Salem (1991, 1992) show that concentrations of this order of magnitude do not occur outside the sheet flow layer.

![Figure 8.4.7 Influence of hindered settling effect on concentration profile](image)

Examples of computed concentration profiles based on Equation (8.4.34) are shown in Figures 8.4.8 to 8.4.10. Experimental results of Bosman (1982) and Van Rijn (1987) were used for comparison. The experiments of Bosman were performed in a small-scale wave flume with water depths in the range of 0.1 to 0.6 m. The bed \( (d_{so} = 100 \) \( \mu \)m) was covered with ripples (height \( \Delta_r \approx 0.01 \) m). The measurements of Van Rijn (1987) were performed in a large-scale wave flume (length \( = 150 \) m, width \( = 5 \) m, depth \( = 7 \) m). Water depths were in the range of 2 to 3 m. The bed material size was about 220 \( \mu \)m.

Figure 8.4.10 shows a considerable underprediction of the concentrations in the upper layers, which is caused by the presence of the finer size fractions in the upper layers of the water column. This effect can only be represented by using a multi-fraction method.
Figure 8.4.8 Measured and computed concentration profile, experiment of Bosman (1982)

Figure 8.4.9 Measured and computed concentration profile, experiment of Bosman (1982)

Figure 8.4.10 Measured and computed concentration profile, experiment of Van Rijn (1987)
8.5 Computation of sediment transport rates

8.5.1 Introduction

Measured transport rates have been analyzed in Section 8.3.3. Herein, the attention is focussed on the computation methods. Basically, two types of approaches are applied:

- sediment transport models representing both the instantaneous fluid velocity and concentration profiles,
- sediment transport formulae similar to the current-related bed-load transport formulae.

Based on the experimental results of Ribberink and Al-Salem (1991, 1992), simple formulae can be applied when phase differences between the instantaneous velocities and concentrations are relatively small.

A practical applicability range for the transport formulae is: $d_{50} > 200 \mu m$, $T > 6 s$ and $U_{crest} > 0.5 m/s$.

8.5.2 Sediment transport models

A considerable amount of literature deals with the modelling of the sediment concentrations within the wave period as an intermediate step to compute the wave induced transport rate. This approach is useful when the phase differences between the instantaneous velocities and sediment concentrations at different elevations above the bed cannot be neglected.

Dividing the instantaneous velocity ($U$) and concentration ($C$) in a time-averaged part ($u$, $c$) and a time-dependent part ($\bar{U}$, $\bar{C}$) as follows:

\[ U = u + \bar{U} \]  
\[ C = c + \bar{C} \]

the net total time-averaged total transport rate can be expressed as:

\[ q_t = \int_0^h uc \, dz + \int_0^h \bar{U}\bar{C} \, dz \]  

(8.5.3)

The first term on the right hand side represents the current-related part ($q_c$) of the sediment transport and the second part represent the wave-related part ($q_w$). Thus:

\[ q_c = \int_0^h uc \, dz \]  

(8.5.4)

\[ q_w = \int_0^h \bar{U}\bar{C} \, dz \]  

(8.5.5)

To compute the sediment concentrations within the wave period, the usual approach is to solve the (simplified) convention-diffusion equation. Neglecting the horizontal convection, the horizontal diffusion and the vertical convection, the simplified equation reads as:

\[ \frac{\partial C}{\partial \tau} - w_{s,m} \frac{\partial C}{\partial z} - \frac{\partial}{\partial z} \left( \varepsilon_{s,w} \frac{\partial C}{\partial z} \right) = 0 \]

(8.5.6)
Equation (8.5.6) is only valid for plane bed regime (sheet flow). Analytical or numerical solution requires specification of the fall velocity \( (w_{fa}) \), the sediment diffusion coefficient \( (\varepsilon_{sm}) \) and boundary conditions at the bed and at the water surface. Usually, a separate oscillatory flow model is applied to compute the sediment diffusion coefficient and the instantaneous bed-shear stress. This latter parameter is then used to compute the instantaneous bed concentration. Based on this, the concentration profiles can be computed as a function of time.

Applying Equation (8.5.5), the net wave-induced transport rate can be computed. Various models are proposed in the literature. Bakker (1974) applied a numerical model and used a mixing length concept to determine the sediment diffusion coefficient. The expressions of Einstein were applied to specify the bed concentration. Recently, the theories of Bagnold (1954) related to particle interactions in case of high concentrations were incorporated in the model of Bakker. The influence of the sediment particles on the fall velocity and turbulence were taken into account (Bakker and Van Kesteren, 1986).

Fredsoe et al (1985) applied a numerical model based on the eddy viscosity concept. The eddy viscosity coefficients were related to the instantaneous bed-shear stresses and the boundary layer thickness. The bed-boundary condition for the sediment was specified as a concentration function, yielding the bed concentration at a height of two particle diameters above the bed as a function of the instantaneous bed-shear stress. At the water surface the vertical sediment flux was set to zero. The influence of the sediment particles on the fall velocity and eddy viscosity coefficient was not taken into account. The method of Fredsoe et al (1985) is described in more detail in Section 8.4.7.

Hagatum and Eidsvik (1986) presented a numerical model based on a two-equation turbulence model to represent the eddy viscosity coefficient. It is questionable whether such a sophisticated improvement is necessary considering the experimental problems related to the measurement of the concentrations in the sheet flow layer. Analytic solutions of the convection-diffusion equation were given by Nielsen (1979) assuming that the eddy viscosity coefficient is constant in space and time.

Finally, it is noted that all transport models are only valid for flat bed regime (sheet flow). The transport process over bed forms cannot be represented because Equation (8.5.6) is not valid for the bed form regime.

### 8.5.3 Sediment transport formulae

Since the major part of the sediment suspension in wave conditions is confined to a region close to the bed (within 3 to 5 times the ripple height or the sheet flow layer thickness), it seems logical to compute the wave-related sediment transport by a simple formula in analogy with the bed-load transport formulae applied in steady currents. A division between bed load and suspended load is only of academic interest.

The existing formulae are generally based on empirical concepts (as used in steady uniform flow) using experimental data of oscillating flow (wave tanks, wave tunnels, oscillating plates in still water).

The sediment transport in half a period of the oscillatory flow \( (q_{w,0.5}) \) or the net transport rate per cycle \( (q_{w,n}) \) is expressed as a function of the horizontal peak velocity just outside the boundary layer, the grain-related friction factor and the sediment parameters. In some formulae (Madsen and Grant 1976, Bailard 1981) the sediment transport is related to the instantaneous fluid velocity and integrated over the complete wave cycle to obtain the net transport rate. In this latter approach the following assumptions are implicitly made:
• no phase differences between the instantaneous bed shear stress and the velocity outside the boundary layer,
• no phase differences between instantaneous bed shear stresses and instantaneous transport rates.

These type of formulae seem to be best applicable in the flat bed sheet flow regime where the net transport is in the direction of the largest peak velocity (onshore). When steep ripples are present, large phase differences between the instantaneous velocities and the concentrations do occur which are caused by eddy motions at the lee-side of the ripples. This may lead to a net transport rate against the wave direction. A similar process may occur in sheet flow over a very fine sand bed (50 to 150 \( \mu \text{m} \)). In all formulae the effect of a net drift velocity is neglected.

Herein, the following formulae are summarized: Madsen and Grant (1976), Bailard-Bagnold (1981), Hallermaier (1982), Sato-Horikawa (1986), Sawamoto-Yamashita (1987) and Van Rijn.

1. **Madsen and Grant (1976)**

\[
q_{w,\text{half}} = 12.5 \ w_s \ d_{s0} \ (\theta')^3 
\]

in which:
\( q_{w,\text{half}} \) = time-averaged transport rate over a half cycle \( \left( \text{m}^3/\text{s} \right) \)
\( w_s \) = particle fall velocity of bed material \( \left( \text{m/s} \right) \)
\( d_{s0} \) = median particle size of bed material \( \left( \text{m} \right) \)
\( \theta' = \frac{0.5 \ f_w' (\bar{U}_\delta)^2}{(s^{-1}) \ g \ d_{s0}} \) = mobility parameter \( (-) \)
\( f_w' = \exp \left[-6 + 5.2 \left( \bar{A}_\delta / d_{s0} \right)^{-0.19} \right] \) = grain-related friction factor \( (-) \)
\( \bar{U}_\delta \) = peak value of near-bed orbital velocity \( \left( \text{m/s} \right) \)
\( \bar{A}_\delta \) = peak value of near-bed orbital excursion \( \left( \text{m} \right) \)
\( \rho \) = fluid density \( \left( \text{kg/m}^3 \right) \)
\( \rho_s \) = sediment density \( \left( \text{kg/m}^3 \right) \)
\( g \) = acceleration of gravity \( \left( \text{m/s}^2 \right) \)

The empirical coefficient is based on the calibration of about 110 experiments (Figure 8.5.1) with \( d_{s0} \) in the range of 300 to 2800 \( \mu \text{m} \); wave periods in the range of 1 to 6 s; in most tests the bed was flat.

Equation (8.5.7) yields a net transport rate in the direction of the largest peak velocity (usually onshore).

Figure 8.5.1 shows measured and predicted transport rates. According to Horikawa et al. (1982), Equation (8.5.7) is also valid for sheet flow conditions based on a comparison with measured transport rates in the sheet flow regime.

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2. **Bailard-Bagnold (1981)**

The instantaneous bed load and suspended load transport rates are expressed as:

$$q_{b,w} = \frac{\sqrt{2} \rho f_w e_b}{(\rho_s - \rho) g \tan \gamma} \left[ \frac{|U|^2 U - \frac{\tan \beta}{\tan \gamma} |U|^3 i_\beta}{|U|^3} \right]$$  \hspace{1cm} (8.5.8)

$$q_{s,w} = \frac{\sqrt{2} \rho f_w e_s}{(\rho_s - \rho) g w_s} \left[ \frac{|U|^3 U - \frac{e_s}{w_s} \tan \beta |U|^5 i_\beta}{|U|^5} \right]$$  \hspace{1cm} (8.5.9)

in which:

- $q_{b,w}$ = instantaneous wave-induced bed-load transport (m$^2$/s)
- $q_{s,w}$ = instantaneous wave induced suspended load transport (m$^2$/s)
- $U$ = instantaneous near-bed orbital velocity (m/s)
- $c_b$ = efficiency factor for bed load transport (0.1-0.2)
- $e_s$ = efficiency factor for suspended load transport (0.01-0.02)
- $f_w$ = friction factor based on the particle diameter (-)
- $\beta$ = bottom slope ($^\circ$)
- $\gamma$ = dynamic friction angle ($\tan \gamma = 0.6$, ($^\circ$))
- $w_s$ = fall velocity of bed material (m/s)
- $i_\beta$ = unity vector component in direction of bed slope ()

Originally, the formulae of Bagnold were developed for steady unidirectional flow. Bailard applied these formulae to oscillatory flow over a plane sloping bottom. Usually, the coefficients are taken as $e_b = 0.1$ and $e_s = 0.02$. Equations (8.5.8) and (8.5.9) yield a net transport rate in the direction of the largest peak velocity (usually onshore).

\[ q_{w,\text{half}} = \omega (d_{50})^2 (0.1 \psi)^{1.5} \]  \hfill (8.5.10)

in which:

- \( q_{w,\text{half}} \) = time-averaged transport rate over half cycle (m\(^2\)/s)
- \( \psi = \frac{(\hat{U}_a)^2}{(s-1)g d_{50}} \) = mobility parameter
- \( \omega = \frac{2\pi}{T} \) = angular frequency

Equation (8.5.10) is based on about 700 experiments with \( d_{50} \) in the range of 150 to 4200 \( \mu \)m and wave periods in the range of 1 to 9 s. The predicted transport rates are in agreement with measured transport rates for \( 30 < \psi < 200 \). The predicted rates are too large for \( 0.05 < \psi < 30 \) (see Figure 8.5.2). Equation (8.5.10) predicts a net transport rate in the direction of the largest peak velocity.

![Figure 8.5.2 Wave-related transport, Hallermeier (1982)](image-url)

Based on tunnel experiments with regular asymmetric wave motion over a rippled sand bed of 180 \( \mu \text{m} \), Sato and Horikawa found:

\[
q_{w,\text{net}} = -7 \; w_s \; d_{50} \; (\theta'_{\text{crest}} - \theta'_{\text{eq}}) (\theta'_{\text{crest}})^{0.5} \quad \text{for} \quad \theta_{\text{crest}}' \leq 0.6
\]  

\hspace{1cm} (8.5.11)

in which:

- \( q_{w,\text{net}} \) = net transport rate over a wave cycle (m\(^2\)/s)
- \( \theta' \) = mobility parameter (-)
- \( \theta'_{\text{crest}} \) = critical mobility parameter according to Shields (-)
- \( f_w \) = friction coefficient, see Equation (8.5.12), with \( k_s = d_{50} \) as grain roughness (-)
- \( w_s \) = particle fall velocity of bed material (m/s)
- \( \hat{U}_{\text{crest}} \) = peak value of near-bed orbital velocity under wave crest (m/s)

Equation (8.5.11), which is based on 36 experimental data, predicts a net transport rate against the direction of the largest peak velocity and is only valid for the ripple regime (see Figure 8.5.3).

![Figure 8.5.3 Wave-related transport, Sato-Horikawa (1986)](image)


Based on experiments performed in a wave tunnel with sand (\( d_{50} = 200, 700 \) and \( 1800 \ \mu \text{m} \)), coal and plastic material, the following empirical formula for the sheet flow regime is proposed:

\[
q_{w,\text{half}} = 2.2 \; d_{50} \; (w_s)^{-2} \; (\hat{u}_{s,w})^3
\]  

\hspace{1cm} (8.5.12)
in which:

\[ q_{w,\text{half}} = \text{time-averaged transport rate over half cycle (m}^2/\text{s}) \]

\[ \hat{u}_{\ast,w} = [0.5 f_w']^{0.5} \hat{U}_h \]

\[ f_w = \exp[-6 + 5.2(\hat{A}_h/d_{50})^{-0.19}] \]

\[ d_{50} = \text{median particle size of bed material (m)} \]

\[ w_s = \text{particle fall velocity of bed material (m/s)} \]

\[ \hat{U}_h = \text{peak value of near-bed orbital velocity (m/s)} \]

\[ \hat{A}_h = \text{peak value of near-bed orbital excursion (m)} \]

Equation (8.5.12) yields a net transport rate in the direction of the largest peak velocity.

6. Van Rijn

The following transport processes are considered:
- asymmetric regular swell waves
- asymmetric irregular wind waves
- mean (weak) current in presence of waves
- bound long waves

A. Transport due to asymmetric regular swell waves

Based on analysis (see Section 8.3.3) of the data of Sato-Horikawa (1986) it is assumed that steep ripples are generated by regular swell waves over sandy beds with particle sizes > 200 \( \mu \text{m} \). The net total transport rate due to regular swell waves in the steep ripple regime is directed against the wave propagation direction and can be predicted by:

\[ q_{w,\text{net}} = -0.00063((s-1)g^{0.5}(d_{50})^{1.5} [\alpha_s(\psi_{\text{crest}}-\psi_{\text{cr}})^{1.7} - \alpha_s(\psi_{\text{trough}}-\psi_{\text{cr}})^{1.7}] \]

(8.5.13)

for rippled sand bed, \( d_{50} \geq 200 \mu \text{m} \), \( \psi_{\text{crest}} \leq 100 \), and \( T \geq 15 \text{ s} \).

in which:

\[ q_{w,\text{net}} = \text{net total transport rate against wave direction (negative sign) (m}^2/\text{s}) \]

\[ s = \text{relative density (-)} \]

\[ d_{50} = \text{median particle size of bed material (m)} \]

\[ \hat{U}_{\text{crest}} = \text{near-bed peak orbital velocity under wave crest (m/s)} \]

\[ \hat{U}_{\text{trough}} = \text{near-bed peak orbital velocity under wave trough (m/s)} \]

\[ \psi_{\text{crest}} = \text{angle of repose (} = 30^\circ) \]

\[ \beta = \text{local bed slope} \]

\[ \psi_{\text{crest}} = \frac{(\hat{U}_{\text{crest}})^2}{(s-1)g d_{50}} = \text{mobility parameter (-)} \]

\[ \psi_{\text{trough}} = \frac{(\hat{U}_{\text{trough}})^2}{(s-1)g d_{50}} = \text{mobility parameter (-)} \]

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\[ \psi_{cr} = \frac{(\dot{U}_{cr})^2}{(s-1) \ g \ d_{s0}} \quad = \text{critical mobility parameter (-)} \]

\[ \alpha_s = \frac{\tan \gamma}{\cos \beta (\tan \gamma \pm \tan \beta)} \quad = \text{slope factor of Bagnold} \]

\[ (+ \text{for upsloping flow, } \ - \text{ for downsloping flow}) \]

The \( \dot{U}_{\text{crest}} \) and \( \dot{U}_{\text{trough}} \) parameters can be estimated by using second order wave theory of Stokes. This yields approx: \( \dot{U}_{\text{crest}}/\dot{U}_{\text{trough}} = 1.1 \) for relatively high wind waves (\( H/h = 0.2 \), \( h = 10 \text{ m}, \ T = 7 \text{ s} \)) and 1.7 for high swell waves (\( H/h = 0.2 \), \( h = 10 \text{ m}, \ T = 14 \text{ s} \)).

In the case of combined waves and current conditions, the near-bed component of the current in the wave direction should be added or subtracted, depending on the current direction (\( \dot{U}_{\text{crest}} + u_c \cos \phi; \dot{U}_{\text{trough}} - u_c \cos \phi \), with \( \phi = \text{angle between waves and current,} \ u_c = \text{steady current strength near the bed} \)).

Equation (8.5.13) is valid for steep rippled bed conditions and \( \psi_{\text{crest}} \leq 100 \). For \( \psi_{\text{crest}} \approx 200 \) the ripples will be washed out (flattened), resulting in a decreasing transport rate. For \( \psi_{\text{crest}} > 200 \) plane bed sheet flow under swell waves will be established with a net transport in the wave direction. In this latter case it is recommended to use Equation (8.5.14).

\[ \text{B. Transport due to asymmetric irregular wind waves} \]

The net total transport rate due to irregular wave motion in the ripple and sheet flow regime is directed in the wave propagation direction and can be predicted by:

\[ q_{w,\text{net}} = 0.000063 ((s-1) \ g)^{0.5} (d_{s0})^{1.5} \left[ \alpha_s (\psi_{\text{crest}} - \psi_{cr})^{1.7} - \alpha_s (\psi_{\text{trough}} - \psi_{cr})^{1.7} \right] \quad (8.5.14) \]

for rippled sand bed and sheet flow bed, \( d_{s0} \geq 200 \mu \text{m}, \ \psi_{\text{crest}} \geq 15 \)

in which:

- \( q_{w,\text{net}} \) = net total transport rate in wave direction (\( \text{m}^2/\text{s} \))
- \( \alpha_s \) = slope factor of Bagnold (-)
- \( s \) = relative density (-)
- \( d_{s0} \) = median particle size of bed material (m)
- \( \dot{U}_{\text{sig.crest}} \) = near-bed significant peak orbital velocity under wave crest (m/s)
- \( \dot{U}_{\text{sig.trough}} \) = near-bed significant peak orbital velocity under wave trough (m/s)
- \( \psi_{\text{crest}} \) = \( \frac{(\dot{U}_{\text{sig.crest}})^2}{(s-1) \ g \ d_{s0}} \) = mobility parameter (-)
- \( \psi_{\text{trough}} \) = \( \frac{(\dot{U}_{\text{sig.trough}})^2}{(s-1) \ g \ d_{s0}} \) = mobility parameter (-)
- \( \psi_{cr} \) = \( \frac{(\dot{U}_{cr})^2}{(s-1) \ g \ d_{s0}} \) = critical mobility parameter (-)
Equation (8.5.14) may also be used to compute the net transport for regular waves (swell) under sheet flow conditions.

In the case of combined wave and current conditions, the near-bed component of the current in the wave direction should be added or subtracted, depending on the current direction ($U_{\text{crest}} + u_c \cos \phi$; $U_{\text{rough}} - u_c \cos \phi$, with $\phi = \text{angle between waves and current}$, $u_c = \text{steady current strength near the bed}$).

Another method used by Van Rijn is based on the computation of the instantaneous bed-shear stress and the instantaneous bed-load transport rate (see Appendix A).

C. Transport due to weak currents in presence of waves

The total transport rate due to a weak near-bed current ($u < 0.2 \text{ m/s}$) in the presence of waves can be obtained by multiplying the sediment load according to Equation (8.3.4) with the near-bed current velocity ($u_{b,c}$) yielding:

$$q_c = 0.0006 \, d_{50} \, u_{b,c} \left( \Psi_{\text{crest}} - \Psi_{cr} \right)^{1.3} \quad (8.5.15)$$

in which:
- $q_c$ = total current-related transport rate ($\text{m}^2/\text{s}$)
- $d_{50}$ = median particle diameter of bed material ($\text{m}$)
- $u_{b,c}$ = current-velocity at edge of wave-boundary layer ($\text{m/s}$)
- $\Psi_{\text{crest}}$ = mobility parameter, see Equation (8.3.4)
- $\Psi_{cr}$ = critical mobility parameter, see Equation (8.3.4)

The near-bed current-velocity can be represented as:

$$u_{b,c} = (1 - p) \, u_{\text{nbr}} - p \, u_{br} + u_{\text{external}}$$

in which:
- $p$ = fraction of breaking waves (0 to 1)
- $u_{\text{nbr}}$ = mean near-bed current velocity due to non-breaking waves
- $u_{br}$ = mean near-bed current velocity due to breaking waves (undertow)
- $u_{\text{external}}$ = external mean near-bed current velocity (wind, tide etc.)

The near-bed velocity is defined at the edge of the wave boundary layer. Equation (8.5.15) is only valid for weak currents ($< 0.2 \text{ m/s}$) when vertical mixing by turbulence is not very important. The transport processes in strong currents are described in Chapter 9.

D. Transport due to bound long waves

The instantaneous (offshore-directed) transport can be represented as:

$$q_w(t) = \tilde{U}_L \, \tilde{L}_L \quad (8.5.16)$$

Assuming a sinusoidal low-frequency variation of the velocity and the sediment load, as follows:

$$\tilde{U}_L = \hat{U}_L \sin \omega t \quad \text{and} \quad \tilde{L}_L = \hat{L}_L \sin(\omega t)$$

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(sediment load out of phase with velocity), the instantaneous transport rate is:

$$q_w(t) = \hat{U}_L \hat{L}_L \sin^2(\omega t)$$  \hspace{1cm} (8.5.17)

The time-averaged value (in m$^2$/s) is:

$$q_w = \frac{\hat{U}_L \hat{L}_L}{2\pi \rho_s} \int_0^T \sin^2(\omega t)dt = \frac{\hat{U}_L \hat{L}_L}{2\rho_s}$$  \hspace{1cm} (8.5.18)

Assuming $\hat{L} = \frac{1}{2} L_t$ with $L_t$ according to Equation (8.3.4), it follows that:

$$q_w = -0.00015 \ d_{50} \ \hat{U}_L \ (\psi_{\text{crest}} - \psi_{\text{cr}})^{1.3}$$  \hspace{1cm} (8.5.19)

in which:

$q_w$ = offshore-directed net transport rate due to bound long waves (m$^2$/s)
$\hat{U}_L$ = peak value of near-bed bound long wave velocity (m/s)
$\psi_{\text{crest}}$ = mobility parameter, see Equation (8.3.4)

7. Ribberink-Al Salem (1992)

Based on analysis of measured transport rates in asymmetric waves, they found:

$$q_w(t) = 5 \ w_s \ d_{50} \ \left(\frac{u_\sigma(t)}{w_s}\right)^3$$  \hspace{1cm} (8.5.20)

in which: $u_\sigma(t) = (0.5 f_w)^{0.5} U(t) = instantaneous \ grain \ bed-shear \ velocity$

8. Comparison of computed and measured transport rates

Ribberink and Al-Salem (1991, 1992) performed experiments in a wave tunnel to determine the net transport rate over a wave cycle (see Tables 8.5 and 8.6). Irregular and regular asymmetric oscillatory flow was generated. The bed consisted of sand with $d_{50} = 210 \ \mu m$ and $d_{90} = 320 \ \mu m$. The measured net transport rates were compared with the computed values according to various methods.

The formula of Madsen and Grant (using $k_s = d_{50}$) was found to overpredict (factor 5 to 8) the measured net transport rate, especially in the high velocity regime. The velocity power of 6 was found to be much too large.

The formula of Bailard-Bagnold (using $k_s = d_{50}$) was found to give very good results for sheet flow conditions. The predicted values were within a factor of 2 of the measured values. Using a $k_s$-value based on the method of Wilson (Eq. 6.3.4) resulted in a 50%-overprediction. The formula of Sawamoto and Yamashita (using $k_s = d_{50}$) also gave good results (within factor 2) for sheet flow conditions.

8.5.4 Influence of bed-slope on bed-load transport

King (1991) studied the influence of bed slope on the bed-load transport ($d_{50} = 443 \ \mu m$ and 1081 \ $\mu m$) in oscillatory flow. Five experiments in a wave tunnel were performed with sloping bed. A run was made by moving the water through a half cycle of sinusoidal motion (from zero velocity to maximum velocity and back to zero). The transport rate was obtained from the amount of sand accumulated in a trap at the end of the bed. The bed-load transport was found to increase by about 20% when the slope was changed from 0° to 5.5° (downslope).
Analysis of the results showed that the observed transport rates can be represented, as:

\[ q_{w,\text{slope}} = \alpha_s q_{w,0} \]  \hspace{1cm} (8.5.21)

in which:

- \( q_{w,\text{slope}} \) = bed-load transport in half cycle on sloping bed
- \( q_{w,0} \) = bed-load transport in half cycle on horizontal bed
- \( \alpha_s \) = \( \frac{\tan \gamma}{(\tan \gamma \pm \tan \beta)} \) = slope factor of Bagnold
  
  (+ for upsloping flow, — for downsloping flow, see Equation 7.2.49)
- \( \gamma \) = dynamic friction angle (\( \approx 30^\circ \))

The \( \gamma \)-value was used as a calibration parameter. A value of \( \gamma = 30^\circ \) gave the best agreement between measured and computed values.

Equation (8.5.21) yields \( \alpha_s = 1.2 \) for flow on a downsloping bed of 5.5\(^\circ\).

The experimental results of King for oscillatory flow over a sloping bed are in agreement with those for unidirectional flow (see Equation 7.2.49).

### 8.6 Examples and problems

1. Waves are propagating normal to the coastline. The water depth is \( h = 8 \) m. The significant wave height is \( H_s = 2 \) m; the peak wave period is \( T_p = 7 \) s. The bed consists of sand with \( d_{50} = 250 \) \( \mu \)m, \( d_{90} = 350 \) \( \mu \)m. The water temperature is \( T_e = 10^\circ \text{C} \), \( v = 1.3 \times 10^{-6} \) m\(^2\)/s. Other data: \( \rho = 1025 \) kg/m\(^3\), \( \rho_s = 2650 \) kg/m\(^3\).

Compute the time-averaged concentrations at \( z = 0.05, 0.15, 0.2, 0.5 \) and 1 m above the bed according to the method of Van Rijn (take \( \delta_s = 3 \Delta_t \)).

**Solution:**

- Relative density \( s = 2.58 \)
- Particle size parameter \( D_* = 5.2 \)
- Critical bed-shear stress \( \tau_{b,cr} = 0.2 \) N/m\(^2\) (Figure 4.1.5)
- Wave length \( L = 55 \) m
- Peak orbital velocity near bed \( \tilde{U}_\delta = \frac{\pi H_s}{T_p \sinh(2\pi h/L)} = 0.86 \) m/s
- Peak orbital excursion near bed \( \tilde{A}_\delta = (T_p/2\pi) \tilde{U}_\delta = 0.96 \) m
- Mobility parameter \( \psi = \frac{(\tilde{U}_\delta)^2}{(s-1)g d_{50}} = 190 \)
- Ripple height, Equation (5.4.6) \( \Delta_r = 2.8 \times 10^{-3} (250 - \psi)^5 \tilde{A}_\delta = 0.09921 \) m (almost sheet flow)
- Ripple steepness, Equation (5.4.7) \( \Delta_r/\lambda_r = 2.7 \times 10^{-7} (250 - \psi)^{1.5} = 0.0075 \)
- Ripple length \( \lambda_r = 0.028 \) m
- Bed roughness (see Section 6.3.1) \( k_{s,w} = 3d_{50} + 3\Delta_r = 0.0017 \) m
  
  Take minimum value \( k_{s,w} = 0.01 \) m
- Friction factor \( f_w = \exp[-6.52(\tilde{A}_\delta/k_{s,w})^{-0.19}] = 0.022 \)
- Bed-shear stress \( \tau_{b,w} = 0.25 \rho f_w (\tilde{U}_\delta)^2 = 4.17 \) N/m\(^2\)
- Efficiency factor \( \mu_{w,a} = 0.6/D_* = 0.115 \)

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Bed-shear stress parameter: \[ T_s = \frac{\mu_{w,a} \tau_{b,w} - \tau_{b,cr}}{\tau_{b,cr}} = 1.4 \]

Reference concentration: \[ a = \frac{1}{2} \Delta_r = 0.0001 \text{ m} \]
Take minimum value \( a = 0.01 \text{ m} \)

\[ c_s = 0.015 \rho_s \frac{d_{50}}{a} \left( \frac{T_s}{D_{0.3}} \right)^{1.5} = 1.01 \text{ kg/m}^3 \]

Mixing layer thickness: \[ \delta_s = 3\Delta_i = 0.00063 \text{ m}, \]
Take minimum value \( \delta_{s,\text{min}} = 0.05 \text{ m} \)

Mixing coefficients: \[ \varepsilon_{s,w,\text{bed}} = 0.004 D_s \hat{U}_\delta \delta_s = 0.0009 \text{ m}^2/\text{s} \]
\[ \varepsilon_{s,w,\text{max}} = 0.035 \frac{H_s h}{T_\nu} = 0.08 \text{ m}^2/\text{s} \]

Suspended sediment size: \[ d_s = 0.8 d_{50} = 200 \text{ \mu m} \]

Particle fall velocity: \[ w_s = 0.022 \text{ m/s (Figure 3.2.6)} \]

Coefficients, Equation (8.4.34):
\[ \gamma = 178 \]
\[ \alpha = 0.376 \]
\[ \beta = 1.1 \]
\[ \eta = 0.0078 \]

Concentrations:
\[ z = a = 0.01 \text{ m} \quad c_s = 1.01 \text{ kg/m}^3 \]
\[ z = 0.05 \text{ m} \quad c = 0.38 \text{ kg/m}^3 \]
\[ z = 0.10 \text{ m} \quad c = 0.17 \text{ kg/m}^3 \]
\[ z = 0.15 \text{ m} \quad c = 0.11 \text{ kg/m}^3 \]
\[ z = 0.20 \text{ m} \quad c = 0.08 \text{ kg/m}^3 \]
\[ z = 0.50 \text{ m} \quad c = 0.03 \text{ kg/m}^3 \]
\[ z = 1.00 \text{ m} \quad c = 0.01 \text{ kg/m}^3 \]

2. Same problem as 1, with \( H_s = 1 \text{ m} \)

Solution:
\[ z = a = 0.0225 \text{ m} \quad c_s = 1.5 \text{ kg/m}^3 \]
\[ z = 0.05 \text{ m} \quad c = 0.91 \text{ kg/m}^3 \]
\[ z = 0.10 \text{ m} \quad c = 0.36 \text{ kg/m}^3 \]
\[ z = 0.15 \text{ m} \quad c = 0.15 \text{ kg/m}^3 \]
\[ z = 0.20 \text{ m} \quad c = 0.07 \text{ kg/m}^3 \]
\[ z = 0.50 \text{ m} \quad c = 0.01 \text{ kg/m}^3 \]
\[ z = 1.00 \text{ m} \quad c = 0.002 \text{ kg/m}^3 \]

3. Waves are propagating normal to the shore in a water depth of \( h = 9 \text{ m} \).
The horizontal bed consists of sand with \( d_{50} = 250 \text{ \mu m} \), \( d_{90} = 350 \text{ \mu m} \). The water temperature is 20°C. Other data: \( \rho = 1025 \text{ kg/m}^3 \), \( \rho_s = 2650 \text{ kg/m}^3 \). The fall velocity is \( w_s = 0.034 \text{ m/s} \). The onshore and offshore peak velocities near the bed are:

- case 1 (swell waves): \( \hat{U}_{\text{cr}} = 0.47 \text{ m/s}, \quad \hat{U}_{\text{off}} = 0.40 \text{ m/s}, \quad T = 7 \text{ s} \)
- case 2 (wind waves): \( \hat{U}_{\text{on}} = 1.75 \text{ m/s}, \quad \hat{U}_{\text{off}} = 1.55 \text{ m/s}, \quad T_p = 7 \text{ s} \)
What is the net wave-induced transport rate for both cases using the formulae of Madsen-Grant, Hallermeier and Van Rijn?

Solution:

**Madsen and Grant, Equation (8.5.7)**

Case 1:
- Orbital excursion: \( \Delta_{on} = 0.524 \text{ m}, \quad \Delta_{off} = 0.446 \text{ m} \)
- Friction coefficients: \( f_{w,on} = 0.0084, \quad f_{w,off} = 0.0087 \)
- Mobility parameters: \( \theta_{on} = 0.238, \quad \theta_{off} = 0.179 \)
- Transport rates: 
  - \( q_{w,on} = 12.5 \, w_s \, d_{so} (\theta_{on})^3 = 1.4 \times 10^{-6} \text{ m}^3/\text{s} \)
  - \( q_{w,off} = 12.5 \, w_s \, d_{so} (\theta_{off})^3 = -0.6 \times 10^{-6} \text{ m}^3/\text{s} \)
  - \( q_{w,net} = (q_{w,on} + q_{w,off})/2 = 0.4 \times 10^{-6} \text{ m}^3/\text{s} \) (onshore)

Case 2 (similar method):
  - \( q_{w,on} = 1.8 \times 10^{-3} \text{ m}^3/\text{s} \)
  - \( q_{w,off} = -0.9 \times 10^{-3} \text{ m}^3/\text{s} \)
  - \( q_{w,net} = 0.45 \times 10^{-3} \text{ m}^3/\text{s} \) (onshore)

**Hallermeier, Equation (8.5.10)**

Case 1:
- Mobility parameter: \( \psi_{on} = 57 \)
  - \( \psi_{off} = 41 \)
- Transport rates: 
  - \( q_{w,on} = \omega \, d_{so}^2 (0.1 \psi_{on})^{1.5} = 7.6 \times 10^{-7} \text{ m}^3/\text{s} \)
  - \( q_{w,off} = \omega \, d_{so}^2 (0.1 \psi_{off})^{1.5} = -4.7 \times 10^{-7} \text{ m}^3/\text{s} \)
  - \( q_{w,net} = 1.45 \times 10^{-7} \text{ m}^3/\text{s} \) (onshore)

Case 2 (similar method):
  - \( \psi_{on} = 797 \)
  - \( \psi_{off} = 618 \)
  - \( q_{w,on} = 4.0 \times 10^{-5} \text{ m}^3/\text{s} \)
  - \( q_{w,off} = -2.7 \times 10^{-5} \text{ m}^3/\text{s} \)
  - \( q_{w,net} = 0.65 \times 10^{-5} \text{ m}^3/\text{s} \) (onshore)

**Van Rijn**

Case 1, Equation (8.5.13): \( \hat{U}_{cr} = 0.23 \text{ m/s, Figure 4.2.1} \)

\[
\psi_{cr} = \frac{(\hat{U}_{cr})^2}{(s-1)g \, d_{50}} = 14
\]

\[
\psi_{on} = 57 \quad \psi_{off} = 41 \\
q_{w,net} = -3.3 \times 10^{-6} \text{ m}^3/\text{s} \) (offshore)

Case 2, Equation (8.5.14): \( \theta_{cr} = 14 \)

\[
\psi_{on} = 797 \quad \psi_{off} = 618 \\
q_{w,net} = 3.0 \times 10^{-5} \text{ m}^3/\text{s} \) (onshore)
The results are summarized in Table 8.8. Large differences can be observed both in direction and magnitude.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Net transport rate (m²/s)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1 (swell waves)</td>
<td>Case 2 (wind waves)</td>
<td></td>
</tr>
<tr>
<td>Madsen-Grant</td>
<td>4 10⁻⁷ (onshore)</td>
<td>45 10⁻⁵ (onshore)</td>
<td></td>
</tr>
<tr>
<td>Hallermeier</td>
<td>1.45 10⁻⁷ (onshore)</td>
<td>0.65 10⁻⁵ (onshore)</td>
<td></td>
</tr>
<tr>
<td>Van Rijn</td>
<td>33 10⁻⁷ (offshore)</td>
<td>3.0 10⁻³  (onshore)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 8.8 Computed net transport rates**

4. Same example as 3.
What is the net wave-induced transport rate for case 2, using the formula of Bailard-Bagnold (divide each half period in 10 steps and assume sinusoidal distribution of instantaneous velocities; use \( e_b = 0.1 \) and \( e_s = 0.02 \)) and the formula of Sawamoto and Yamashita?

Solution:

Bailard-Bagnold, \( q_{w, \text{net}} = 11 \times 10^{-5} \text{ m}^3/\text{s} \) (onshore)
Sawamoto-Yamashita, \( q_{w, \text{net}} = 6.5 \times 10^{-5} \text{ m}^3/\text{s} \) (onshore)

5. Waves are propagating normal to the shore in a water depth of \( h = 8 \text{ m} \). The annual distribution of the significant wave height is given in Table 8.8 with \( p_l \) = percentage of occurrence. The horizontal bed consists of sand with \( d_m = 250 \mu\text{m} \), \( d_m = 350 \mu\text{m} \). The fall velocity is \( w_s = 0.034 \text{ m/s} \). The onshore and offshore peak velocities are: \( \dot{U}_{on} = 1.2 \dot{U}_{\text{mean}} \), \( \dot{U}_{off} = 0.8 \dot{U}_{\text{mean}} \) and \( \dot{U}_{\text{mean}} = \frac{1}{2} (\dot{U}_{on} + \dot{U}_{off}) \) according to linear wave theory. Other data: \( \rho = 1025 \text{ kg/m}^3 \), \( \rho_s = 2650 \text{ kg/m}^3 \), \( \psi_r = 14 \).

What is the annual transport rate according to the method of Van Rijn?

Solution:

The results are given in Table 8.9. The annual net transport rate = 1.37 \times 10^{-6} \text{ x 24 x 3600 x 365} = 43 \text{ m}^3 \) (onshore), negative sign means offshore.

<table>
<thead>
<tr>
<th>Wave type</th>
<th>( p_l )</th>
<th>( \Delta H_l )</th>
<th>( H_l )</th>
<th>( T )</th>
<th>( L )</th>
<th>( \dot{U}_{\text{mean}} )</th>
<th>( \dot{U}_{on} )</th>
<th>( \dot{U}_{off} )</th>
<th>( q_{w, \text{net}} )</th>
<th>( P_l )</th>
<th>( q_{w, \text{net}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swell waves</td>
<td>0.10</td>
<td>0.25 - 0.75</td>
<td>0.5</td>
<td>10</td>
<td>80</td>
<td>0.23</td>
<td>0.28</td>
<td>0.18</td>
<td>-0.2 \times 10^4</td>
<td>-0.02</td>
<td>10^4</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.75 - 1.25</td>
<td>1.0</td>
<td>10</td>
<td>100</td>
<td>0.50</td>
<td>0.60</td>
<td>0.40</td>
<td>-14.0 \times 10^4</td>
<td>-0.72</td>
<td>10^4</td>
</tr>
<tr>
<td>Wind waves</td>
<td>0.24</td>
<td>0.25 - 0.75</td>
<td>0.5</td>
<td>4</td>
<td>26</td>
<td>0.12</td>
<td>0.14</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>0.75 - 1.25</td>
<td>1.0</td>
<td>5</td>
<td>35</td>
<td>0.32</td>
<td>0.38</td>
<td>0.25</td>
<td>0.2 \times 10^4</td>
<td>0.08</td>
<td>10^4</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
<td>1.25 - 1.75</td>
<td>1.5</td>
<td>6</td>
<td>45</td>
<td>0.58</td>
<td>0.70</td>
<td>0.46</td>
<td>2.5 \times 10^4</td>
<td>0.42</td>
<td>10^4</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>1.75 - 2.25</td>
<td>2.0</td>
<td>7</td>
<td>53</td>
<td>0.82</td>
<td>0.98</td>
<td>0.66</td>
<td>8.6 \times 10^4</td>
<td>0.43</td>
<td>10^4</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>2.25 - 2.75</td>
<td>2.5</td>
<td>8</td>
<td>65</td>
<td>1.15</td>
<td>1.38</td>
<td>0.92</td>
<td>27.9 \times 10^4</td>
<td>0.56</td>
<td>10^4</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>2.75 - 3.25</td>
<td>3.0</td>
<td>9</td>
<td>75</td>
<td>1.45</td>
<td>1.74</td>
<td>1.16</td>
<td>60.5 \times 10^4</td>
<td>0.62</td>
<td>10^4</td>
</tr>
</tbody>
</table>

**Table 8.9 Computation annual transport rate due to wave asymmetry**
6. Same example as 5, with $\bar{U}_{on} = 1.1 \bar{U}_{mean}$ and $\bar{U}_{off} = 0.9 \bar{U}_{mean}$.

Solution: Annual net transport = 22 m² (onshore)

7. Waves are propagating normal to the shore in a water depth of $h = 8$ m. The annual distribution of the significant wave height of the wind waves is given in Table 8.10. Bound long waves are generated by the wave groups. The peak long wave velocities ($\bar{U}_L$) are given in Table 8.10. The peak short wave velocities are given by second order Stokes theory (Eq. 2.3.4). The horizontal bed consists of sand with $d_{50} = 250$ μm, $d_{90} = 350$ μm. The fall velocity is $w_f = 0.034$ m/s. Other data: $\rho = 1025$ kg/m³, $\rho_s = 2650$ kg/m³, $\bar{U}_{cr} = 0.25$ m/s.

What is the direction and magnitude of the annual cross-shore transport due to bound long waves?

Solution:

The results, based on Equation (8.5.19), are given in Table 8.10. The offshore-directed transport rate is about 20 m² per year (negative sign means offshore).

<table>
<thead>
<tr>
<th>Wave type</th>
<th>$P_i$</th>
<th>$\Delta H_{i,j}$</th>
<th>$H_{i,j}$</th>
<th>$T_p$</th>
<th>$\bar{U}_L$</th>
<th>$\bar{U}_{on}$</th>
<th>$\bar{U}_{off}$</th>
<th>$\bar{U}_L$</th>
<th>$q_w,off$</th>
<th>$P_i q_w,off$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind waves</td>
<td>0.07</td>
<td>0.25</td>
<td>0.25</td>
<td>4</td>
<td>0.06</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.34</td>
<td>0.75</td>
<td>0.75</td>
<td>4</td>
<td>0.18</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.23</td>
<td>1.25</td>
<td>1.25</td>
<td>5</td>
<td>0.38</td>
<td>0.38</td>
<td>0.05</td>
<td>-0.1 10⁻⁶</td>
<td>-0.02 10⁻⁴</td>
<td>-0.02 10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>0.13</td>
<td>1.75</td>
<td>1.75</td>
<td>6</td>
<td>0.72</td>
<td>0.66</td>
<td>0.1</td>
<td>-0.6 10⁻⁶</td>
<td>-0.08 10⁻⁴</td>
<td>-0.08 10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>2.25</td>
<td>2.25</td>
<td>6</td>
<td>0.94</td>
<td>0.84</td>
<td>0.2</td>
<td>-1.9 10⁻⁶</td>
<td>-0.13 10⁻⁴</td>
<td>-0.13 10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>0.03</td>
<td>2.75</td>
<td>2.75</td>
<td>7</td>
<td>1.30</td>
<td>1.06</td>
<td>0.5</td>
<td>-6.6 10⁻⁶</td>
<td>-0.20 10⁻⁴</td>
<td>-0.20 10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>3.25</td>
<td>3.25</td>
<td>7</td>
<td>1.56</td>
<td>1.22</td>
<td>1.0</td>
<td>-18.8 10⁻⁶</td>
<td>-0.19 10⁻⁴</td>
<td>-0.19 10⁻⁴</td>
</tr>
</tbody>
</table>

| Total     |      |                 |         |      |             |               |               | -0.62 10⁻⁶   |          | -20 m²/yr  |

Table 8.10 Computed annual transport rates related to bound long waves
REFERENCES


REFERENCES (continued)


REFERENCES (continued)


REFERENCES (continued)


8.76
REFERENCES (continued)


9 BED MATERIAL SUSPENSION AND TRANSPORT IN COMBINED WAVES AND CURRENTS

9.1 Introduction

In Chapter 8 it has been shown that wave motion can generate sediment suspensions with large concentrations in the near-bed region. When strong tide-induced, wind-induced or wave-induced currents are present, the vertical mixing due to turbulence will result in additional upward transport of particles yielding larger concentrations in the upper layers. The basic mechanism is the entrainment of particles by the stirring wave action and the transport of the particles by the current motion.

The transport of particles by the mean current velocities (in the presence of waves) is defined as the current-related transport rate. A detailed discussion of wave-related and current-related transport processes is given in Section 8.2.

In combined current and wave conditions the current-related transport usually is dominant and can be derived from the time-averaged variables.

Bijker (1967, 1971) presented a detailed method (based on time-averaged variables) to compute the local transport rate for combined current and (breaking) wave conditions following the approach of Einstein (1950).

Another formula-type approach was proposed by Grant and Madsen (1976) and Van Rijn (1989). More complicated mathematical models based on instantaneous variables have been developed by Fredsøe et al. (1985) for non-breaking waves and by Deigaard et al. (1986) for breaking waves in the surf zone.

Simple prediction methods have been proposed to compute the width-integrated longshore sediment transport (CERC, Kamphuis et al, 1986). These integral methods are based on the longshore component of the wave energy flux at the breaker line.

The following subjects are presented in this chapter:
• analysis of measured concentrations and transport rates,
• computation of concentrations,
• computation of current-related transport rates in non-breaking waves,
• computation of current-related transport rates in breaking waves.

9.2 Analysis of measured concentration profiles and transport rates

9.2.1 Time-averaged concentration profiles

Time-averaged concentrations (averaged over many waves) are most relevant for current-related transport processes (Section 8.2).

Experimental research in flumes and basins has been performed by various researchers (Bosman, 1982; Nieuwjaar and Van der Kaaij, 1987; Nap and Van Kampen, 1988; Havinga, 1992). Field data in non-breaking waves with a current over an intertidal flat have been collected by Van Vesse (see Van Rijn, 1987). Field data in the surf zone with breaking waves and longshore currents have been collected by Jaffe et al. (1984), by Van Rijn and Kroon (1992), by Kroon and Van Rijn (1993) and by others.

1. Concentrations in non-breaking waves with a current over a rippled bed

Figure 9.2.1 shows time-averaged concentration profiles in a current with following and opposing waves (flume) for a sediment bed of 200 μm and a water depth of h = 0.5 m (Nieuwjaar and Van der Kaaij, 1987). Similar results were obtained for a sediment bed of 100 μm (Nap and Van Kampen, 1988). In all tests the bed was covered with ripples.
Irregular waves were generated. Detailed results were presented by Van Rijn et al. (1993).

Based on these results, the following phenomena can be identified:
- rapid decrease of concentrations from the bed upwards in case of waves alone ($\bar{u} = 0 \text{ m/s}$),
- transport of sediment to upper layers by mixing processes in case of combined waves and currents,
- mixing effects are small in case of a weak current ($\bar{u} = 0.1 \text{ m/s}$), and large in case of a strong current ($\bar{u} = 0.4 \text{ m/s}$),
- influence of current direction (following or opposing) on concentration profile is relatively small,
- influence of current velocity on the near-bed concentrations, which are in the range of 0.1 to 10 kg/m$^3$, is only significant in case of small waves.

*Figure 9.2.1 Concentration profiles in non-breaking waves and currents, $d_{50} = 200 \mu m$, Van Rijn et al. (1993)*
Havinga (1992) measured time-averaged concentration profiles in combined current and wave conditions for a sediment bed of 100 μm. The experiments were performed in a basin. The water depth was 0.4 m. Irregular non-breaking waves were generated. The angle between the current and wave direction was φ = 60°, 90° and 120°. Figures 9.2.2 to 9.2.5 present time-averaged concentration profiles showing the influence of wave height, current strength and angle between current and waves.

The following phenomena can be observed:
- relatively strong increase of the concentrations with wave height (factor 5 to 10),
- transport of sediment to upper layers by mixing processes in combined currents and waves,
- mixing effects are small for a weak current (0.1 m/s) and large for a strong current (0.3 m/s),
- near-bed concentrations are hardly influenced by the current velocity,
- influence of the wave direction on the concentrations is relatively small (factor 2); concentrations are largest for φ = 90°.

Van Vessem measured concentrations (see Van Rijn, 1989) at an intertidal flat in the Eastern Scheldt estuary, The Netherlands. The bed material had a median particle size of about 150 μm. The bed was covered with small-scale ripples (Δr ≈ 0.03 m). For a relative wave height of Hs/h ≈ 0.17 the sediment concentrations were extremely small, probably because the bed-shear stresses were just beyond the critical values for initiation of suspension. The sediment concentrations showed an increase of more than a factor 10 when the wave height increased by a factor 2 (Hs/h from 0.17 to 0.29). The generation of wave and current-induced mixing effects was also observed.

2. Concentrations in surf zone with breaking waves and longshore currents


The results of Jaffe et al (1984) are presented in Fig. 9.2.6, showing concentrations in the range of about 0.5 to 2 kg/m³ in the lowest measuring point (≈ 0.13 m) above the bed. The largest near-bed concentrations have been measured in stations 3 and 7, where the wave height is largest. The lowest concentrations occur in stations 1 and 2 (longshore trough area) where the wave height is relatively small. The largest concentrations over the depth can be observed in station 3 which is close to the crest of the longshore bar where strong wave breaking occurs (Hs/h = 0.55). The vertical concentration gradients are largest in station 7 just seaward of the longshore bar, where wave breaking is less intensive. The concentration gradients are smallest (which means a rather uniform profile) in stations 1 and 2 where the current-related mixing probably is largest (longshore trough area) and also in station 3 where strong wave-related mixing is important (longshore bar crest). Information of the bed form has not been reported by Jaffe et al. (1984).

Figures 8.3.13 and also 9.3.5 show measured time-averaged concentrations in the surf and swash zone of the Dutch coast near Egmond. The longshore currents were in the range of 0.1 to 0.8 m/s. The median bed material size was about 300 μm. The bed was covered with flat small-scale undulations (height = 0.01 m) Spilling and plunging breaking waves did occur. The near-bed concentrations are in the range of 0.1 to 1.0 kg/m³ for Hs/h = 0.4 to 0.6 (spilling breakers) and in the range of 1 to 10 kg/m³ for Hs/h = 0.6 to 1.1 (plunging breaking waves). The concentration profiles become more uniform for increasing relative wave heights.
Figure 9.2.2 Concentration profiles in non-breaking waves and currents, $d_{50} = 100$ µm. Influence of wave height, Havinga (1992)
Figure 9.2.3  Concentration profiles in non-breaking waves and currents, $d_{50} = 100 \, \mu m$. 
Influence of current strength, Havinga (1992)
Figure 9.2.4 Concentration profiles in non-breaking waves and currents, $d_{so} = 100 \ \mu m$. Influence of current-wave angle, Havinga (1992)
Figure 9.2.5 Concentration profiles in non-breaking waves and currents, $d_{50} = 100 \mu m$. Influence of current-wave angle, Havinga (1992)
Figure 9.2.6 Concentration profiles in surf zone, USA (Jaffe et al., 1984)

Summarizing, the following phenomena have been observed in the surf zone:

- near-bed concentrations in the range of 0.1 to 1 kg/m³ for relative wave heights of $H_r/h - 0.4$ to 0.6 (spilling breakers),
- near-bed concentrations in the range of 1 to 10 kg/m³ for relative wave heights of $H_r/h - 0.6$ to 1.1 (plunging breakers),
- relatively large vertical mixing due to wave-related and current-related mixing in the longshore trough area and longshore bar area.
The main cause for the relatively small near-bed concentrations (0.1 to 1 kg/m²) in spilling breaking waves is the absence of the typical wave-induced bed ripples and the associated eddies. Wave-induced ripples are strong stirring mechanisms yielding large near-bed concentrations (1 to 10 kg/m³) at relatively small wave heights, as shown by laboratory experiments (Figs. 9.2.1 and 9.2.5)

9.2.2 Sediment transport rates

In Section 8.2 it has been shown that the total sediment transport in combined waves and currents can be divided into a:
- current-related sediment transport rate \( q_c \),
- wave-related sediment transport rate \( q_w \).

The current-related sediment transport is defined as the transport of particles by the time-averaged current velocities, the latter being modified by the wave motion. The wave-related sediment transport is defined as the transport of particles by the oscillating fluid motions (orbital velocities).

Quantitative information of measured transport rates is presented by Nieuwjaar and Van der Kaaij (1987), by Nap and Van Kampen (1988) and by Havinga (1992) for non-breaking waves combined with a current under an angle of \( \phi = 60^\circ, 90^\circ \) and \( 120^\circ \). Kroon and Van Rijn (1993) analyzed cross-shore and longshore transport rates measured in the surf zone and in the swash zone of the Dutch coast near Egmond.

1. Current-related transport rates

Non-breaking waves

Figure 9.2.7 shows current-related transport rates for the 100 \( \mu \)m and 200 \( \mu \)m-experiments in a laboratory flume (Nieuwjaar and Van der Kaaij, 1987; Nap and Van Kampen, 1988). Currents with following and opposing waves (\( \phi = 0^\circ \) and \( 180^\circ \)) were generated. These results and those of Havinga (1992) were analyzed to determine the influence of the wave height and depth-averaged velocity, yielding:
- relatively large increase of the transport rate with increasing wave height for a weak current: \( q_{c} \approx H^{3.5} \),
- relatively small increase of the transport rate with increasing wave height for a strong current: \( q_{c} \approx H^{1.2} \),
- relatively large increase of the transport rate with increasing depth-averaged velocity for low waves: \( q_{c} \approx \bar{u}^{2} \),
- relatively small increase of the transport rate with increasing depth-averaged velocity for high waves: \( q_{c} \approx \bar{u}^{1.2} \).

Nieuwjaar-Van der Kaaij (1987) and Nap-Van Kampen (1988) did not find a clear influence of the wave-current angles (\( \phi = 0^\circ, 180^\circ \)), on the transport rate. The experimental results of Havinga (1992) however show a significant influence of the current-wave angle \( \phi \) on the transport rate (100 \( \mu \)m sediment). Figure 9.2.8 shows the current-related transport rate \( q_{c} \) as a function of the current-wave angle (\( \phi \)) for a depth-averaged velocity of \( \bar{u} = 0.1 \) m/s and \( \bar{u} = 0.3 \) m/s. For all wave conditions (\( \bar{U}_{b} = 0.15 \) to \( 0.3 \) m/s) the transport rate is maximum for \( \phi = 90^\circ \) (waves perpendicular to current). The effect is most pronounced for relatively high waves (\( \bar{U}_{b} = 0.3 \) m/s).

It is not quite clear why the transport rates are maximum for \( \phi = 90^\circ \). It may be related to the influence of the waves on the velocity profiles which is also maximum for \( \phi = 90^\circ \); apparent roughness is maximum for \( \phi = 90^\circ \) (Section 2.4.3. and Fig. 9.2.9). It may also be related to the geometry of the bed ripples.

9.9
**Figure 9.2.7** Measured current-related transport rate in currents with following and opposing waves ($\phi = 0^\circ$, $180^\circ$), Van Rijn et al. (1993)

**Figure 9.2.8** Influence of current-wave angle on current related transport rate, Havinga (1992)
Breaking waves

Kroon and Van Rijn (1993) measured instantaneous and time-averaged velocities and concentrations in the surf zone and in the swash zone near Egmond, The Netherlands. The bed material consisted of sand with $d_{50} = 300 \mu m$. The water depths were in the range of 0.5 to 1.5 m. The longshore currents were ranging from 0.1 to 1 m/s. The mean cross-shore currents were ranging from 0.1 to 0.5 m/s and directed offshore.

Figure 9.2.10 shows the cross-shore suspended sediment fluxes at $z = 0.05 \pm 0.02 m$ above the bed as a function of the relative wave height $H_s/h$. As can be observed, the current-related suspended sediment fluxes are dominant because the mean cross-shore current is relatively strong in the surf zone (offshore directed). The wave-related suspended sediment fluxes are quite small at $z = 0.05 m$ above the bed. The wave-related fluxes will become more important closer to the bed.

The depth-integrated current-related suspended sediment transport rates ($q_c$) in cross-shore and in longshore direction are shown in Figure 9.2.11. The transport rates show an increasing trend for increasing values of $H_s/h$, particularly for plunging breaking waves ($H_s/h > 0.6$).

To demonstrate the relative importance of the wave-related transport ($q_w$) close to the bed, Equation (8.5.14) was used to compute this value based on the measured peak onshore and offshore orbital velocities and this value was compared with the measured current-related suspended sediment transport ($q_c$). Figure 9.2.12 shows the ratio of the computed wave-related transport (onshore) and the measured current-related suspended transport (offshore) as a function of the ratio $H_s/h$. The wave-related transport in the near-bed layer may be quite important when the return current velocity ($u$) is smaller than the orbital velocity difference ($\dot{U}_{on} - \dot{U}_{off}$). In that case the net transport is directed onshore. The net transport is zero when $q_w$ is equal and opposite to $q_c$. A clear influence of the relative wave height ($H_s/h$) cannot be detected.
Figure 9.2.10 Cross-shore sediment fluxes at $z = 0.05$ m above the bed, surfzone Egmond, The Netherlands

Figure 9.2.11 Cross-shore and longshore depth-integrated current-related suspended sediment transport, surfzone Egmond, The Netherlands
2. Wave-related transport rate

Various researchers have observed that the wave-related transport can be directed against the wave direction, even when there is a weak following current. The results of Nap and Van Kampen (1988) provide quantitative information of these phenomena based on special sand balance experiments. The experiments consisted of measuring the sediment mass eroded from the bed by combined current and wave action between the flume entrance and the measurement location (= 15 m downstream from the entrance). Since there was no sediment feed at the flume entrance, the total time-averaged transport rate \( q_b \) passing the measurement location is equal to the eroded mass divided by the elapsed time. Simultaneously, time-averaged current velocity and sand concentration profiles were measured at the measurement location, yielding the time-averaged current-related transport rate \( (q_{b,c} = \int u c d z) \). Three experiments have been executed: two experiments with a following current (T15,10 and T15,20) and one experiment with an opposing current (T15,-10). The experimental results are presented in Fig. 9.2.13 showing the ratio \( q_b/q_{b,c} \) as a function of \( U_y/u \). The most important results are:

- the total transport \( q_b \) is smaller than the current-related transport \( q_{b,c} \) in case of a following current which means that the wave-related transport \( q_{w,c} \) is directed against the wave and current direction,

- the wave-related transport rate (against the current and wave direction) is largest in case of a weak following current, \( q_{w,c}/q_{b,c} \approx 0.4 \) for \( U_y/u \approx 2.5 \) and \( q_{w,c}/q_{b,c} \approx 0.25 \) for \( U_y/u \approx 1.5 \),

- the wave-related transport seems to be negligible small in case of an opposing current.
The underlying mechanism of wave-related transport against the wave direction is strongly related to the near-bed eddy motions (see Fig. 9.2.14). When the wave crest superimposed on a following current passes a ripple, the near-bed velocities will be relatively large and a strong eddy with high sand concentrations is generated at the ripple back. When the wave trough passes the ripple, the eddy is lifted and carried against the wave propagation direction, whereas the eddy also desintegrates at higher levels. Some part of the sediment particles will fall back on the bed during this process. Simultaneously, another eddy is generated at the ripple front when the wave trough passes. During this phase of the wave cycle the near-bed velocities are relatively small and thus the eddy velocities and the corresponding sand concentrations in the eddy are also relatively small. Thus, near the bed there is a process of relatively large concentrations transported by relatively small velocities against the wave direction and a process of relatively small concentrations transported in the wave direction by relatively large velocities. Quantitative estimates of both processes indicate a net wave-related transport against the wave direction in case of a following current (Nap and Van Kampen, 1988).

Finally, the importance of the wave-related transport \( (q_{w,w}) \) in or against the current direction is discussed. Although only three experiments were carried out, the wave-related transport seems to be of less importance for \( \tilde{U}_w / \tilde{U} < 2 \). Neglecting the \( q_{w,w} \) component with respect to the \( q_{u,w} \)-component results in a systematic error of about 30%, which is hardly significant in sediment transport predictions.
WAVE CREST PASSES ($u > 0$)

WAVE TROUGH PASSES ($u < 0$)

Figure 9.2.14  Eddy motions over a rippled bed
9.3 Computation of time-averaged concentration profiles

9.3.1 Methods

Various models to compute the time-averaged concentration profiles are available in the literature. The Bijker model (1967, 1971) is based on the time-averaged convection-diffusion Equation, (8.4.1). Other models such as that of Fredsøe et al. (1985) and that of Deigaard et al. (1986) are based on the instantaneous convection-diffusion Equation, (8.4.2). These latter models are only valid for plane bed conditions with sheet flow.


The concentration profile is described as:

\[
\frac{c}{c_a} = \left( \frac{a}{h-a} \right) \frac{h-z}{z} \exp \left( \frac{w_s}{w_{scw}} \right) \tag{9.3.1}
\]

\[
c_a = \frac{b \rho_s d_{50}}{6.34 a} \exp \left[ - \frac{0.27 (\rho_s - \rho) g d_{50}}{\mu \tau_{b,scw}} \right] \tag{9.3.2}
\]

in which:

- \( c \) = time-averaged concentration (kg/m³)
- \( c_a \) = time-averaged reference concentration at \( z = a \) (kg/m³)
- \( a \) = \( k_s \) = reference level or thickness of bed-load layer (m)
- \( k_n \) = effective bed roughness height (m)
- \( h \) = water depth (m)
- \( \tau_{b,scw} = \tau_{b,w} + \tau_{b,c} \) = bed-shear stress (N/m²)
- \( \tau_{b,w} = (1/4) \rho f_w (\bar{U})^2 \) = wave-related bed-shear stress (N/m²)
- \( \tau_{b,c} = (1/8) \rho f_c \bar{V}_c^2 \) = current-related bed-shear stress (N/m²)
- \( \mu = [C/C']^{1.5} \) = efficiency factor (-)
- \( C = 18 \log(12h/k_n) \) = Overall Chézy-coefficient (m⁰.⁵/s)
- \( C' = 18 \log(12h/d_{50}) \) = Chézy-coefficient related to grains (m⁰.⁵/s)
- \( f_c = (8g)/C^2 \) = current-related friction factor (-)
- \( f_w = \exp[-6 + 5.2 (\bar{A}_b/k_d)^{-0.19}] \) = wave-related friction factor (-)
- \( w_s \) = particle fall velocity of suspended sediment (m/s)
- \( \kappa \) = Von Karman constant (= 0.4)
- \( u_{scw} = [\tau_{b,scw}/\rho]^{0.5} \) = bed-shear velocity (m/s)
- \( \rho \) = fluid density (kg/m³)
- \( \rho_s \) = sediment density (kg/m³)
- \( b \) = empirical coefficient (≈ 5)
- \( d_{50} \) = median particle size of bed material (m)
- \( \bar{A}_b \) = peak orbital excursion near the bed (based on \( H_{max} \)) (m)
- \( \bar{U}_s = \tau_{b,scw} \) = peak orbital velocity near the bed (based on \( H_{max} \)) (m/s)
- \( \bar{V}_c \) = depth-averaged value of current velocity vector (m/s)

2. Fredsøe et al. (1985)

The computation of the concentration profile in case of waves alone is given in Section 8.4.7. In case of a combination of waves and current (under an angle \( \phi \)) the instantaneous velocity profile is assumed to consist of two parts: a steady component \( u_c \) due to the mean current and an unsteady component \( U_{scw} \) due to the wave motion. The unsteady component is represented by potential flow theory outside the wave boundary layer and by a logarith-
mic velocity profile inside the wave boundary layer. The influence of the wave boundary layer on the mean current velocity is taken into account by an apparent roughness. The concentration profile is computed by numerical integration applying the convection-diffusion equation.

3. **Van Rijn**

The concentration profile can be obtained from numerical integration of the time-averaged convection-diffusion Equation (8.4.1) applying Equation (8.4.25) as reference concentration:

\[
\frac{dc}{dz} = -\frac{c \cdot w_{s,m}}{\varepsilon_{s,cw}}
\]

\[
c_s = 0.015 \rho_s \frac{d_{50}}{a} \frac{T_{w}^{1.5}}{D_{w}^{0.3}}
\]

in which:

- \( c \) = time-averaged concentration at height \( z \) above the bed (kg/m³)
- \( w_{s,m} = (1-c^2) w_s \) = particle fall velocity of suspended sediment in fluid-sediment mixture (m/s)
- \( w_s \) = particle fall velocity of suspended sediment in clear water (m/s)
- \( \varepsilon_{s,cw} = \left[ \varepsilon_{s,c}^2 + \varepsilon_{s,w}^2 \right]^{0.5} \) = sediment mixing coefficient in combined currents and waves (m²/s)
- \( \rho_s \) = sediment density (= 2650 kg/m³)
- \( a \) = reference level (m)
- \( d_{50} \) = median particle diameter of bed material (m)
- \( T_{w} \) = bed-shear stress parameter (-), see Appendix A
- \( D_{w} \) = particle parameter (-), see Appendix A

The sediment mixing coefficient in combined currents and waves is assumed to be given by the sum of the squares of the current-related and the wave-related values. Thus:

\( \varepsilon_{s,cw}^2 = \varepsilon_{s,c}^2 + \varepsilon_{s,w}^2 \). Since \( \varepsilon \approx \ell \cdot k^{0.5} \) with \( k \) = kinetic energy and \( \ell \) = length scale, this approach corresponds to the summation of the kinetic energy of both types of motions, which seems to be more realistic than assuming \( \varepsilon_{s,cw} = \varepsilon_{s,c} + \varepsilon_{s,w} \).

The reference level is assumed to be equal to \( a = k_{s,w} \) or \( a = 0.5 \Delta_w \) in case of a rippled bed \( (\Delta_w = \text{ripple height}) \) or \( a = \delta_w \) in case of a plane sheet flow bed \( (\delta_w = \text{wave boundary layer thickness}, \text{see Eq. (2.3.8))} \).

The variables are specified in detail in Appendix A.

### 9.3.2 Comparison of measured and computed concentration profiles

1. Non-breaking waves

Van Rijn et al. (1993) have compared measured and computed concentrations; the latter based on the methods of Bijker, Nielsen and Van Rijn. Figures 9.3.1 and 9.3.2 show the results for 100 µm and 200 µm sediment.

The Bijker method overestimates the sediment concentrations in the major part of the depth for a weak current (0.1 m/s, 0.2 m/s). The predicted near bed concentrations are somewhat too small for the 100 µm-sediment, but show good agreement for the 200 µm-
sediment. For a relatively strong current (0.4 m/s) the shape of the concentration profile is better represented, but now the predicted near-bed concentrations are too small. Based on these results, it is concluded that the parabolic mixing coefficient distribution applied by Bijker is not correct for the combination of waves and a weak current, whereas the bed concentration prediction is not correct when a strong current is present.

Figure 9.3.1 Measured and computed concentration profiles, \( d_{50} = 100 \ \mu m \)
Van Rijn et al., 1993
Figure 9.3.2 Measured and computed concentration profiles, $d_{so} = 200 \mu m$

Van Rijn et al., 1993
The Nielsen method largely overestimates the sediment concentrations for the 200 µm-sediment, particularly for weak currents (0.1, 0.2 m/s). The near-bed concentrations are reasonably well predicted for the 100 µm-sediment. The method of Nielsen is rather sensitive to the particle size of the bed material.

The method of Van Rijn yields good results for the 100 and 200 µm-sediment, particularly in the near-bed region. The concentrations in the upper part of the water column are overpredicted for relatively high waves (T100,18,-20), which may be an indication that the wave-related mixing coefficient is somewhat too large for relatively high waves.

Havinga (1992) found similar results using the methods of Bijker and Van Rijn, see Figure 9.3.3.

Van Rijn (1989) found good agreement between the measured and computed concentrations in the near-bed region of an intertidal flat.

2. **Breaking waves**

Measured concentrations (Jaffe et al., 1984) were compared with computed concentrations using the method of Van Rijn, see Figure 9.3.4. The computed concentrations based on a fall velocity of \( w_s = 0.011 \) m/s and an assumed roughness \( k_{s,w} = k_{s,w} = 0.01 \) m are generally a factor of 2 to 3 too large.

Van Rijn and Kroon (1992) and Kroon and Van Rijn (1993) compared measured and computed concentration profiles for the surf and swash zone near Egmond, The Netherlands, see Figure 9.3.5. The fall velocity was found to be ranging from 0.030 to 0.04 m/s. The bed roughness was assumed to be \( k_{s,c} = k_{s,w} = 0.01 \) m for spilling and plunging breaking wave conditions and 0.05 m for non-breaking waves (ripple regime). The computed concentration profiles show good agreement with the measured values for \( H_s/h \leq 0.75 \). For \( H_s/h > 0.75 \) (plunging waves in the swash zone) the computed concentrations are systematically too small. Based on this, it is concluded that the time averaged concentrations in the swash zone cannot be computed using an equilibrium model concept. The hydrodynamic processes in the swash zone (steep sloping bed) are strongly non uniform. Vertical and horizontal convective processes are important and should be modelled properly.
Figure 9.3.3 Measured and computed concentration profiles, $d_{50} = 100 \, \mu m$

Havinga (1992)
Figure 9.3.4 Measured and computed concentrations in surf zone, USA

- measured in surf zone, Jaffe et al (1984)
- computed Van Rijn
Figure 9.3.5 Measured and computed concentrations in surf and swash zone near Egmond, The Netherlands, Van Rijn and Kroon (1992)
9.4 Computation of sediment transport in non-breaking waves

9.4.1 Methods

Methods available for the computation of the transport rate in combined current and wave conditions are those of Bijker (1967, 1971), Grant and Madsen (1976), Ballard (1981), Fredsøe et al. (1985) and Van Rijn.

The formulae generally yield a net transport rate in the current direction, which may be unrealistic in case of a weak current combined with waves \( \bar{U}_d > u \) over a rippled bed. In that case the wave-related transport may dominate the current-related transport yielding a net transport rate against the current direction, as observed by Nap and Van Kampen (1988).


The method of Bijker yields the current-related bed-load and suspended-load transport rates; the wave-related transport rate is not taken into account.

Based on the concept of Einstein (1950), Bijker proposed:

\[
q_{b,c} = 1.83 \ q_{b,c} \ [1_2 + 1_1 \ ln(33 \ h/k_s)] \tag{9.4.1}
\]

\[
q_{b,c} = b \ u_{*c} \ d_{50} \ cexp \left[ \frac{-0.27(\rho_s - \rho)g \ d_{50}}{\mu \ \tau_{b,cw}} \right] \tag{9.4.2}
\]

in which:

\( q_{b,c} = \) time averaged suspended load transport, incl. pores (m²/s)

\( q_{b,c} = \) time-averaged bed-load transport, incl. pores (m²/s)

\( 1_1 = \) integral of Einstein (Eq. 7.3.34), see Fig. 7.3.14

\( 1_2 = \) integral of Einstein (Eq. 7.3.35), see Fig. 7.3.14

\( b = \) coefficient (b=1 for non-breaking waves; b=5 for breaking waves).

The other variables are specified in Section 9.3.1, see Equations (9.3.1) and (9.3.2).

2. Grant-Madsen (1976)

The instantaneous total transport rate (m²/s) is represented as:

\[
q_t = 40 \ w_s \ d_{50} \ \theta^3 \ \frac{U}{|U|} \tag{9.4.3}
\]

in which:

\( w_s = \) particle fall velocity

\( d_{50} = \) median particle diameter of bed material

\( U = [U_w^2 + u_c^2 + 2 U_w u_c \cos \phi]^{0.5} = \) instantaneous velocity vector near the bed

\( U_w = \bar{U}_d \sin(\omega t) = \) near-bed orbital velocity

\( u_c = \) near-bed current velocity

\( \theta = \frac{0.5 f_w U^2}{(s \ g) \ d_{50}} = \) particle mobility parameter

\( f_w = \) friction factor

\( \phi = \) angle between current direction and wave propagation direction

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Averaging over the wave period yields the mean transport rate and direction. The transport rate comprises both the current-related and the wave-related contributions.

3. **Bailard-Bagnold (1981)**

The instantaneous bed load and suspended load transport rates are expressed as:

\[
q_b = \frac{1}{2} \rho f_w e_b \left[ \frac{|U|^2}{(\rho_s - \rho) g \tan \gamma} U \frac{\tan \beta}{\tan \gamma} |U|^3 i_\beta \right]
\]

\[
q_s = \frac{1}{2} \rho f_w e_s \left[ \frac{|U|^3}{(\rho_s - \rho) g w_s} U - \frac{e_s}{w_s} \tan \beta |U|^5 i_\beta \right]
\]

(9.4.4)

(9.4.5)

in which:

- \(q_b\) = instantaneous bed-load transport rate (m³/s)
- \(q_s\) = instantaneous suspended-load transport rate (m³/s)
- \(f_w\) = friction factor
- \(e_b\) = efficiency factor for bed-load transport (= 0.1 to 0.2)
- \(e_s\) = efficiency factor for suspended load transport (= 0.01 to 0.02)
- \(\beta\) = local bottom slope
- \(\gamma\) = dynamic friction factor \((\tan \gamma = 0.6)\)
- \(w_s\) = particle fall velocity
- \(U\) = \(U^*_w + u_c^2 + 2 U_w u_c \cos \phi \)^{0.5} = instantaneous velocity vector near the bed
- \(U_w\) = \(U_b \sin(\omega t)\) = near-bed orbital velocity
- \(u_c\) = near-bed current velocity
- \(\phi\) = angle between current direction and wave propagation direction
- \(i_\beta\) = unity vector component in direction of bed slope

Originally, the formulae of Bagnold were developed for steady unidirectional flow. Bailard applied these formulae to oscillatory flow in combination with a steady current over a plane sloping bottom. The near-bed current velocity is not clearly defined. The terms containing \(\tan \beta\) represent the gravity-related transport rates which are directed in the direction of the local bed slope \((\partial z_b/\partial x, \partial z_b/\partial y)\) and may differ from the direction of the local time-averaged velocity.

Averaging over the wave period, the total transport rate and direction are obtained (both the wave-related and the current-related contributions). Usually, the coefficients are taken as \(c_b = 0.1\) and \(c_s = 0.02\). Kachel and Sternberg (1971) have shown that the efficiency factors \((e_b\) and \(e_s\)) are not constant, but are strongly related to the bed-shear stress and the particle diameter.

The Bailard-Bagnold formula is only valid for plane bed conditions.

4. **Fredsøe et al (1985)**

The computation of current velocities, orbital velocities, bed-shear stresses and sediment concentrations is given in Sections 8.4.7, 8.5.2 and 9.3.1.

The instantaneous transport rate is computed as the sum of the instantaneous bed-load transport \((q_b)\) and the instantaneous suspended load transport \((q_s)\), as follows:

\[
q_t = q_b + q_s - q_b + \int_{a}^{h} V_R C \, dz
\]

(9.4.6)
in which:
$q_b = \text{instantaneous bed-load transport by a formula}$
$q_s = \text{instantaneous suspended load transport}$
$V_R = \text{instantaneous resultant velocity at height } z \text{ above bed}$
$C = \text{instantaneous concentration at height } z \text{ above bed}$
$a = \text{reference level } (= 2 \ d_{50})$
$h = \text{water depth}$

5. Van Rijn

This method yields the bed-load transport and the suspended load transport due to combined currents and waves (see TRANSPOR-diskette and Appendix A).

**Bed-load transport**

An instantaneous approach is used to compute the instantaneous bed-load transport using Eq. (7.2.45). The time-averaged value is obtained by averaging over the wave period. The instantaneous bed-load transport reads as:

$$q_b(t) = 0.25 \ \alpha \ \ d_{50} \ \ D_s^{-0.3} \ \left[ \tau_{bcw} / \rho \right]^{0.5} \left[ \left( \tau_{bcw} / \tau_{b,cr} \right)^{1.5} \right] \ (9.4.7)$$

in which:
$q_b(t) = \text{instantaneous bed-load transport } (m^2/s)$
$D_s = \text{dimensionless particle parameter } (-)$, see Appendix A
$d_{50} = \text{median particle diameter of bed material } (m)$
$\alpha = \text{calibration factor } = 1 - (H/h)^{0.5}$
$\tau_{bcw} = \text{grain-related instantaneous bed-shear stress due to combined current and waves } (N/m^2)$
$\tau_{b,cr} = \text{critical bed-shear stress according to Shields } (N/m^2)$

**Suspended load transport**

A time-averaged approach is used to compute the suspended load transport by integration over the depth of the product of velocity and concentration, as follows:

current direction  :  \[ q_s = \int_{a}^{h} u \ c \ dz \]  \hspace{1cm} (9.4.8)

wave direction :  \[ q_s = \int_{a}^{h} v \ c \ dz \]  \hspace{1cm} (9.4.9)

reference concentration :  \[ c = c_a \text{ at } z = a \]  \hspace{1cm} (9.4.10)
in which:

\( q_s \) = suspended load transport (m³/s)
\( u \) = current velocity at height \( z \) above bed in the direction of the velocity vector (m/s)
\( v \) = wave-induced velocity (m/s) at height \( z \) above the bed in the wave direction
  (return velocity due to mass transport between wave crest and trough)
\( c \) = sediment concentration (volume) at height \( z \) above the bed computed numerically from the convection-diffusion equation
\( a \) = reference level (m)
\( h \) = water depth (m)

The computation procedure is given in Appendix A.
A diskette (TRANSPOR) is available for numerical computations.

**9.4.2 Comparison of measured and computed transport rates**

Van Rijn et al. (1993) have compared computed depth-integrated current-related suspended sediment transport rates \((q_{s_c})\) according to the methods of Nielsen (1985), Bijker (1967, 1971) and Van Rijn with measured transport rates. The measured transport rates are based on velocity and concentration measurements in a flume with a current superimposed by irregular non-breaking waves over a sand bed of 100 µm and 200 µm. The water depth was about 0.5 m in all tests. The current velocities were in the range of 0.1 to 0.4 m/s. The significant wave height was varied in the range of 0.07 to 0.18 m.

The peak wave period was about 2.5 s. The bed was covered with small-scale ripples in all tests.
Figures 9.4.1 and 9.4.2 show the measured and computed transport rates according to the three methods. The Bijker method shows the least scatter. The computed values are much too large (factor 3 to 10) at small transport rates and much too small (factor 3 to 5) at large transport rates in both cases (100 and 200 µm). The deviations are largest for the 200 µm-experiments.

Bed-load transport rates under combined current and waves were measured in a flume with a sand bed \((d_{50} = 560 \mu m)\) at ISVA, Denmark (Gray et al, 1991). The water depth was about 0.3 m. The significant wave heights were 0.091 to 0.017 m (period of 2 s). The current velocities were 0.17 to 0.43 m/s. The data of run 11 to 17 were used for comparison with the predicted values according to the method of Van Rijn. All predicted values were within a factor 2 of the measured values. The predicted values were, on the average, about 30% too small.
Figure 9.4.1 Measured and computed suspended sediment transport rates, $d_{50} = 100 \, \mu m$

Figure 9.4.2 Measured and computed suspended sediment transport rates, $d_{50} = 200 \, \mu m$
The Nielsen method yields results that are systematically too large (factor 5) in case of the 200 \( \mu \text{m} \) tests. In case of the 100 \( \mu \text{m} \) tests the computed values are a factor 3 too small at small transport rates and a factor 5 to 10 too large at large transport rates.

The computed transport rates according to the Van Rijn method (using \( k_{s,c} = k_{s,w} = 3\Delta \)) show reasonably good agreement with measured values. Most (80\%) of the computed values are within a factor 2 of the measured values. On the average the computed values are somewhat too large.

Havinga (1992) using the methods of Bijker and Van Rijn, found similar results.

9.5 Computation of sediment transport in breaking waves (surfzone)

9.5.1 Methods

The longshore sediment transport over a fine sand bed inside the surfzone (breaking waves) mainly consists of suspended load transport as a result of the mixing properties of the breaking waves and the generated longshore currents. The transport process inside the surf-zone typically has a sawtooth motion because the wave direction is almost perpendicular to the current velocity direction.

Basically, two methods are available to compute the longshore transport rate in the surf-zone: the local method and the integral method. The first method attempts to represent the physics of the sediment transport process, taking all relevant parameters into account and is therefore more universal.

1. Local method

The local method is based on the application of a sediment transport formula related to local parameters (\( \bar{u} \), \( H_s \), \( T_p \), etc.) and integration of the local transport rate (\( q_t \)) over the width of the surf zone, yielding:

\[
Q_t = \int_{o}^{b} q_t \, dy \quad \text{with} \quad q_t = F(\bar{u}, H_s, T_p, d_{50}, w_{g\ldots})
\]  

(9.5.1)

Detailed prediction of the wave height distribution and the current velocity distribution across the surf-zone is required. Tidal currents can also be taken into account.

The methods of Bijker (1967, 1971), Fredsøe and Deigaard et al. (1985, 1986) and Van Rijn can be applied in breaking wave conditions (see Section 9.4.1).

2. Integral methods

The integral method relates the total longshore sediment transport to the longshore component of the wave energy flux at the breaker line (\( Q_i \approx E \cdot c_{e,br} \sin \theta_{br} \cos \theta_{br} \)), as follows:

\[
Q_i = F(H_{br}, c_{e,br}, \theta_{br})
\]  

(9.5.2)

Three integral methods are described herein:

CERC

The CERC-formula developed by the US-Corps of Engineers relates the immersed weight (I) of the longshore sediment transport rate to the longshore wave energy flux factor (Shore Protection Manual, 1984):
\[ I = K E c_{g,br} \sin \theta_{br} \cos \theta_{br} \]  
\hspace{1cm} (9.5.3)

in which:
\[ I \quad \text{longshore transport rate (immersed weight)} \]
\[ E = 1/8 \rho g (H_{rms,br})^2 \quad \text{wave energy at breaker line} \]
\[ H_{rms,br} \quad \text{rms wave height at breaker line} \]
\[ c_{g,br} \quad n_{br} c_{br} = \text{wave group celerity at breaker line} \]
\[ \theta_{br} \quad \text{wave angle at breaker line (between wave crest line and coastline)} \]
\[ K \quad \text{coefficient} = 0.77. \]

Early calibration of the CERC-formula was based upon nine field data points and 150 laboratory data points resulting in \( K = 0.42 \). Subsequent calibration based on the original nine field data points plus fourteen additional field data points with all laboratory data deleted resulted in \( K = 0.77 \), which is the recommended value to date for use with \( H_{rms,br} \) (Shore Protection Manual, 1977, 1984). Since then, more field data points have become available. The \( K \)-values reported for these studies range from 0.2 to 1.6 (Bodge and Kraus, 1991).

Using the significant wave height \( H_s = \sqrt{2} H_{rms} \), Equation (9.5.3) can be rearranged to:

\[ Q_t = 0.025 H_{br}^2 n_{br} c_{br} \sin(2\theta_{br}) \]  
\hspace{1cm} (9.5.4a)

or

\[ Q_t = 0.05 H_{br}^2 n_{br} c_{br} \sin \theta_{br} \cos \theta_{br} \]  
\hspace{1cm} (9.5.4b)

in which:
\[ Q_t \quad \text{longshore sediment transport by volume (m}^3/\text{s}, \text{including pores); the sediment transport by weight is } Q_{t,weight} = (1-p)\rho_s Q_{t,volume} \]
\[ H_{br} \quad \text{significant wave height at the breaker line (m)} \]
\[ c_{br} \quad \text{phase velocity of the waves at the breaker line } = (g h_{br})^{0.5} \]
\[ n_{br} = 1/2[1 + (2kh)/\sinh (2kh)] = \text{coefficient at breaker line} = 1 \]
\[ \theta_{br} \quad \text{wave angle at the breaker line (°)} \]
\[ h_{br} \quad \text{water depth at the breaker line (m)} \]
\[ p \quad \text{porosity factor} = 0.4 \]
\[ \rho_s \quad \text{sediment density} \approx 2650 \text{ kg/m}^3 \].

Applying \( n_{br} \approx 1, c_{br} \approx (g h_{br})^{0.5} \), and \( \gamma_{br} = H_{br}/h_{br} \), Equation (9.5.4a) can be rearranged to:

\[ Q_t = 0.025 g^{0.5} \gamma_{br}^{-0.5} H_{br}^{2.5} \sin(2\theta_{br}) \]  
\hspace{1cm} (9.5.5)

The most important parameters are the wave height and the wave angle. An error of 10% in the wave height and in the wave angle at the breaker line yields a 30% error in the transport rate. Equation (9.5.4) is a rather crude formula showing no influence of the particle diameter and the beach slope inside the surf-zone. Therefore, the CERC-formula is only valid for a narrow range of conditions as represented by the calibration data. According to Kamphuis et al. (1986), the CERC-formula shows reasonable results for particle sizes in the range of 200 to 600 \( \mu \text{m} \) and beach slopes in the range \( \tan \beta = 0.015 \) to 0.15, (see Fig. 9.5.1). The CERC-formula cannot be applied when tidal current velocities are significant.

9.30
A critical examination of the CERC-formula is given by Bodge and Kraus (1991). It is noted that for a uniform coast:

\[(H_{br})^2 n_{br} c_{br} \cos \theta_{br} = (H_o)^2 n_o c_o \cos \theta_o\]

with subscript \(o\) referring to deep water.

Assuming \(n_o = 0.5\), \(n_{br} = 0.95\), \(c_{br} = (gh_{br})^{0.5}\), \(\cos \theta_{br} = 0.95\) and \(\gamma = H_{br}/h_{br}\), it can be derived that (Van Rijn, 1990):

\[l_{br} = \left[\frac{H_o^2 c_o \cos \theta_o}{1.8 g^{0.5} \gamma^2}\right]^{0.4}\] \hspace{1cm} (9.5.6)

The wave angle at the breaker line follows from:

\[\sin \theta_{br} = \frac{c_{br}}{c_o} \sin \theta_o\] \hspace{1cm} (9.5.7)

*Kamphuis et al. (1986)*

Based on the analysis of laboratory and field data, the following formula is proposed by Kamphuis et al (1986):

\[Q_t = 1.28 \frac{H_{br}^{2.5}}{d_{50}} \tan \beta \sin(2\theta_{br})\]

\hspace{1cm} (9.5.8)

in which:

\[Q_t = \text{longshore sediment (mass) transport (kg/s)} \]
\[H_{br} = \text{significant wave height at breaker line (m)} \]
\[\theta_{br} = \text{wave angle at breaker line (°)} \]
\[d_{50} = \text{median particle size in surf zone (m)} \]
\[\tan \beta = \text{beach slope defined as the ratio of the water depth at the breaker line and the distance from the still water beach line to the breaker line; the beach slope may be represented by } \tan \beta = 1.8 (H_{br}/d_{50})^{-0.5}, \text{ see Fig. 9.5.1.} \]

The value 1.28 is a dimensional coefficient related to the SI system assuming salt water. Equation (9.5.8) takes into account the beach slope and the particle size which is an improvement to the CERC-formula The transport rate increases for increasing steepness of the beach slope because the breaking process will be more intensive at steeper slopes resulting in larger concentrations and transport rates. The calibration data consist of data with particle sizes in the range of 200 to 600 \(\mu\)m and beach slopes in the range of 0.015 to 0.15.

A similar influence of the beach slope based on small-scale experiments was found by Quick (1991).

Kamphuis (1991) studied the alongshore sediment transport distributions as measured in hydraulic model tests. A trap was used to measure the bed load and suspended load transport across the swash and surf zone. Based on analysis of the test results, Kamphuis concluded that the sediment transport rate is maximum in the swash zone and near the seaward breaker line.

\[9.31\]
A. LONGSHORE SEDIMENT TRANSPORT ACCORDING TO CERC-FORMULA (KAMPHUIS ET AL, 1986)

B. BEACH SLOPE VERSUS GRAINSIZE (KAMPHUIS ET AL, 1986)

Figure 9.5.1 Measured and computed longshore sediment transport, Kamphuis et al (1986)
Van der Meer (1990)

Van der Meer reanalyzed longshore transport data of gravel beaches and proposed:

\[ Q_t = 0.0012 \ g \ d_{50} \ T_p \ H_s (\cos \theta_{br})^{0.5} \left( \frac{H_s (\cos \theta_{br})^{0.5}}{d_{50}} - 11 \right) \sin \theta_{br} \]  \hspace{1cm} (9.5.9)

9.5.2 Comparison of measured and computed transport rates

Figure 9.5.1 shows measured longshore transport rates and computed values based on the CERC-method. As can be observed, the general trend is fairly good represented, but the scatter is relatively large.

Kroon and Van Rijn (1993) compared measured and computed depth-integrated current-related suspended sediment transport rates for the surf zone near Egmond, The Netherlands. The computed transport rates are based on the method of Van Rijn (Section 9.4.1). Good agreement between measured and computed values in longshore and cross-shore direction can be observed, see Fig. 9.5.2. The computed transport rates for the swash zone with high plunging breaking waves \((H_s/h \geq 0.75)\) are systematically too small (factor 2 to 5).

![Graph showing measured vs. computed transport rates](image)

**Figure 9.5.2** Measured and computed transport rates, Kroon and Van Rijn (1993)
9.6 Examples and problems

1. Waves are propagating (in the surf zone) normal to the coastline. The local water depth is \( h = 2 \) m. The wave height is \( H = 1.6 \) m. The wave length \( L = 32 \) m. The wave period is \( T = 7 \) s. The longshore current velocity is \( \bar{v} = 0.8 \) m/s. The bed material characteristics are \( d_{50} = 200 \) \( \mu \)m and \( d_{90} = 270 \) \( \mu \)m. The bed roughness is \( k_s = 0.06 \) m. Other data given, are: \( \rho = 1000 \) kg/m\(^3\), \( \rho_s = 2650 \) kg/m\(^3\), temperature = 20\(^\circ\)C.

Compute the current-related longshore sediment transport according to the Bijker method (use \( b = 5 \)) and the method of Van Rijn (use TRANSPOR-program).

Solution:

**Bijker**

Peak orbital velocity near bed :
\[
\hat{U}_b = \frac{\pi H}{T \sinh(2\pi h/L)} = 1.78 \text{ m/s}
\]

Peak orbital excursion near bed :
\[
\hat{A}_b = \frac{(T/2\pi)\hat{U}_b}{1.98 \text{ m}}
\]

Friction factor :
\[
f_w = \exp[-6 + 5.2(\hat{A}_b/k_s)^{0.19}] = 0.036
\]

Chézy-coefficients :
\[
C = 18 \log(12h/k_s) = 46.8 \text{ m}^{0.5}/\text{s}
\]
\[
C' = 18 \log(12h/d_{50}) = 89.1 \text{ m}^{0.5}/\text{s}
\]

Efficiency factor :
\[
\mu = (C/C')^{1.5} = 0.381
\]

Wave-related bed-shear stress :
\[
\tau_{b,w} = (1/4) \rho f_w(\hat{U}_b)^2 = 28.2 \text{ N/m}^2
\]

Current-related bed-shear stress :
\[
\tau_{b,c} = \rho g(\bar{v}/C)^2 = 2.8 \text{ N/m}^2
\]

Total bed-shear stress :
\[
\tau_{b,cw} = \tau_{b,w} + \tau_{b,c} = 31 \text{ N/m}^2
\]

Current-related bed-shear velocity :
\[
u_{*,c} = (\tau_{b,c}/\rho)^{0.5} = 0.053 \text{ m/s}
\]

Total bed-shear velocity :
\[
u_{*,cw} = (\tau_{b,cw}/\rho)^{0.5} = 0.176 \text{ m/s}
\]

Longshore bed-load transport
Eq. (9.4.2)
\[
q_{b,c} = \Lambda \exp[B] = 4.9 \times 10^{-5} \text{ m}^2/\text{s}
\]
\[
\Lambda = b u_{*,c} d_{50} = 5.3 \times 10^{-5} \text{ m}^2/\text{s}
\]
\[
B = \exp[-0.27(\rho_s - \rho)g d_{50}/(\mu \tau_{b,cw})] = 0.929
\]

Longshore suspended load transport, see Fig. 7.3.14
\[
q_{b,w} = \alpha q_{b,c}
\]
\[
\alpha = F(Z \text{ and } k_s/h)
\]

Suspension number :
\[
Z_r = w_s/(\kappa u_{*,w})
\]

Fall velocity, see Fig. 3.2.6
\[
w_s = 0.025 \text{ m/s for } d_{50} = 200 \mu \text{m}
\]

Suspension number :
\[
Z = 0.025/(0.4 \times 0.176) = 0.355
\]

Relative bed roughness :
\[
k_s/h = 0.06/2 = 0.03
\]

\( \alpha \)-coefficient :
\[
\alpha = 30 \text{ (Fig 7.3.14)}
\]

Longshore total load transport :
\[
q_{t,c} = q_{b,c} + q_{b,b} = 152 \times 10^{-5} \text{ m}^2/\text{s}
\]
The input data of the TRANSPOR-program are:

Water depth : \( h = 2 \text{ m} \)
Mean current velocity : \( \bar{\nu}_R = 0.8 \text{ m/s} \)
Mean current velocity in wave direction : \( \bar{\bar{u}}_R = 0 \text{ m/s} \)
Near-bed velocity in wave direction : \( u_b = 0 \text{ m/s} \)
Wave height : \( H = 1.6 \text{ m} \)
Wave period : \( T = 7 \text{ s} \)
Angle between current and wave direction : \( \phi = 90^\circ \)
Bed material characteristics : \( d_{50} = 0.0002 \text{ m} \)
\( d_{90} = 0.00027 \text{ m} \)
Suspended sediment size : \( d_s = 0.0002 \text{ m} \)
Current-related roughness : \( k_{s,c} = 0.06 \text{ m} \)
Wave-related roughness : \( k_{s,w} = 0.06 \text{ m} \)
Fluid temperature : \( T_e = 20^\circ \text{C} \)
Salinity fluid : \( s_a = 0\% \)

Computed transport rates in current direction
Bed-load transport : \( q_{b} = 0.19 \text{ kg/sm} = 7.2 \times 10^{-5} \text{ m}^2/\text{s} \)
Suspended load transport : \( q_{s} = 2.11 \text{ kg/sm} = 79.6 \times 10^{-5} \text{ m}^2/\text{s} \)
Total load transport : \( q_{t} = 2.30 \text{ kg/sm} = 86.8 \times 10^{-5} \text{ m}^2/\text{s} \)

2. Waves are propagating in deep water. The wave characteristics are: significant wave height \( H_{sw} = 2 \text{ m} \), peak wave period \( T_p = 7 \text{ s} \), angle between wave crest line at deep water and coast line \( \theta_0 = 30^\circ \). The breaker coefficient is \( \gamma = H_{br}/h_{br} = 0.8 \). The sediment characteristics of the beach are: \( d_{50} = 300 \mu\text{m} \). The beach slope is \( \tan\beta = 0.03 \).

What is the longshore sediment transport according to the methods of CERC and Kamphuis et al.?

Solution:

Wave phase velocity deep water : \( c_0 = (g/2\pi) T_p = 10.9 \text{ m/s} \)
Water depth at breaker line : \( h_{br} = \left[ \frac{H_{sw}^2 c_0 \cos\theta_0}{1.8 g^{0.5} \gamma^2} \right]^{0.4} = 2.56 \text{ m} \)
Wave height at breaker line : \( H_{br} = \gamma h_{br} = 0.8 \times 2.56 = 2.05 \text{ m} \)
Wave phase velocity at breaker line : \( c_{br} = (g h_{br})^{0.5} = 5.01 \text{ m/s} \)
Wave angle at breaker line : \( \sin\theta_{br} = (c_{br}/c_0) \sin\theta_0 = 0.23 \)
\( \theta_{br} = 13.3^\circ \)
Longshore transport CERC : 

\[ Q_l = 0.025 \, g^{0.5} \, \gamma^{-0.5} \, H_{br}^{2.5} \, \sin(2\theta_{br}) \]

= 0.24 m³/s (incl. pores)

Longshore transport kamphuis : 

\[ Q_l = 1.28 \, H_{br}^{3.5} \, d_{s0}^{-1} \, \tan \beta \, \sin(2\theta_{br}) \]

= 707 kg/s = 0.44 m³/s (incl. pores)
REFERENCES


9.37
REFERENCES (continued)


Shore Protection Manual, 1984. CERC, Waterways Experiment Station, Vicksburg, Mississippi, USA.


10. **BED MATERIAL TRANSPORT, EROSION AND DEPOSITION IN NON-STEADY AND NON-UNIFORM FLOW**

### 10.1 Introduction

The sediment transport capacity of a stream is defined as the quantity of sediment that can be carried by the flow without net erosion or deposition. In non-steady and non-uniform flow the actual sediment transport rate may be smaller (underload) or larger (overload) than the transport capacity resulting in net erosion or deposition assuming sufficient availability of bed material (no armour layers).

Bed-load transport in non-steady and non-uniform flow can be modelled by a formula type of approach because the adjustment of the transport of sediment particles close to the bed proceeds rapidly to the new hydraulic conditions. Suspended load transport, however, does not have such a behaviour because it takes time to transport the particles upward and downward over the depth and therefore it is necessary to model the vertical convection-diffusion process.

This chapter presents the basic features and equations of sediment transport in non-steady and non-uniform flow.

### 10.2 Sediment transport in non-steady flow

The following subjects are presented:

- Transport in river flow
- Transport in tidal flow
  - time lag
  - salinity stratification
  - coastal shelf
  - formulae
  - neap/spring cycle

#### 10.2.1 River flow

In general, a river flood wave is a relatively slow process with a time scale of a few days. Consequently, the sediment transport process in river flow can be represented as a quasi-steady process. Therefore, the bed-load transport formulae and the suspended load transport formulae as presented in Chapter 8 can be applied for transport rate predictions. An exception to this may be the sediment transport process in a flash flood wave (banjir) and in the tide-influenced lower reach of the river.

#### 10.2.2 Tidal flow

1. *Time lag*

Tidal flow is characterized by a daily ebb and flood cycle with a time scale of 6 to 12 hours (semidiurnal or diurnal tide) and by a neap-spring cycle with a time scale of about 14 days. Sediment concentration measurements in tidal flow over a fine sand bed (50 to 300 \(\mu\)m) show a continuous adjustment of the concentrations to the flow velocities with a lag period in the range of 0 to 60 minutes, as shown in Figure 10.2.1 for a station in the entrance of the Eastern Scheldt, The Netherlands. Close to the bed the time lag between the concentration and the depth-averaged velocity is negligible, but it increases to about 45 minutes near the water surface.
Figure 10.2.1 Measured concentrations in the Eastern Scheldt Estuary, The Netherlands
The basic transport process in tidal flow is shown in Fig. 10.2.2. Sediment particles go into suspension when the current velocity exceeds a critical value. In accelerating flow there is always a net vertical upward transport of sediment particles due to turbulence-related diffusive processes, which continues as long as the sediment transport capacity exceeds the actual transport rate. The time lag period \( \Delta T_1 \) is the time period between the moment of maximum flow and the moment of equal transport capacity and actual transport rate. After this latter moment there is a net downward sediment transport because settling dominates yielding smaller concentrations and transport rates. In case of very fine sediments (silt) or a large \( \Delta T_2 \), depth the settling process can continue during the slack water period giving a large time lag which is defined as the period between a zero transport capacity and the start of a new erosion cycle. Figure 10.2.2 shows that the suspended sediment transport during decelerating flow is always larger than during accelerating flow, as described by Eq. (12.4.52).

![Figure 10.2.2 Time lag of suspended sediment concentrations in tidal flow](image)

### 2. Salinity stratification

In a stratified estuary a high-density salt wedge exists in the near-bed region resulting in relatively high near-bed regions resulting in relatively high near bed densities and relatively low near-surface densities. Stratified flow will result in damping of turbulence because turbulence energy is consumed in mixing of heavier fluid from a lower level to a higher level against the action of gravity (see section 7.3.9). The usual method to account for the salinity-related stratification effect on the velocity and concentration profiles is the reduction of the fluid mixing coefficient by introducing a damping factor \( \phi \) related to the Richardson number (Ri), as follows:

\[
\epsilon_f \to \phi \epsilon_{f0}
\]  
(10.2.1)

in which:
- \( \epsilon_{f0} \) = fluid mixing coefficient in fresh water
- \( \phi \) = \( F(Ri) \) = damping factor (\(<1\))
- \( \rho \) = fluid density (including salt)
- \( u \) = local fluid velocity
- \( Ri = (-g \partial \rho / \partial z) / (\partial u / \partial z)^2 \) = Richardson number

Terms (1990) studied the influence of the vertical density gradient on the sand concentration profile. Three different density profiles were considered: L-, S- and J-profiles, see Figure 10.2.3. The L-density profile represents a linear distribution between the bed density of 1020...
kg/m³ and the surface density of 1000 kg/m³. A typical flood velocity profile was used to represent the effect of the horizontal density gradient (landward-directed) resulting in an additional acceleration force near the bed and a relatively full velocity profile, see Figure 10.2.3.

Figure 10.2.3  Density and velocity profile

The $\phi$-factor was represented by a function given by Munk-Anderson: $\phi = (1 + 3.3 \text{ Ri})^{1.5}$. Based on these input variables, the concentration profiles for particles of 200 $\mu$m were computed for fresh water (no salinity gradient) and for stratified conditions (L-, S- and J-density profiles). The results, presented in Figure 10.2.4, show a considerable reduction of the concentrations due to damping of turbulence. The influence is largest for the J-density profile and smallest for the S density profile.

Figure 10.2.4  Sand concentration profiles
3. Transport on coastal shelf

Van de Meene (1991) performed sediment transport measurements on the inner shelf of the Dutch sector of the North Sea in August 1991. The water depth varied in the range between 13 and 15 m, yielding a tidal range of 2 m (spring tide). The maximum near-bed velocity at approximately 0.5 m above the bed was 0.5 m/s. Waves did not occur (calm weather). The bed material consisted of medium fine sand with \( d_{50} = 280 \mu m \) and \( d_{90} = 340 \mu m \). Mega-ripples with a height of about 0.2 m and a length of about 10 m were observed (echo soundings). Mini-ripples were superimposed on the mega-ripples.

Bed-load transport measurements were performed with a bag-type trap sampler, lowered and raised from a ship. The sampling period was 3 min. The median particle size of the bed-load catches was 270 \( \mu m \), which is in good agreement with the median size of the bed material \( (280 \mu m) \). The measured bed-load transport rates are shown in Figure 10.2.5. The measured values are extremely small. Transport did occur during a relatively short period of not more than 2 hours during the flood period when the near-bed velocities were larger than 0.4 m/s.

Figure 10.2.5  Bed load and suspended load transport in North Sea during springtide (flood)

Suspended sediment concentrations at \( z = 0.07 \) m, 0.105 m, 0.145 m and 0.255 m were measured by means of a pump sampler operated from the survey vessel. The measured values are shown in Figure 10.2.6. The maximum concentration is 40 mg/l at \( z = 0.07 \) m, which is extremely small. Based on the measured concentrations and velocities, the depth-integrated suspended load transport was computed, see Figure 10.2.5. The maximum suspended load transport is somewhat larger than the bed-load transport. The period during which the sediments are transported as suspended load is considerably shorter (about 1 hour).

Based on these results it is concluded that the bed-load transport is slightly dominant at low tidal velocities. Transport will occur for near-bed velocities larger than 0.4 m/s in the summer period. More intensive sediment transport will occur in the winter period when waves are generated.

Figure 10.2.6  Suspended sediment concentrations in North Sea during springtide (flood)
4. Application of transport formulae

The sediment transport formulae of Engelund-Hansen (1967), Ackers-White (1973) and Van Rijn (1984) yield reasonably good results in tidal conditions for bed material sizes larger than 300 \( \mu \text{m} \), as shown by Voogt et al. (1991). These formulae which yield the transport capacity, tend to overpredict for accelerating conditions and underpredict for decelerating conditions, which is related to the time lag effect (Fig. 10.2.2).

Formulae based on the slope parameter (I) have to be rearranged in terms of the mean velocity (\( \bar{u} \)) using the Chézy-equation \( \bar{u} = C(hl)^{0.4} \). The effective roughness (\( k_e \)) must be known. Estimation of this latter parameter is problematic in tidal conditions. The best approach is to measure the bed-form dimensions by echo-soundings and to use the equations presented in Chapter 6. Voogt et al. (1991) have shown that the formulae of Engelund-Hansen (1967) and Ackers-White (1973) are rather sensitive to the \( k_e \)-parameter and yield therefore somewhat less accurate results.

The formulae do not give realistic results when the salinity-related damping effect is significant (vertical density gradient in stratified flow).

5. Representation of neap-spring cycle

The tidal range of a neap-spring tidal cycle varies as a function of time which is mainly caused by astronomical effects. Apart from astronomical effects, there are also climatological effects (wind effect) which are superimposed on the astronomical variations.

Sand transport computations in tidal conditions requires representation of the neap-spring cycle. This can be simply done by multiplying the velocities of the mean tidal cycle by a (correction) factor (\( \zeta \)) to account for the higher velocities and hence higher transport rates (power-law relationship of transport and velocity) during springtide conditions.

In the following Sections the correction factor (\( \zeta \)) is derived, assuming a power law relationship between the sediment transport (\( q_s \)) and the depth-averaged velocity (\( \bar{u} \)), \( q_s = a \bar{u}^b \) (zero time lag).

Astronomical effect

The depth-averaged flow velocity (\( \bar{u} \)) of each daily tidal cycle is assumed to vary sinusoidally in time:

\[
\bar{u} = \bar{u} \sin(\omega t) \tag{10.2.2}
\]

in which:
- \( \bar{u} \) = current velocity at time \( t \) of a daily tidal cycle (flood/ebb)
- \( \bar{u} \) = maximum current velocity of a daily tidal cycle (flood/ebb)
- \( \omega = 2\pi/T_m \) = angular frequency
- \( T_m \) = duration of tidal cycle

Assuming a power-law relationship between the sediment transport (\( q_s \)) and the maximum depth-averaged flow velocity (\( \bar{u} \)), the tide-integrated sediment transport (\( Q_s \)) can be related to the maximum velocity (\( \bar{u} \)) of a tidal cycle. The variation of the tidal range (\( H \)) during a neap-spring cycle is assumed to be sinusoidal.
The tidal range \((H)\) is expressed as:

\[
H = \alpha \ H_m
\]  
(10.2.3)

in which:
- \(H\) = tidal range at time \(\tau\)
- \(H_m\) = tidal range of mean tide
- \(\alpha = 1 + \dot{\alpha} \sin(\omega \tau)\) = tidal coefficient
- \(\omega = 2\pi/T\) = angular frequency
- \(T\) = duration of a neap-spring cycle
- \(\hat{H}\) = tidal range of spring tide

The maximum depth-averaged velocity of each tide is assumed to be related to the tidal range, as follows:

\[
\hat{u} \approx H^n
\]  
(10.2.4)

For the mean tide it follows that:

\[
\hat{u}_m \approx (H_m)^n
\]  
(10.2.5)

In most estuarine channels there is a nearly linear relationship between the maximum flow velocity \((\hat{u})\) and the tidal range \((H)\) resulting in \(n \approx 1\). In coastal conditions (open sea) the \(n\)-coefficient may be as small as \(n = 0.5\). Measurements should be analysed to determine the exact value of the \(n\)-coefficient for each specific location.

The astronomical correction factor \(\zeta_a\), which is a multiplication factor for the velocities of the mean tide, can be determined, as follows:

\[
(\zeta_a \hat{u}_m)^b - \frac{1}{T} \int_T (\hat{u})^b \ d\tau
\]  
(10.2.6)

Substitution of Equations (10.2.2), (10.2.3), (10.2.4), (10.2.5) in Equation (10.2.6) yields:

\[
(\zeta_a H_m^n)^b - \frac{1}{T} \int_T (a^n H_m^n)^b \ d\tau
\]  
(10.2.7)

or

\[
(\zeta_a)^b - \frac{1}{T} \int_T \alpha^{nb} \ d\tau - \frac{1}{2\pi} \int_0^{2\pi} (1 + \dot{\alpha} \sin(\omega \tau))^{nb} \ d\omega \tau
\]  
(10.2.8)
Equation (10.2.8) yields:

\[ \begin{align*}
\text{nb} = 2 & \quad \zeta_a = \left(1 + \frac{1}{2} (\hat{\alpha})^2\right)^{1/2} \\
\text{nb} = 3 & \quad \zeta_a = \left(1 + \frac{3}{2} (\hat{\alpha})^2\right)^{1/3} \\
\text{nb} = 4 & \quad \zeta_a = \left(1 + 3 (\hat{\alpha})^2 + \frac{3}{8} (\hat{\alpha})^4\right)^{1/4} \\
\text{nb} = 5 & \quad \zeta_a = \left(1 + 5 (\hat{\alpha})^2 + \frac{15}{8} (\hat{\alpha})^4\right)^{1/5} \\
\text{nb} = 6 & \quad \zeta_a = \left(1 + \frac{15}{2} (\hat{\alpha})^2 + \frac{45}{8} (\hat{\alpha})^4 + \frac{5}{16} (\hat{\alpha})^6\right)^{1/6}
\end{align*} \]  

(10.2.9)

Figure 10.2.7 shows the correction factor \( \zeta_a \) as a function of \( \hat{\alpha} \) and nb.

**Climatological effects**

The maximum current of the mean tide is assumed to have a normal distribution with an average value \( \hat{u}_m \) and standard deviation \( \sigma_u \).

Thus:

\[ f(\hat{u}) = \frac{1}{(2\pi)^{0.5} \sigma_u} e^{-\left(\hat{u} - \hat{u}_m\right)^2 / 2\sigma_u^2} \]  

(10.2.10)

or,

\[ f(x) = \frac{1}{(2\pi)^{0.5} \frac{\sigma}{\hat{u}_m}} e^{-x^2 / 2\sigma^2} \]  

(10.2.11)

in which:

\[ x = \frac{\hat{u} - \hat{u}_m}{\hat{u}_x} \]

\[ \sigma = \frac{\sigma_u}{\hat{u}_x} \]

The probability that \( \hat{u}_1 < \hat{u} < \hat{u}_2 \) or \( x_1 < x < x_2 \) is:

\[ F(\hat{u}) = \int_{\hat{u}_1}^{\hat{u}_2} f(\hat{u}) \, d\hat{u} = \int_{x_1}^{x_2} f(x) \, dx = \frac{1}{(2\pi)^{0.5} \sigma} \int_{x_1}^{x_2} e^{-x^2 / 2\sigma^2} \, dx \]  

(10.2.12)
The correction factor $\zeta_c$ due to climatological effects follows from:

$$\left(\zeta_c \hat{u}_m\right)^b = \int_{\hat{u}_1}^{\hat{u}_x} (\hat{u})^b F(\hat{u}) \, d\hat{u}$$

resulting in:

$$\left(\zeta_c\right)^b = \left(\hat{u}_m\right)^b \int_{\hat{u}_1}^{\hat{u}_x} (1+x)^b f(x) \, dx = \frac{1}{(2\pi)^{0.5} \sigma} \int_{\hat{u}_1}^{\hat{u}_x} (1+x)^b e^{-x^2/2\sigma^2} \, dx$$

The $\zeta_c$-correction coefficient has been determined for various values of $b$ and $\sigma$, by using a numerical integration method. The results are presented in Figure 10.2.7.

**Figure 10.2.7** Correction factor representing astronomical and climatological effects (Van Rijn, 1989)

**Characteristic tide for sediment transport**

The characteristic tide can be determined by multiplying the velocities of the mean tidal cycle with a correction factor ($\zeta$). The correction factor ($\zeta$) is defined as:

$$\zeta = \zeta_a \zeta_c$$

In this method the neap-spring tidal cycle is assumed to vary sinusoidally in time superimposed by a normal probability distribution to represent the climatological effects. The long period tidal components have been neglected. Furthermore, in practice it will be difficult to separate the various effects (astronomical and climatological).
10.3 Sediment transport in non-uniform conditions

The following subjects are presented:
- adjustment length of suspended sediment transport
- erosion and scour near structures
  - bridge piers
  - groynes, spurdikes, bridge abutments
  - weirs, barrages
- deposition in channels
  - current, waves, transport
  - formulae
  - trapping efficiency graphs

10.3.1 General

In non-uniform flow conditions the sediment transport will adjust to the new hydraulic conditions.

The adjustment of the bed-load transport proceeds almost instantaneously because the transport takes place close to the bed. The adjustment of the suspended load transport proceeds relatively slow because it takes time and hence distance for the particles to settle out from the suspension or to be mixed into the flow, depending on the ratio of the bed-shear velocity and the fall velocity.

The adjustment of the suspended load transport was studied by Van Rijn (1987) using a two-dimensional vertical mathematical model. The adjustment length \( L \) was defined as the length after which the suspended load transport differs less than 5% from the new equilibrium suspended transport.

In dimensionless form the adjustment length was given as:

\[
\frac{L}{h} = F\left(\frac{q_s}{q_{s,e}}, \frac{w_s}{u_*}, \frac{k_s}{h}\right)
\]

(10.3.1)

in which:
- \( L \) = adjustment length
- \( h \) = water depth
- \( q_s \) = actual suspended load transport
- \( q_{s,e} \) = equilibrium suspended load transport
- \( w_s \) = fall velocity
- \( u_* \) = bed-shear velocity
- \( h_s \) = effective bed-roughness height

The \( k_s/h \)-parameter was held constant at a value of 0.01. Computer runs for overloading and underloading conditions were made. The results were presented in graphical form, see Figure 10.3.1

As an example a uniform irrigation channel with a depth of \( h = 2 \) m is considered. The flow entering the channel has a suspended load transport of \( q_s = 0.1 \) kg/sm. The equilibrium transport is \( q_{s,e} = 1 \) kg/sm. The \( w_s/u_* \)-ratio is assumed to be 0.2. For \( q_s/q_{s,e} = 0.1 \) and \( w_s/u_* = 0.2 \) the dimensionless adjustment length is \( L/h = 150 \), see Figure 10.3.1. Thus, after \( L = 300 \) m the suspended load transport has adjusted from \( q_s = 0.1 \) to \( q_{s,e} = 1 \) kg/sm. Hence, the bed will be eroded over a length of about 300 m.

10.10
A typical example of non-uniform sediment transport is the generation of concentration profiles in a clear flow (without initial sediment load) entering a channel with an erodible channel bed. Figure 10.3.2 shows concentration profiles at various locations in a flume in case of a current ($\bar{u} = 0.18$ m/s) superimposed by following waves ($H = 0.11$ m) over a fine sediment bed ($d_{50} = 100$ $\mu$m). The horizontal adjustment length of the concentration profiles and the transport rate to the equilibrium values is approximately equal to 20 times the water depth.

Another typical example of non-uniform suspended load transport is the transport of fine sediments across a trench or channel perpendicular to the flow. Figure 10.3.3 shows the experimental results of a trench with side slopes of 1:3 in a flume (Van Rijn, 1985, 1986).

Figure 10.3.1 Adjustment length of suspended sand transport (Van Rijn, 1985)
A. EXPERIMENTAL SET-UP

B. SUSPENDED SEDIMENT TRANSPORT

C. CONCENTRATIONS

Figure 10.3.2 Sediment concentrations in an eroding uniform current with waves
The bed material consisted of fine sand with a $d_{50}$ of 160 $\mu$m. The upstream water depth was $h_0 = 0.39$ m. The current velocity was $u_0 = 0.51$ m/s. Small-scale ripples with a height of about 0.025 m and a length of about 0.2 m were present on the bed in the upstream section. The sand feed rate at the inlet was equal to $q_s = 0.04$ kg/sm. The measured suspended load transport was $q_s = 0.03$ kg/ms. Hence, the bed load transport rate was $q_b = 0.01$ kg/sm. Upstream of the trench the sediment concentrations vary from about 1000 mg/l near the bed to about 20 mg/l near the water surface. There are large concentration gradients in the near-bed layer. In the deceleration zone (profile 4) where the flow velocity is reduced, there is a decrease of the concentrations and the vertical gradients near the bed.

In the middle of the trench (profile 6) a further decrease of the concentrations can be observed. Thus, the settling process is dominant in both the deceleration and the middle zone of the trench. In the acceleration zone (profile 7) the erosion process is dominant and the concentrations show an increasing trend. Downstream of the trench (profile 8) the concentration profile is similar to that in the upstream section.
Sediment transport in non-uniform conditions over an erodible bed will result in bed-level changes which can be determined from the sediment mass balance equation, which reads as:

$$\frac{\partial z_p}{\partial t} + \frac{\partial (hc)}{\partial t} + \frac{\partial q_{t,x}}{\partial x} + \frac{\partial q_{t,y}}{\partial y} = 0$$

(10.3.2)

in which:
- $z_p$ = bed level above a horizontal datum
- $t$ = time
- $h$ = water depth
- $c$ = depth-averaged suspended sediment concentration
- $q_{t,x}$ = total load transport in x-direction
- $q_{t,y}$ = total load transport in y-direction

An increase of the transport rate in flow direction yields erosion. A decrease of the transport rate in flow direction yields deposition.

10.3.2 Erosion and scour near structures

Scour can be long-term erosion such as bank or channel erosion due to meandering processes or local erosion caused by an increase of the sediment transport capacity due to an increase of the local velocity and/or turbulence intensity.

Examples of local scour are:
- scour near bridge piers,
- scour near bridge abutments, groynes and spurdikes (structures connected to bank)
- scour downstream of weirs, barrages (structures perpendicular to flow).

Generally, a distinction is being made between clear water scour and live-bed scour. The former is related to conditions with no upstream sediment transport ($\bar{u} < \bar{u}_{critical}$); the latter is related to conditions with sediment transport ($\bar{u} > \bar{u}_{critical}$).

Literature reviews have been given by Breusers et al. (1977) and Melville (1988).

1. Scour near bridge piers

The scouring process is related to three effects:
- local disturbance of the flow field,
- local reduction of cross-section,
- general lowering of river bed near bridge site during floods.

The flow pattern around a cylindrical pier is characterized by (see Figure 10.3.4):
- water surface roller in front of pier,
- downflow in front of pier,
- vortex-shedding in separation zone,
- wake flow downstream of pier,
- generation of horseshoe-vortices in scourhole.

Based on analysis of field and flume data, Breusers et al. (1977) have found for a single pier in uniform bed material:

$$h_{b,\text{max}} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 b$$

(10.3.3)
in which:

\( h_{u, \text{max}} \) = maximum scour depth below original river bed
\( b \) = width of pier in plane of river cross-section
\( \alpha_1 \) = coefficient related to \( \bar{u}/\bar{u}_c \)
\( \alpha_2 \) = coefficient related to \( h_u/b \)
\( \alpha_3 \) = coefficient related to shape of pier
\( \alpha_4 \) = coefficient related to angle of attacking flow.
\( \bar{u} \) = depth-averaged flow velocity upstream of pier
\( \bar{u}_c \) = critical depth-averaged flow velocity (upstream)
\( h_0 \) = flow depth (upstream)

\[ \begin{align*}
\alpha_1 &= 0 & \text{for } \bar{u}/\bar{u}_c < 0.5 \text{ (no upstream transport)} \\
\alpha_1 &= 2(\bar{u}/\bar{u}_c - 0.5) & \text{for } \bar{u}/\bar{u}_c = 0.5 \text{ to } 1.0 \text{ (no upstream transport)} \\
\alpha_1 &= 1 & \text{for } \bar{u}/\bar{u}_c \geq 1 \\
\alpha_2 &= 2 \tanh(h/b) \text{ yielding } & \alpha_2 = 2 \quad \text{for } h/b \geq 3 \\
\alpha_2 &= 1.5 \quad \text{for } h/b < 1 \\
\alpha_3 &= 1 & \text{for circular and round-nosed piers,} \\
\alpha_3 &= 0.75 & \text{for streamlined piers,} \\
\alpha_3 &= 1.3 & \text{for rectangular piers.} \\
\alpha_4 &= 1 & \text{for flow normal to bridge piers,} \\
\alpha_4 &= 1.3 & \text{for flow under angle of } 15^\circ \text{ and length-width ratio of } 4, \\
\alpha_4 &= 2 & \text{for flow under angle of } 15^\circ \text{ and length-width ratio of } 8.
\end{align*} \]

Other methods were proposed by Melville and Sutherland (1988) and by Kothyari et al (1992).

The gradation of the bed material has a strong influence on the scour depth. In graded bed materials \((\alpha_s \geq 3)\) the scour depth can be reduced significantly (up to 50\% of that in uniform material) due to armouring of the top layer of the scour hole by coarse particles. The channel bed may also be composed of a series of layers of different resistance to scour. Where a relatively resistant material overlies a more readily erodible material, large scour depths may result if the scour breaks through the more resistant layer. Conversely, if a highly resistant material is known to exist at a particular level it may be unnecessary to extend the bridge foundation below that level.

![Flow pattern and scour near bridge pier (Melville, 1988)](image)

\( Figure \ 10.3.4 \) Flow pattern and scour near bridge pier (Melville, 1988)
When bed forms are present, an extra foundation depth equal to 0.5 times the maximum dune height to be expected should be taken into account.

The length of the scour hole is about \(1b\) (\(b = \text{width of pier}\)) upstream of the pier and about \(5b\) downstream of the pier. The width of the scour hole is about \(2b\) on each side of the pier. The time scale of the scouring process (time at which \(h = b\)) depends primarily on the approach velocity, the sediment size and the width of the pier.

A group of bridge piers yields a larger scour depth when the piers are spaced closely. De Bruyn (1988) studied the scour process near a (platform) pier in current and wave conditions. The bed material was sand with \(d_{50} = 200\ \mu\text{m}\). The water depth was 0.3 m. The depth-averaged velocity upstream of the pier was 0.4 m/s (mobile bed, \(\overline{u}/\overline{u}_{cr} > 1\)). The maximum scour depth was found to be:

\[
h_{b,\text{max}} = \alpha \ b
\]  \hspace{1cm} (10.3.4)

with:

\[
\alpha = 1.3 \quad \text{for a current alone.}
\]

\[
\alpha = 1 \quad \text{for current and non-breaking waves,}
\]

\[
\alpha = 1.9 \quad \text{for current and breaking waves.}
\]

In case of non-breaking waves the increased upstream bed-load transport probably resulted in a decrease of the scour depth.

The length of the scour hole was \(3b\) upstream and \(5d\) downstream of the pier for a current alone. For combined current and waves the scour length upstream was \(4b\) and \(6b\) downstream of the pier.

Usually, the bed near a pier has to be protected by a layer of stones (rip-rap) on a filter layer or mat to prevent erosion of fine sediments through the protection layer of stones. The protection layer should be placed below the lowest river bed level to prevent the creation of extra obstruction. The stone size can be determined by the design rules given in Chapter 4.

The design velocity should be taken 2 times the average approach velocity to account for the local increase of the velocity near the pier.

Model tests are recommended for complicated situations.

2. Scour near groynes, spurdikes, bridge abutments

The flow pattern around groynes is characterized by curvature of the streamlines resulting in a spiral type motion like flow in a river bend. Figure 10.3.5 shows velocity vectors computed by a two-dimensional horizontal mathematical model. The approach depth-averaged velocity is 0.67 m/s. The water depth upstream of the groyne is 6 m. The maximum velocity near the groyne is about 2 m/s. The length \(L_1\) over which the flow field is disturbed in the contracted cross-section is approximately equal to the length of the groyne \((L_1 = L)\) when the total river width is larger than twice the groyne length.

Based on analysis of field data, Breusers (1988) proposed:

\[
h_{b,\text{max}} = \alpha (q_1)^{2/3} - h_1
\]  \hspace{1cm} (10.3.5)
in which:
\( h_{s,\text{max}} \) = maximum scour depth near groyne
\( h_l \) = mean water depth of contracted section before scour
\( q_l \) = discharge per unit width in contracted section
\( \alpha \) = coefficient depending on groyne and river geometry (≈ 2 for straight channel and groyne normal to bank).

Another method is to assume that the cross-section area of the contracted section ultimately will be equal to that without the groyne (see Figure 10.3.5). This means that the scoured area \( (A_s) \) will be equal to the area blocked by the groyne. Thus: \( A_s = h_l L \).

\[ \text{Figure 10.3.5 Flow pattern and scour near a groyne} \]
Assuming that \( A_s = \frac{1}{2}(h_{s, \text{max}}L_1) \) for a long groyne (\( L > 10 \ h_1 \)) and \( L_1 = L \), it follows that:

\[ h_{s, \text{max}} \approx 3 \ h_1 \quad \text{for} \quad L > 10 \ h_1 \]  
(10.3.6)

This is in good agreement with values observed by Richardson et al. (1988), who found \( h_{s, \text{max}} \leq 4h \) for rock dikes (\( L > 25 \ h_1 \)) in the Mississippi river. Equation (10.3.6) is valid for a relatively long groyne (\( L/h_1 \geq 10 \)) resulting in a significant increase of the velocities in the contracted section. The channel bed is assumed to be composed of sandy material and the approach velocity is assumed to be larger than the critical velocity for initiation of motion \( (\tilde{u}/\tilde{u}_c) > 1 \). Armouring which may occur in course bed material, will result in reduced scour depths.

The scour near a short groyne will be considerably smaller than according to Equation (10.3.6). The maximum scour depth is of the order the:

\[ h_{s, \text{max}} = 0.5 \text{ to } 1.5 \ h_1 \quad \text{for} \quad L = 1 \text{ to } 3 \ h_1 \]  
(10.3.7)

The shape of the groyne will also affect the scour depth. Scour is maximum near a vertical wall (rectangular cross-section). The scour depth may be reduced with about 30% in case of a rock-type groyne with a trapezoidal cross-section.

Model studies are recommended for a complicated groyne geometry and river alignment (bends). Bed protection measures should be considered to ensure the stability of the groyne head (if necessary).

3. Scour downstream of weirs and barrages

Two-dimensional scour downstream of a structure such as a weir or a barrage (see Figure 10.3.6) has been studied by many researchers (see Schopmann, 1972). The maximum scour depth in the equilibrium situation as well as the development in time of the scour depth has been studied.

Delft Hydraulics (Breusers, 1967) focussed on the time-dependent behaviour of scour holes (in sandy beds) related to closure works in tidal channels, where scour is primarily important during the relatively short construction period.

Based on experimental research in flumes, the time-dependent development of the scour depth in clear water flows was found to be:

\[ \frac{h_s(t)}{h_o} = \left( \frac{t}{T} \right)^{0.38} \]  
(10.3.8)

in which:

- \( h_s(t) \) = maximum depth at time \( t \)
- \( h_o \) = upstream water depth
- \( T \) = time at which \( h_s = h_o \)

Equation (10.3.8) is not valid close to the equilibrium situation.

The time-scale \( T \) was found to be:

\[ T = \frac{330 \ (s - 1)^{1.7} \ (h_o)^2}{(\alpha \ \tilde{u}_o - \tilde{u}_c)^{4.3}} \]  
(10.3.9)

10.18
in which:

\[ \bar{u}_0 = \text{depth-averaged velocity just upstream (x = 0) of scour hole} \]

\[ \bar{u}_{cr} = \text{critical depth-averaged velocity (initiation of motion)} \]

\[ s = \text{pecific density (} \rho_s / \rho \text{)} \]

\[ \alpha = \text{coefficient depending on flow and turbulence structure at the upstream end of scour hole (} \alpha = 1.5 \text{ for two-dimensional flow without structure, } \alpha = 3 \text{ for very violent three-dimensional flow, Van der Meulen and Vinjé, 1975)} \]

Generally-accepted formulae for the maximum scour depth in the equilibrium situation are not available. A rough estimate can be obtained from:

\[ h_{s,\text{max}} = \left( \frac{\alpha \bar{u}_0 - \bar{u}_{cr}}{\bar{u}_{cr}} \right) h_0 \]  \hspace{1cm} (10.3.10)

Usually, the river bed downstream of a weir or barrage is protected over a certain distance to reduce the maximum scour depth which is strongly dependent on the \( \alpha \)-factor (\( \alpha \) decreases with distance due to the decay of turbulence). The bed protection length is of the order of 10 to 20 \( h_0 \).

Model studies are recommended for complicated geometries.

---

**Figure 10.3.6 Two-dimensional scour downstream of structure**

10.3.3 Deposition in channels

Deposition in channels may occur due to: geological processes (time scale of centuries); shifting of shoals and banks (time scale of decades) and reduction of the local transport capacity (short term effect).

When a current crosses a channel, the sediment transport capacity decreases. As a result the bed-load particles and a certain amount of the suspended sediment particles will be deposited in the channel. The settling process is dominant in the downsloping (deceleration) section and the middle section of the channel. In the case of a steep-sided channel with flow separation resulting in the generation of extra turbulence energy, the settling process may be reduced considerably. In the upsloping (acceleration) section of the channel the dominant process is sediment pick-up from the bed into the accelerating flow, resulting in an increase of the suspended sediment concentrations. The most relevant processes in the deposition and erosion regions are: convection of sediment particles by the horizontal and vertical fluid velocities, mixing of sediment particles by turbulence and orbital motions, settling of the particles due to gravity and pick-up of the particles from the bed by current and wave-induced bed-shear stresses. The effect of waves is that of an intensified stirring action in the near-bed region resulting in larger sediment concentrations, while the current is responsible for the transportation of the sediment. These processes are schematically shown in Figure 10.3.7.

10.19
Gravitational effects may lead to a smoothing of the side slopes of the channel. When a sediment particle resting on a side slope is set into motion by waves or currents, the resulting movement of the particle will, due to gravity, have a component in downwards direction (see Equation 7.2.52). By this mechanism sediment material will always be transported to the deeper part of the channel yielding a smoothing of the side slopes.

![Diagram of sediment transport processes in channel](image)

**Figure 10.3.7 Sediment transport processes in channel**

1. **Currents**

Most important for the deposition process is the local current pattern. The influence of the channel on the local current pattern is determined by: channel dimensions (length, width, depth), angle between channel axis and direction of approaching current, strength of local current and bathymetry of local area.

Usually, the dimensions of the channel are so small that there is no significant influence of the channel on the macro-scale current pattern. In most cases the current pattern is only changed in the direct vicinity of the channel.

When the channel is situated **parallel** to the local current, the velocities in the channel may increase considerably due to the decrease of the bottom friction while the water surface slope remains approximately constant.

When the channel is situated **perpendicular** to the local current, the velocities in the channel are reduced due to the increased water depth. This influence is most significant in the near-bed layer where positive pressure gradients are acting, causing a strong reduction of the flow. In case of steep side slopes (1:5 and steeper) flow separation and reversal will occur introducing a rather complicated flow pattern. The velocities in the recirculation zone are small compared with those in the main flow. The flow velocities in the near-water surface layers are hardly influenced by the presence of the channel (inertial effect). The layer between the near water surface region and the recirculation region is dominated by production of turbulence energy (mixing layer). In case of narrow channel the flow pattern will be completely dominated by flow separation and flow reversal phenomena. Most probably, a large vortex will be generated in the channel.
When a channel is situated oblique to the local current, the effects of parallel and perpendicular flow situations are occurring simultaneously. The velocity component perpendicular to the channel is inversely proportional to the local water depth, while the velocity component parallel to the channel may increase due to a reduction of bottom friction. As a result, the streamlines show a refraction-type pattern in the channel (see Figure 10.3.8). This effect is more pronounced in the bottom region where the velocities are relatively small. Usually, there is an overall increase of the velocities in the channel when the angle between the approaching current and the channel axis is smaller than about 20° to 30°, depending on channel dimensions and bottom roughness.

Physical model studies have been executed at the Hydraulic Research Station Wallingford (Lean, 1973) and at Delft Hydraulics (Boer, 1985). The tests at HRS Wallingford were executed in relatively wide channels inclined to the current at angles of $\alpha_0 = 0°, 10°, 20°$ and $30°$. The water depth on the banks ranged from 0.015 to 0.098 m and the difference in the depth between the channel and the banks was 0.02 m. Based on these results, it can be concluded that for angles smaller than $30°$, the velocity in the channel is generally larger than outside the channel.

The tests at Delft Hydraulics were also executed in relatively wide channels inclined to the flow of $\alpha_0 = 45°, 60°$ and $90°$. The water depth outside the channel ranged from 0.1 to 0.2 m. The difference in depth between the channel and the banks was 0.1 m. In all cases, the measured velocities inside the channel were smaller than those outside the channel.

![Figure 10.3.8 Definition sketch for deposition in channel](image)

**Figure 10.3.8** Definition sketch for deposition in channel
2. Waves

Waves are important for the morphological processes because of the stirring action of the orbital motions in the near-bed region. This effect is most pronounced in the nearshore area, where the waves are affected by the limited water depth and possibly by an irregular bottom bathymetry, shoals and channels. Wave propagation and deformation in the nearshore area are governed by refraction (shoaling), diffraction, and energy dissipation by breaking and bottom friction. Wave breaking in the surf zone may result in the generation of outward-directed currents in the channel due to longitudinal differences in wave set-up (rip-type currents).

When the waves are propagating parallel with a wide channel, the wave height above the channel may be reduced somewhat due to the increased water depth. The wave propagation velocity in the middle of the channel will increase yielding curved wave crests.

When the waves are propagating perpendicular to the channel, the wave height above the channel reduces due to the increased water depth. Secondary effects are reflection at the edges of the side slopes of the channel.

When the waves are propagating oblique to a wide channel, shoaling and refraction effects occur and a rather irregular wave pattern may be generated above the channel depending on the depth and wave approach angle. This process is even more pronounced when there is a strong current-wave interaction. For a certain (critical) wave approach angle and channel depth the incoming waves can be trapped in the channel or even return in the direction of shallower depths. As a result the wave height in the channel may be reduced considerably.

3. Sediment transport

Various methods to determine the deposition rate in relatively wide channels are presented in the literature; from relatively simple formulae to sophisticated mathematical models based on the convection-diffusion equation. Herein, the formulae of Mayor-Mortensen-Fredsoe (1976), Lean (1980) and Eysink-Vermaas (1983) are presented, while also results of the two-dimensional mathematical model SUSTRA (Van Rijn, 1986, 1987) are presented.

The most general situation is that of deposition by a current oblique to the channel, as shown in figure 10.3.8. Because of the streamline refraction effect, the flow is contracted inside the channel ($b_1 < b_0$).

The simple sedimentation formulae are based on a rather strong schematization of the flow situation. Usually, the channel is schematized to a rectangular cross-section making intersections half way along the side slopes of the channel. The length between the intersection points is assumed to be the effective sedimentation length ($L \approx B/\sin \alpha$), see Figure 10.3.8.

Assuming an exponential behaviour of the decreasing suspended sediment transport, the expression for the local suspended sediment transport rate ($q_{s,x}$) in the channel is:

\[
\frac{b_0 q_{s,o} - b_1 q_{s,x}}{b_0 q_{s,o} - b_1 q_{s,1}} = (1 - e^{-Ax})
\]  

(10.3.11)

in which:

$q_{s,0}$ = incoming suspended sediment transport per unit width

$q_{s,1}$ = equilibrium suspended sediment transport per unit width in the channel

$b_0$ = width of streamtube of approaching flow

$b_1$ = width of streamtube in channel

10.22
\( x \) = longitudinal coordinate
\( A \) = coefficient

Basically, the formulae of Mayor-Mortensen-Fredsøe and Lean are only valid for a channel almost perpendicular to the current, because the current refraction effect has been neglected by these authors. Application of these formulae to oblique channels leads to unrealistic results. Therefore, the formulae are given in a modified form by introducing the \( h_0/b_1 \)-ratio to account for the refraction effect.

Further, it is noted that the formulae do not represent the erosion effect at the downstream channel slope where the velocities are increasing. The erosion effect reduces the overall deposition volume, especially in tidal flow. Thus, application of the formulae may lead to an overestimation of the deposition rate.

**Formula of Mayor-Mortensen-Fredsøe (1976), Fredsøe (1978)**

The suspended sediment transport in the channel is expressed as:

\[
q_{s,x} = \left( \frac{b_0}{b_1} \right) q_{s,0} \exp \left( -A \frac{h_0 b_0}{h_1 b_1} x \right) + q_{s,1} \left( 1 - \exp(-A \ x) \right) \tag{10.3.12}
\]

in which:

- \( h_0 \) = water depth upstream of channel
- \( h_1 \) = water depth in channel
- \( A = \frac{(w_0^2/[\epsilon_1 \ u_1])}{\epsilon_1} \) = coefficient
- \( w_s \) = particle fall velocity
- \( \epsilon_1 \) = depth-averaged mixing coefficient in channel (= 0.085 \( u_{*1} h_1 \))
- \( u_1 \) = depth-averaged current velocity in channel

The deposition rate \( (\Delta s) \) per unit channel length due to the suspended load transport (see Figure 10.3.8) is

\[
\Delta s = \left[ q_{s,0} \left( \frac{b_0}{b_1} \right) \right] \left[ 1 - \exp \left( -A \frac{h_0 b_0}{h_1 b_1} \frac{B}{\sin \alpha_1} \right) \right] - q_{s,1} \left[ 1 - \exp \left( -A \frac{B}{\sin \alpha_1} \right) \right] \sin \alpha_1 \tag{10.3.13}
\]

Fredsøe (1978) proposed a method to compute the deposition due to bed-load transport in a channel for the special case of a current parallel to the channel axis.

He considered the effect of gravity forces and drag forces on the sediment particles moving on side slopes of the channel. The resulting force is inclined towards the channel bottom and thus causes deposition. The angle between the direction of the force and the axis is a function of the inclination of the side slopes of the channel and the dynamic friction angle of the bed material which, normally, is slightly less than the angle of static friction.

Assuming a small variation of the bed-load transport over the side slope, an equation for the diffusion of the sediment can be derived, yielding the following formula for gravity deposition:

\[
\Delta s = 2d \left( \frac{q_{b,0}}{\pi \ \tan \phi} \right)^{0.5} (t + t_v)^{0.5} - (t_v)^{0.5} \tag{10.3.14}
\]

10.23
in which:
\( \Delta s \) = solid volume of sand transported from the two banks into the channel during a period of \( t \) seconds (m\(^3\)/m)
\( q_{b,0} \) = longitudinal bed-load transport at middle of slope (m\(^3\)/sm)
\( \phi \) = dynamic friction angle (generally taken 25°)
\( t_0 = \frac{\pi d^2}{64 (\tan \gamma)^2} \) = time-scale parameter(s)
\( \gamma \) = angle of initial side slope of the channel
\( d \) = channel depth, see Figure 10.3.8.

Fedose states that this formula, though it has been derived for parallel flow, is also valid for currents crossing the channel under a small angle relative to the channel axis. In that case the longitudinal component of the bed load on the side slope should be substituted into Equation 10.3.14.

*Formula of Lean (1980)*

The suspended sediment transport in the channel is:

\[
q_{s,x} = \left( \frac{b_0}{b_1} q_{s,0} \right) - \left( \frac{b_0}{b_1} q_{s,0} - q_{s,1} \right) \left( 1 - \exp \left( 1 - \frac{x}{A} \right) \right)
\]

in which:

\[
A = \frac{2 Z_0 \bar{U}_0}{w_s}
\]

\[
Z_0 = \frac{\varepsilon_0}{w_s} \left( 1 - \frac{w_s h_0}{\varepsilon_0 \left( -1 + \exp(w_s h_0/\varepsilon_0) \right)} \right)
\]

\( \varepsilon_0 \) = depth-averaged mixing coefficient upstream of channel
\( \bar{U}_0 \) = depth-averaged velocity upstream of channel

The deposition rate (\( \Delta s \)) per unit channel length due to the suspended load transport is:

\[
\Delta s = \left[ \left( \frac{b_0}{b_1} q_{s,0} \right) - q_{s,1} \right] \left[ 1 - \exp \left( - \frac{B}{A \sin \alpha} \right) \right] \sin \alpha
\]

The A-factor is completely expressed in parameters which are valid outside the channel. This may lead to unrealistic results in case of oblique flows.

*Formula of Eysink Vermaas (1983)*

Basically, this method is a parametrization of computer results of the SUSTRA model (Van Rijn, 1987).

The suspended sediment transport in the channel is expressed as:

\[
q_{s,x} = \left( \frac{b_0}{b_1} q_{s,0} \right) - \left( \frac{b_0}{b_1} q_{s,0} - q_{s,1} \right) \left( 1 - \exp \left( -A \frac{x}{h_1} \right) \right)
\]

10.24
in which

\[ A = 0.015 \left( \frac{2w_s}{u_{s,1}} \right) \left( 1 + \frac{2w_s}{u_{s,1}} \right) \left( 1 + 4.1 \left( \frac{k_s}{h_1} \right)^{0.25} \right) \]  \hspace{1cm} (10.3.20)

The backfilling rate (\( \Delta s \)) per unit channel length is:

\[ \Delta s = \left[ \left( \frac{b_0}{b_1} q_{s,0} - q_{s,1} \right) \left( 1 - \exp \left( -A \frac{B}{h_1 \sin \alpha_1} \right) \right) \right] \sin \alpha_1 \]  \hspace{1cm} (10.3.21)

**Mathematical model SUSTRA (Van Rijn, 1987)**

The transport of the suspended sediment particles is described by the convection-diffusion equation which represents the transport by convection, diffusion (or mixing) and gravity (settling). Assuming steady-state conditions and neglecting the transport by longitudinal diffusion, which usually is small compared to the other terms, the convection-diffusion equation can be expressed, as (Van Rijn, 1986, 1987):

\[ \frac{\partial}{\partial x} (buc) = \frac{\partial}{\partial z} \left( b(w - w_s)c \right) - \frac{\partial}{\partial z} \left( b \varepsilon_s \frac{\partial c}{\partial z} \right) = 0 \]  \hspace{1cm} (10.3.22)

in which:
- \( c \) = sediment concentration
- \( u \) = longitudinal velocity
- \( w \) = vertical velocity
- \( w_s \) = particle fall velocity
- \( \varepsilon_s \) = mixing or diffusion coefficient
- \( b \) = width
- \( x \) = longitudinal coordinate
- \( z \) = vertical coordinate

The width of the flow has been included to represent the suspended sediment transport in a laterally diverging or converging flow.

The convection-diffusion equation can be solved numerically when the flow velocities, the fall velocity, the mixing coefficients and the flow width are known.

The general input data are the bathymetry, the flow, wave and sediment characteristics. At the inlet boundary the sediment concentration profile should be known. At the outlet boundary no specifications are required since the horizontal diffusion has been neglected. At the water surface the net vertical sediment transport is set to zero. At the bed boundary the sediment concentration or the concentration gradient can be prescribed as a function of local hydraulic and sediment parameters.

The model has been used to determine the trapping efficiency of the channel. The channel is assumed to be infinitely long (see Fig. 10.3.8). The current velocities inside the channel were computed by a mathematical model (Boer, 1985) representing the refraction effect.

The sediment trapping efficiency factor is defined as the relative difference of the incoming suspended load transport and the minimum suspended load transport in the channel, as follows:

\[ e = \frac{t_0 q_{s,0} - t_1 q_{s,1,\text{minimum}}}{b_0 q_{s,0}} \]  \hspace{1cm} (10.3.23)
in which:
\(b_0\) = width of approaching streamtube,
\(h_0\) = width of streamtube in channel,
\(q_{s,0}\) = incoming suspended load transport per unit width,
\(q_{s,1}\) = suspended load transport in channel per unit width.

The backfilling rate (\(\Delta s\)) per unit channel length can be computed by:

\[
\Delta s = e q_{s,0} \sin \alpha_0
\]  
(10.3.24)

The basic parameters which determine the trapping efficiency factor, are (see Fig. 10.3.8): the approach angle (\(\alpha_{0}\)), the approach velocity (\(v_{r,0}\)), the approach depth (\(h_0\)), the approach bed-shear velocity (\(u_0\)), the particle fall velocity (\(w_s\)), the wave height (\(H\)), the channel depth (\(d\)), the channel width (\(B\)), the channel side slope (\(\tan \gamma\)) and the bed roughness (\(k_s\)). The functional relationship can be described as follows:

\[
e = F\left(\alpha_{0}, v_{r,0}, \frac{w_s}{u_0}, \frac{H}{h_0}, \frac{k_s}{h_0}, \frac{d}{h_0}, \frac{B}{h_0}, \tan \gamma\right)\]  
(10.3.25)

A sensitivity analysis has shown that the influence of the approach velocity \(v_{r,0}\), the relative wave height \(H/h_0\) and the relative roughness \(k_s/h_0\) is relatively small compared to the influence of the other parameters.

To reduce the number of computations, the former parameters were therefore not varied. In all, 300 computations were executed, using the following data:

- approach velocity (\(v_{r,0}\)) = 1 m/s;
- approach water depth (\(h_0\)) = 5 m;
- approach angles (\(\alpha_0\)) = 15°, 30°, 60°, 90°;
- channel depth (\(d\)) = 2, 2½, 5, 10 m;
- channel width (normal to axis), (\(B\)) = 50, 100, 200, 500 m;
- channel side slope (\(\tan \gamma\)) = 0.2, 0.1, 0.05;
- particle fall velocity (\(w_s\)) = 0.0021, 0.005, 0.0107, 0.0142, 0.025, 0.036 m/s;
- bed roughness (\(k_s\)) = 0.2 m.

The results are presented in Figs. 10.3.9 and 10.3.10. As can be observed, the trapping efficiency factor (\(e\)) increases for an increasing approach angle \(\alpha_0\). The maximum trapping efficiency does occur for \(\alpha_0 = 90\degree\), because there is a maximum reduction of the current velocity in the channel for this situation.

The applicability range of the graphs is limited to the values of the parameters not varied (approach velocity, bed roughness, wave height and dimensions). Additional computations have been carried out to extend the applicability range of the graphs accepting an error in the trapping efficiency factor of about 25%, which resulted in: \(v_{r,0} = 0.8\) to 1.2 m/s, \(k_s/h = 0.02\) to 0.06 and \(H/h_0 = 0\) to 0.3. More computations are necessary to extend the graphs to a velocity range from 0.2 to 2.0 m/s.

As an example the following data are given:

\[
\begin{align*}
\alpha_0 & = 30\degree \\
u_0 & = 1 \text{ m/s} \\
h_0 & = 5 \text{ m} \\
B & = 200 \text{ m} \\
u_{r,0} & = 0.05 \text{ m/s} \\
d & = 5 \text{ m} \\
w_s & = 0.01 \text{ m/s} \\
\tan \gamma & = 0.1 \\
q_{s,0} & = 1 \text{ kg/sm.}
\end{align*}
\]
The dimensionless parameters are computed, yielding \( w_s/u_{\infty,0} = 0.2, \ d/h_0 = 1, \ B/h_0 = 40. \) The trapping efficiency factor can be obtained from Fig. 10.3.9, yielding: \( e = 0.5. \)

The deposition per unit channel length due to an incoming suspended load transport of \( q_{b,0} = 1 \) kg/m is:

\[
\Delta s = e q_{b,0} \sin \alpha_0 = (0.5) (1) (\sin 30^\circ) = 0.25 \text{ kg/m}
\]

The total deposited mass in a channel with a length of 1000 m during one month is:

\[
M_s = (0.25) (30 \times 24 \times 3600) (1000) = 648 \times 10^6 \text{ kg}.
\]

\[\text{Figure 10.3.9} \quad \text{Trapping efficiency of suspended load for } \alpha_0 = 15^\circ, 30^\circ \text{ and } 60^\circ \text{ (Van Rijn, 1987)}\]
Figure 10.3.10 Trapping efficiency of suspended load for $\alpha_0 = 90^\circ$, channel normal to current (Van Rijn, 1987)
REFERENCES


REFERENCES (continued)


11. TRANSPORT OF COHESIVE MATERIALS

11.1 Introduction

Sediment mixtures with a fraction of clay particles (d < 4 µm, Am. Geoph. Union Scale) larger than about 10% have cohesive properties because electro-static forces comparable to or higher than the gravity forces are acting between the particles. Consequently, the sediment particles do not behave as individual particles but tend to stick together forming aggregates known as flocs whose size and settling velocity are much larger than those of the individual particles.

Most clay minerals have a layered (sheet-like) structure. The most important types of clay minerals are:
- kaolinite (two-layer structure)
- montmorillonite (three-layer structure)
- illite (three-layer structure)
- chlorite (four-layer structure)

Mud is herein defined as a fluid-sediment mixture consisting of (salt)water, sands, silts, clays and organic materials. Proper classification requires the determination (using standardized methods) of a minimum set of parameters, as follows:

sampling location : coordinates, water depth, maximum current velocity, maximum wave height,
bed structure : dry sediment density with depth, critical shear stress for erosion of surface,
native fluid : temperature, salinity, pH, chemical composition,
sediment : clay, silt, sand percentage, organic content, mineralogy, grain sizes, fall velocities, specific area, cation exchange capacity (CEC),
pore fluid : temperature, salinity, pH, sodium absorption ratio (SAR), cation/anion composition.

The composition of samples of natural muds found in The Netherlands and that of kaolinite material are presented in the following Table 11.1.

<table>
<thead>
<tr>
<th>Mud type</th>
<th>Ill. &lt; 2 µm (%)</th>
<th>Clay &lt; 16 µm (%)</th>
<th>Sand &gt; 63 µm (%)</th>
<th>Organic material</th>
<th>Stokes diameter of deflocculated material d50 (µm)</th>
<th>Fall velocity of deflocculated material w50 (mm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaolinite</td>
<td>40</td>
<td>95</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0.008</td>
</tr>
<tr>
<td>Hollands Diep 1</td>
<td>30</td>
<td>75</td>
<td>9</td>
<td>10</td>
<td>5</td>
<td>0.02</td>
</tr>
<tr>
<td>Hollands Diep 2</td>
<td>25</td>
<td>60</td>
<td>23</td>
<td>9</td>
<td>10</td>
<td>0.1</td>
</tr>
<tr>
<td>Ketelmeer</td>
<td>28</td>
<td>67</td>
<td>7</td>
<td>12</td>
<td>7</td>
<td>0.05</td>
</tr>
<tr>
<td>Biesbosch</td>
<td>28</td>
<td>65</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>0.06</td>
</tr>
<tr>
<td>Maas</td>
<td>16</td>
<td>42</td>
<td>36</td>
<td>8</td>
<td>25</td>
<td>0.5</td>
</tr>
<tr>
<td>Breskens Harbour</td>
<td>24</td>
<td>55</td>
<td>27</td>
<td>5</td>
<td>10</td>
<td>0.1</td>
</tr>
<tr>
<td>Delfzijl Harbour</td>
<td>15</td>
<td>30</td>
<td>60</td>
<td>2</td>
<td>80</td>
<td>5.5</td>
</tr>
<tr>
<td>Loswal Noord</td>
<td>14</td>
<td>23</td>
<td>69</td>
<td>2</td>
<td>100</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Table 11.1 Composition of kaolinite and natural muds
In a natural environment there is a continuous transport cycle of mud material which consists of: erosion, settling, deposition, consolidation, erosion and so on. As a result of the complexity and the lack of fundamental knowledge, the description of the various processes is largely empirical. Most information is based on laboratory experiments. Figures 11.1.1 and 11.1.2 show typical laboratory methods to study mud properties. Laboratory results are, however, often not representative because the biogenic mechanisms and the organic materials are missing, the water depths are small, the deposition and the consolidation history of the bed is different from nature. Therefore, more in-situ research should be carried out.

This chapter summarizes the most important properties and processes:

- viscosity and yield stress
- flocculation
- settling
- deposition
- consolidation
- erosion
- transport

11.2 Cohesion, plasticity, viscosity and yield stress

If a cohesive soil sample with a low water content is submitted to shear stresses ($\tau$) under various normal pressures ($\sigma$) to the point of failure, the relationship between $\tau$ and $\sigma$ can be expressed as (Law of Coulomb):

$$\tau = \tau_y + \sigma \tan \phi$$  \hspace{1cm} (11.2.1)

in which:

$\tau_y$ = yield stress

$\phi$ = angle of internal friction

The yield stress is generally interpreted as the "cohesion" of the sample. Thus, a cohesive sediment sample is able to withstand a finite shear stress for $\sigma = 0$ (no deformation). The angle of internal friction represents the mechanical resistance to deformation by friction and interlocking of the individual particles.

Plasticity is the property of cohesive material to undergo substantial permanent deformation without breaking.

Types of basic rheological behaviour of mud are shown in Fig. 11.2.1. Dilute suspensions with concentrations smaller than 10 kg/m³ show a Newtonian behaviour. Deviations from this latter behaviour tend to occur at concentrations larger than 10 kg/m³. High-concentrations ($> 50$ kg/m³) suspensions of water, fine sand, silt, clay and organic material usually have a pseudo-plastic or a Bingham plastic shearing behaviour, which means that the relationship between shear-stress ($\tau$) and shear rate ($\text{du/dz}$) is non-linear (Fig. 11.2.1). The slope of the curve expresses the effective dynamic viscosity ($\eta$) of the material. The intercept with the shear stress axis is called the yield stress ($\tau_y$) and represents the initial particle interaction force which must be overcome to give a distortion of the material. The yield stress of an ideal Bingham type mud is called the Bingham stress ($\tau_B$). Laboratory observation of static slopes of settled mud beds indicates that a yield stress is a physical reality.

11.2
Figure 11.1.1 Circular laboratory flume (carousel)

Figure 11.1.2 Laboratory settling tube
General formulations of rheological models are:

Newtonian : \[ \tau = \eta \frac{du}{dz} \]

Pseudo plastic : \[ \tau = m \left( \frac{du}{dz} \right)^n \]

Ideal Bingham : \[ \tau = \tau_B + \mu \left( \frac{du}{dz} \right) \]

Bingham plastic : \[ \tau = \tau_y + m \left( \frac{du}{dz} \right)^n \]  \hspace{1cm} (11.2.2)

in which:
\[ \tau = \text{shear stress} \]
\[ \tau_y = \text{yield stress} \]
\[ \tau_B = \text{Bingham stress} \]
\[ \mu = \text{dynamic viscosity coefficient} \]
\[ n = \text{empirical coefficient (} < 1 \text{)} \]
\[ m = \text{empirical coefficient} \]
\[ \frac{du}{dz} = \text{velocity gradient} \]

Viscosity and yield stress can be measured in a roto-viscometer. This instrument consists of two concentric cylinders in which the sediment material is placed (Fig. 11.2.2). One cylinder is rotated at a constant rate giving a constant shearing rate \((du/dz)\), while the force is measured on the other cylinder giving the shear stress \((\tau)\). Figure 11.2.3 shows the results for a natural mud based on the use of a Haake roto-viscometer (Winterwerp et al., 1991). The down-curve intercept with the vertical axis is defined as the yield stress \((\tau_y)\). The results show a hysteresis loop for increasing shear rates (up curve) and decreasing shear rates (downcurve). This phenomenon is known as thixotropy (time-dependent changes of the viscosity under constant shear) and is due to shear thinning effects.

Based on experimental data, the yield stress was found to be proportional to the sediment concentration \((\tau_y \approx c^\alpha \text{ with } \alpha = 2 \text{ to } 6)\). Many attempts have been made to relate the yield stress to the chemical and physical properties of the fluid-sediment mixture. General relationships, however, do not exist. Krone (1986) measured yield stresses of various natural muds and found values in the range of 0.01 to 0.1 N/m² for concentrations in the range of 30 to 60 kg/m³. Other values are given in Table 11.2, based on the tests (using native fluid) of Winterwerp et al., 1991. Figure 11.2.4 shows the yield stress as a function of concentration of these latter test results and the experimental ranges of others (Migniot, 1968; Owen, 1975 and Krone, 1985).

Comparison of the results for kaolinite in tap and saline water shows a considerable decrease (factor 2 to 3) of the yield stress due to the effect of salinity. Comparison of the results for the natural muds shows a considerable decrease (factor 2 to 3) of the yield stress when the percentage of fine sand is larger than about 40% (Maas, Delfzijl Harbour, Loswal Noord, see Table 11.1), which seems logical because the sand particles do not take part in the cohesive processes.

The natural muds with the smallest median particle diameter seem to have the highest yield stresses (Hollands Diep 1, Ketelmeer, Biesbosch, see Table 11.2). The functional relationships observed by Migniot (1968), Owen (1975) and Krone (1986) in their experimental ranges are in good agreement with the results of Winterwerp et al (1991), see Fig. 11.2.4.
<table>
<thead>
<tr>
<th>Material</th>
<th>sediment concentration (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Kaolinite (top)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_y = 0.01 - 0.02$ N/m²</td>
</tr>
<tr>
<td>Kaolinite (saline)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_y = -$</td>
</tr>
<tr>
<td>Hollands Diep 1 (lake)</td>
<td>$\tau_y = 0.03 - 0.06$ N/m²</td>
</tr>
<tr>
<td>Hollands Diep 2 (lake)</td>
<td>$\tau_y = 0.04 - 0.08$ N/m²</td>
</tr>
<tr>
<td>Ketelmeer (lake)</td>
<td>$\tau_y = 0.05 - 0.10$ N/m²</td>
</tr>
<tr>
<td>Bleskoog (lake)</td>
<td>$\tau_y = 0.10 - 0.15$ N/m²</td>
</tr>
<tr>
<td>Maas (river)</td>
<td>$\tau_y = 0.02 - 0.05$ N/m²</td>
</tr>
<tr>
<td>Breekse Harbour (estuary)</td>
<td>$\tau_y = -$</td>
</tr>
<tr>
<td>Delfzijl Harbour (estuary)</td>
<td>$\tau_y = 0.01 - 0.02$ N/m²</td>
</tr>
<tr>
<td>Loswal Noord (sea)</td>
<td>$\tau_y = 0.02 - 0.04$ N/m²</td>
</tr>
</tbody>
</table>

**Table 11.2** Yield stresses (N/m²)

Relative dynamic viscosity coefficients ($\eta_m/\eta$, with $\eta_m$ = viscosity of mixture, $\eta$ = viscosity of clear fluid) as a function of concentration for natural muids in The Netherlands are shown in Fig. 11.2.5. The viscosity of a fluid-sediment mixture begins to deviate significantly from that of clear water for concentrations larger than about 50 kg/m³. Mud samples with a high percentage of fine sand have a lower relative viscosity (Loswal Noord, Delfzijl Harbour, Maas river). The relative viscosity of kaolinite in saline water is relatively low compared with that of natural muds (low sand percentage).

Einstein (1906) considered the effect of elastic, spherical particle movements on the viscosity of dilute suspensions, resulting in an increase of the effective viscosity, see Chapter 3. Based on experiments with large volume concentrations, Bagnold (1954) also found an increase of the effective viscosity, see Chapter 3.

Both equations yield values which are considerably too small compared with those of natural muids (Fig. 11.2.5). More realistic values can be obtained by taking the volume concentrations of the particle aggregates or flocs (which are however unknown).

**Figure 11.2.1** Rheological behaviour: A. Pseudoplastic, B. Newtonian, C. Ideal Bingham, D. Bingham plastic
Figure 11.2.2 Examples of roto-viscometers

Figure 11.2.3 Bingham plastic behaviour of natural mud (Hollands Diep 1) at a concentration of 200 kg/m$^3$

Figure 11.2.4 Yield stress as a function of concentration for natural muds in The Netherlands (Winterwerp et al. 1991)
11.3 Flocculation

Most of the individual clay particles have a negative charge. The mutual forces experienced by two or more clay particles in close proximity are the result of the relative strengths of the attractive and repulsive forces. The attractive forces Van der Waals ($V_A$) are due to the interaction of the electrical fields formed by the dipoles in the individual molecules (Fig. 11.3.1). The repulsive forces ($V_R$) are due to ion clouds of similar charge repelling each other. Positive ions present in the fluid form a cloud of ions around the negatively charged clay particles (double layer theory). The result can be either attraction or repulsion depending on the relative strengths (depending on number of positive ions) and the distance between the particles.

![Diagram of electrical forces as a function of distance between particles](image-url)

**Figure 11.3.1** Electrical forces as a function of distance between particles

---

**Figure 11.2.5** Dynamic viscosity coefficient as a function of concentration for natural muds in The Netherlands (Winterwerp et al, 1991)
In fresh water suspensions (few positive ions) the repulsive forces between the negatively charged particles dominate and the particles will repel each other. In saline water the attractive forces dominate due to the (abundant) presence of positive sodium-ions forming a cloud of positive ions (cations) around the negatively charged clay particles resulting in the formation of flocs (aggregates), as shown in Fig. 11.3.2. Other binding forces are chemical forces (hydrogen bonds, cementation, coatings of organic materials).

Figure 11.3.2 Sizes of individual clay particles, flocs and floc groups

Flocculation requires particle collisions. The three collision mechanisms are:

- the Brownian motions of the particles (< 4 μm) due to the random bombardment by the thermally agitated water molecules; the number of collisions is linearly proportional to the concentration,
- turbulent mixing due to the presence of velocity gradients in the fluid and
- differential settling velocities because the larger particles have larger settling velocities and may therefore "fall" on the smaller particles.

Other factors affecting flocculation are: size, concentration of particles, salinity, temperature and organic material.

A small particle size in combination with a large concentration greatly intensifies the flocculation process because these two factors yield a small relative distance between the particles. Experimental research (Krone, 1962) has shown that flocculation quickly reaches an equilibrium situation at a salinity of about 5 to 10 promille which is small compared to that (35 promille) of sea water (Fig. 11.4.1).

A high temperature also enhances the flocculation process because the double layer repulsive energy decreases in magnitude and this leads to a decreased repulsion. Organic materials in and on the flocs significantly intensify the flocculation process, because of the binding properties of the organic materials. The binding forces become larger due to the presence of organic material (biogenic forces) and the flocs are becoming larger.

Break up of the flocs is caused by large shearing forces in the fluid when these forces are larger than the strength of the flocs. The flocs are broken into smaller flocs or particles. Large shearing forces exist close to the bottom where the velocity gradients are largest. Large shearing forces also exist in small-scale eddies everywhere in the fluid. Under the influence of turbulent forces there is a continuous process of flocculation and break-up resulting in a dynamic equilibrium of the flocs (size, density and strength).

In still water (no turbulence) the flocs may grow to larger sizes due to differential settling collisions. However, as the flocs get larger they fall faster until the fluid shear on the flocs becomes greater than the floc strength resulting in break-up. Analysis of under-water photographs shows the presence of macroflocs with sizes in the range of 100 to 1000 μm, miniflocs with sizes in the range of 10 to 100 μm and single mineral particles smaller than about 10 μm.
When the flocs grow larger, the floc size increases but the density of the flocs (consisting of sediment, fluid, organic materials) becomes smaller. Figure 11.3.3 shows the excess floc density as a function of floc diameter based on experimental research. Individual clay particles will have an excess density of about 1600 kg/m$^3$ ($\mu_0 - 2600$ kg/m$^3$, $\mu - 1000$ kg/m$^3$). Large flocs of about 1000 $\mu$m may have a density in the range of 1 to 10 kg/m$^3$ in excess of the fluid density, because most of the floc consists of (porc) fluid.

![Diagram](image)

**Figure 11.3.3** Differential density (floc density minus fluid density) as a function of floc diameter

11.4 Settling

An important parameter in sedimentation studies of cohesive materials is the settling velocity of the flocs. Analysis of laboratory and field data has shown that the settling velocity of the flocs is strongly related to the salinity, the sediment concentration (c), the water depth, the flow velocity and the type of measuring instrument.

11.4.1 Influence of salinity

Experimental laboratory research shows a clear effect of the salinity on the settling velocity for salinities up to 10 ppt when the sediment concentration is smaller than 1000 mg/l (Krone, 1962, see Fig. 11.4.1). When the sediment concentration is larger than 1000 mg/l, an almost linear increase of the settling velocity with the salinity can be observed (Owen, 1970; Allersma, 1967; see Fig. 11.4.1).

11.4.2 Influence of concentration

In saline suspensions with sediment concentrations up to about 1000 mg/l an increase of the settling velocity with concentration has been observed as a result of the flocculation effect both in laboratory and in field conditions (Fig. 11.4.2). When the sediment concentrations are larger than approximately 10000 mg/l, the settling velocity decreases with increasing concentrations due to the hindered settling effect (Fig. 11.4.2). Hindered settling is the effect that the settling velocity of the flocs is reduced due to an upward flow of fluid displaced by the flocs. At very large concentrations the vertical fluid flow can be so strong that the upward fluid drag forces on the flocs become equal to the
downward gravity forces resulting in a temporary state of dynamic equilibrium with no net vertical movement of the flocs. This state which occurs close to bed, generally is called fluid mud. In the laboratory the hindered settling velocity can be quite accurately determined from consolidation tests by measuring the subsidence of the sediment-fluid interface. The settling velocity in the two ranges can be expressed as:

\[ w_{s,m} = k \cdot c^m \quad \text{in flocculating suspensions (10-10000 mg/l)} \quad (11.4.1) \]

\[ w_{s,m} = w_s (1 - \alpha c)^\beta \quad \text{in hindered-settling suspensions (> 10000 mg/l)} \quad (11.4.2) \]

in which:

- \( w_{s,m} \) = settling velocity of flocs in fluid-sediment mixture
- \( w_s \) = settling velocity of individual particles
- \( c \) = volume concentration
- \( m \) = coefficient (= 1 to 2)
- \( k \) = coefficient
- \( \alpha \) = coefficient
- \( \beta \) = coefficient (= 3 to 5)

![Figure 11.4.1 The influence of salinity on the settling velocity (Burt, 1984)](image)

Settling velocities as a function of concentration in saline conditions from all over the world are shown in Fig. 11.4.2 (Severn, Avonmouth, Thames, Mersey in England; Western Scheldt in The Netherlands; River Scheldt in Belgium; Brisbane in Australia; Chao Phya in Thailand, Demerara in South America).
The settling velocities in flocculating suspensions were mostly determined in still water by analyzing fluid-sediment samples taken from the original suspension and may not represent the actual settling velocity in a natural turbulent flow.

\[ \text{Figure 11.4.2 The influence of sediment concentration on the settling velocity} \]

11.4.3 Influence of water depth and flow velocity

Analysis of settling velocities in fluids of different water depths shows a marked influence of the water depth, which is caused by the differential settling effect. Owen (1970) found that a minimum height of 2 m was required to achieve maximum flocculation and settling velocities (by differential settling). Differential settling is the effect that the larger flocs have larger settling velocities and may therefore "fall" on the smaller flocs forming new larger flocs. Consequently, in natural conditions the settling velocities in the lower layers will be larger than those in the upper layers, especially during the slack water period of the tide, when the disruptive turbulent fluid forces are absent. Figure 11.4.3 shows an example of concentration and settling velocity profiles at different times in a laboratory settling column with a height of 4 m (Cornelisse et al., 1990). Low-level turbulence was generated by an oscillating grid \( f = 0.025 \text{ Hz} \) giving an effective mixing coefficient of \( 3 \times 10^{-5} \text{ m}^2/\text{s} \). The settling velocities were computed from the (vertical) mass-balance equation using the measured concentration profiles as input data. As can be observed, the settling velocities are largest close to the bottom. The settling velocity is maximum after about 1600 s, which is the time scale to obtain maximum flocculation.

The settling process in the laboratory column is representative for the settling process in the slack water period (low velocities) of a tidal cycle in an estuary. At higher flow velocities the settling velocities near the bed may be significantly reduced due to the presence of disruptive shear forces (velocity gradients) in the boundary layer, as shown schematically in Fig. 11.4.4. Larger flocs will be broken down in smaller flocs, the latter being resuspended in the flow.
Burt (1984) found no influence of the tidal range (neap-spring cycle) on the settling velocity based on the analysis results of about 200 samples collected during a period of several years in the Thames estuary (England). He concluded that the flocs are mainly affected by eddy scales comparable to the floc size scales. These smaller eddy scales will be present during nearly all tidal conditions with exception of the slack tide periods.

11.4.4 Influence of measuring instrument

Figure 11.4.2 clearly shows that general relationships to determine the settling velocity of the flocs at a particular location are not available. The best approach is to do field measurements when there is no information available. Since, the settling and deposition process is most effective during the slack water period of the tidal flow, the measurements should be concentrated in this period. Sampling positions should be located near the water surface, at middepth and near the bed to determine differential settling effects (Fig. 11.4.4). These type of measurements are usually carried out with an in-situ settling tube, which is lowered horizontally to the sampling position to trap a fluid-sediment sample (by closing two valves on both ends of the tube). The sample container is rotated into a vertical position and the settling process starts (in still water). The settling velocity can be determined by measuring the sediment concentration in the container as a function of time. The length of the settling tube should be small (≈ 0.3 m) to avoid differential settling in the tube.

Van Leussen and Cornelisse (1991) used an in-situ video camera system and an in-situ settling tube to determine floc sizes and settling velocities in the Ems estuary in The Netherlands. The in-situ video system consists of a small vertical tube with a closed end at the bottom in which particles are settling down in still water. Two small windows are present in the tube for enlightening (light beam) and for video recordings (camera). The instrument was connected by a signal cable to the survey vessel which floated with the flow during the sampling period. Floc sizes and settling velocities were obtained from the recordings by computer analysis. Figure 11.4.5 shows settling velocities based on the in-situ video system and the in-situ settling tube. The video recordings showed the presence of a large amount of relatively large flocs with large settling velocities during the maximum flow period. During the slack tide period much smaller flocs and settling velocities were observed.

The large settling velocities around maximum flow were not found from the settling tube results.

Boere (1987) determined the settling velocities in the Western Scheldt Estuary by analyzing sediment concentration profiles measured during the slack water period. Neglecting horizontal diffusive transport \( \varepsilon_h \frac{\partial^2 c}{\partial x^2} \approx 0 \) and vertical convective transport \( (w \approx 0) \), the fall velocity can be determined from integration of the two-dimensional vertical mass balance equation, yielding:

\[
W_{sz} = \frac{1}{c} \left[ - \int_{-h}^{h} (\partial c/\partial t)dz - \int_{-h}^{h} u(\partial c/\partial x)dz - \varepsilon_t \frac{\partial c}{\partial z} \right] \tag{11.4.3}
\]

in which: \( W_{sz} \) = fall velocity at height \( z \), \( c \) = concentration at height \( z \), \( t \) = time, \( u \) = velocity at height \( z \), \( \varepsilon_t \) = mixing coefficient at height \( z \).

The settling velocities based on this method were found to be considerably larger (factor 10) than those derived from settling tube measurements. Based on the above-given results, it may be concluded that the use of an in-situ settling tube may result in relatively small settling velocities because of floc destruction during the sampling procedure. The video camera system may offer promising results, but it should be realized that the larger flocs are better observed by the camera than the smaller flocs. The in-situ settling tube represents the total distribution of particles, but the larger flocs may be destroyed during closing of the valves to trap the fluid-sediment sample.

11.12
Figure 11.4.3 Concentration and settling velocity profiles at various times
Mud Ketelmeer (lake), $c_0 = 0.5 \text{ kg/m}^3$, fresh water, $f = 0.025 \text{ Hz}$
Figure 11.4.4 Settling and flocculation in still and flowing water

Figure 11.4.5 Settling velocities at 2.8 m below the surface over a tidal cycle in the Ems-estuary
11.5 Deposition

11.5.1 Introduction

Deposition is predominant when the bed-shear stress ($\tau_b$) falls below a critical value for deposition ($\tau_d$), as shown by Krone (1962). He studied the deposition process of cohesive material by circulating natural mud at various flow velocities in a laboratory flume with saline water. The material (mud from San Francisco Bay) had a particle size distribution with 60\% smaller than 2 $\mu$m and 50\% smaller than 1 $\mu$m. Figure 11.5.1 shows the deposition processes in case of an initial mud concentration of 20 kg/m$^3$ and a flow velocity of about 0.1 m/s. Three distinct periods depending on the mud concentration can be observed:

- a period of about 10 hours with relatively (slow) deposition (due to the hindered settling effect) and (suspension) concentrations decreasing to about 10 kg/m$^3$,
- a period of about 100 hours with more rapid deposition (due to the flocculation effect) and (suspension) concentrations decreasing from about 10 kg/m$^3$ to about 0.3 kg/m$^3$,
- a final period of long duration with a very slow deposition of concentrations smaller than 0.3 kg/m$^3$. Deposition in this stage could be increased significantly by placing a grid in the flow (to intensify flocculation).

11.5.2 Concentrations larger than 10 kg/m$^3$

A fluid mud of sediment floccs was found to exist at concentrations above 10 kg/m$^3$ in which the floccs are partially supported by the escaping fluid (known as the hindered settling effect) and partially supported by interfloc contacts (Fig. 11.5.1). In nature where the water depths are large, there will be a situation with concentrations increasing towards the bed. The settling velocity reduces for increasing concentrations (hindered settling). Thus, the settling velocity will be relatively small near the bed and relatively large at the upper layers resulting in the formation of a near-bed fluid mud layer (especially in neap-tide conditions) with increasing sediment concentrations in the range of 10 to 300 kg/m$^3$. The thickness of the fluid mud layer will increase as long as the deposition rate at the upper side of the layer is larger than the consolidation rate at the bottom side. Values up to several meters have been observed. The fluid mud layer can be transported horizontally as a turbidity current due to:

- gravity forces on a sloping bottom,
- pressure gradient forces due to horizontal differences in sediment concentrations and fluid densities,
- shear forces at the interface generated by the overflowing water.

Turbulent mixing at the interface (luctoline) between the fluid mud layer and the overlying water generally is small because of the stabilizing effect of the heavier sediment material which has to be moved against gravity. Kirby and Parker (1980, 1983) have observed fluid mud layers with concentrations $>15$ kg/m$^3$ transported with velocities of about 0.3 m/s in the Severn Estuary in England (see also Section 11.8.2).

11.5.3 Concentrations from 0.3 to 10 kg/m$^3$

The deposition process in this concentration range is dominated by flocculation effects, as shown by the increase of the deposition rate in the flume experiment of Krone, 1962 (see Fig. 11.5.1).

The decrease in time of the concentration was, as follows:

$$\log(c/c_0) = -K \log(t/t_0)$$  \hspace{1cm} (11.5.1)
in which:
c_o  = initial concentration (at t = t_o)
t    = time
K    = coefficient

Krone conducted his deposition experiments in a straight flume under steady flows with bed-shear stresses smaller than the critical bed-shear stress for full deposition.

Mehta and Partheniades (1975) performed experiments with cohesive materials (initial concentrations of 1 to 10 kg/m³) under steady flow conditions with bed-shear stresses smaller and larger than the critical bed-shear stress for full deposition (see Figs. 11.5.4, 11.5.5 and 11.5.6).

The experiments were performed in a straight flume and in a circular flume (see Fig. 11.1.1). The circular flume was covered by a circular ring in contact with the water surface. A simultaneous rotation of the two components in opposite directions generated an uniform turbulent flow with minimum rotation-induced secondary currents. Figure 11.5.5 shows some experimental results. After a short period of rapid deposition an equilibrium concentration (c_eq) is obtained. Floccs with a low shear resistance are broken down in the near-bed layer with large velocity gradients whereas the smaller flocs and particles are resuspended. Floccs with a high shear resistance (strong flocs) can be deposited. The equilibrium concentration was found to be dependent on the applied bed-shear stress (τ_b), the type of sediment material and the initial concentration (c_o). Mehta and Partheniades also found a constant ratio c_eq/c_o at a constant bed-shear stress (τ_b) for various initial concentrations (c_o). The ratio c_eq/c_o which is dependent on the τ_b-value is shown in Fig. 11.5.6. The c_eq/c_o-ratio and the variation in time of the concentration c from c_o to c_eq can both be represented by logarithmic normal relationships (Mehta and Partheniades, 1975).

The experimental results can be interpreted by assuming that there are two groups of flocs. The first have sufficiently strong bonds to resist the disruptive near-bed shear stresses and will be able to reach the bottom and form strong bonds with deposited flocs. The second does not have sufficiently strong bonds and will be broken down before reaching the bed resulting in resuspension or are eroded very quickly after being deposited because the bond between the floc and the bed was relatively weak. The ratio c_eq/c_o represents the percentage of the flocs that remains in suspension and that have a floc strength smaller than the bed-shear stress.

The deposited flocs develop bonds with the bed flocs. The bed-shear stress to disrupt these bonds and to overcome the submerged floc weight has to be larger than that at which the floc is deposited. This larger value is called the critical bed-shear stress for erosion (τ_e).

In one experiment Mehta and Partheniades slowly replaced the fluid-sediment mixture (having the equilibrium concentration c_eq) by clear water and found no measurable sediment concentrations in the replaced water. This indicated that there was no erosion of the deposited strong flocs. Thus, there was no process of simultaneous deposition and erosion (equilibrium) for the group of strong flocs. This does not rule out the possibility that there may be simultaneous deposition and erosion for the group of weak flocs or individual particles.

Based on the experimental results of Mehta and Partheniades, the following processes can be distinguished (see also Fig. 11.5.6):
Figure 11.5.1  Deposition experiments of Krone (1962)
\[ c_0 = 20 \text{ kg/m}^3, \tau_b < \tau_{d,\text{full}} \text{ (full deposition)} \]

Figure 11.5.2  Deposition experiments of Krone (1962)
\[ c_0 = 0.1-1 \text{ kg/m}^3, \tau_b < \tau_{d,\text{full}} \text{ (full deposition)} \]

Figure 11.5.3  Deposition rates according to Krone (1962)
\[ c_0 = 0.1-1 \text{ kg/m}^3, \tau_b < \tau_{d,\text{full}} \text{ (full deposition)} \]
Figure 11.5.4 Deposition experiments of Mehta (1984), straight flume \( c_o = 1-5 \text{ kg/m}^3 \), \( \tau_b < \tau_{d, full} \) (full deposition)

Figure 11.5.5 Deposition experiments of Mehta and Partheniades (1975) \( c_o = 1-10 \text{ kg/m}^3 \), \( \tau_b > \tau_{d, full} \) (partial deposition)

Figure 11.5.6 Ratio of equilibrium and initial concentration as a function of bed-shear stress, Mehta and Partheniades (1975)
1. **Full deposition**

All sediment particles and flocs are deposited when the bed-shear stress \( \tau_b \) is smaller than the bed-shear stress for full deposition \( \tau_{d,\text{full}} \).

The deposition rate \( D \) is:

\[
D = c \alpha w_{s,m} \quad \text{for} \quad \tau_b < \tau_{d,\text{full}} \tag{11.5.2}
\]

in which:

- \( c \) = sediment concentration
- \( w_{s,m} \) = (concentration dependent) settling velocity distribution in still water
- \( \alpha \) = coefficient \( \leq 1 \)

The deposition rate \( D \) of Eq. (11.5.2) should be read as a quantity obtained by summation over size fractions based on the settling velocity distribution.

Equation (11.5.2) is valid for uniform concentrations over the depth, otherwise the near-bed concentration should be used.

The \( \alpha \)-coefficient can be interpreted as a reduction factor of the settling velocity (in still water) due to the presence of turbulence and small concentration gradients yielding an upward sediment flux \( \left( \varepsilon_c \text{ dc/dz} \right) \). Consequently, the \( \alpha \)-coefficient will be dependent on the bed-shear stress: \( \alpha = f(\tau_b) \) with \( \alpha = 1 \) for \( \tau_b = 0 \text{ N/m}^2 \) and \( \alpha < 1 \) for \( \tau_b > 0 \text{ N/m}^2 \).

Mehta and Partheniades (1975) found \( \tau_{d,\text{full}} = 0.15 \text{ N/m}^2 \) for kaolinite in distilled water.

2. **Hindered or partial deposition**

The group of relatively strong flocs is deposited, whereas the group of relatively weak flocs with a shear strength smaller or equal to the applied bed-shear stress \( \tau_b \) (see Fig. 11.5.6) remains in suspension. The relative magnitude of both groups depends on the bed-shear stress. The deposition rate can be represented as:

\[
D = (c-c_{eq}) \alpha w_{s,m} \quad \text{for} \quad \tau_{d,\text{full}} < \tau_b < \tau_{d,\text{part}} \tag{11.5.3}
\]

3. **No deposition**

No sediment flocs are deposited when the bed-shear stress is larger than the maximum bed-shear stress for deposition \( \tau_{d,\text{part}} \). Thus, no flocs are present with a shear strength larger than \( \tau_{d,\text{part}} \). The deposition rate is equal to zero:

\[
D = 0 \quad \text{for} \quad \tau_b > \tau_{d,\text{part}} \tag{11.5.4}
\]

### 11.5.4 Concentrations smaller than 0.3 kg/m³

Krone (1962) performed a series of deposition experiments in a flume at low concentrations. The flocculation effect appeared to be of minor importance. In each experiment a constant concentration \( c_0 \) was established in the flume while the water was recirculating. Then, the flow velocity was decreased to start the deposition process with decreasing concentrations, as shown in Fig. 11.5.2.

Assuming a uniform concentration profile, the deposition rate can be expressed as:

\[
\frac{d(c_0 c)}{dt} = c_0 \alpha w_{s,m} \tag{11.5.5}
\]
in which:
\( c \) = concentration at time \( t \)
\( h \) = water depth
\( w_{s,m} \) = settling velocity of particles or flocs at \( \tau_b = 0 \) (zero flow)
\( t \) = time
\( \alpha \) = coefficient

The quantity \((hc)\) represents the volume (or mass) in suspension per unit area.

Integration of Eq. (11.5.5) yields the decrease of the concentration in time (assuming constant settling velocity):

\[
c = c_o \ e^{-\alpha \ w_{s,m} \ h \ t}
\]  
(11.5.6)

in which:
\( c_o \) = initial concentration at \( t = 0 \).

Substitution of Eq. (11.5.6) in Eq. (11.5.5) yields:

\[
D_{\text{net}} = - \frac{d(hc)}{dt} = \alpha \ w_{s,m} \ c_o \ e^{-\alpha \ w_{s,m} \ h \ t}
\]  
(11.5.7)

The deposition rates measured by Krone (1962) are shown in Fig. 11.5.3 as a function of the applied bed shear stress \((\tau_b)\).

Extrapolation of the line indicates:
- no net deposition at \( \tau_b > 0.06 \) N/m²
- maximum deposition rate at \( \tau_b = 0 \) yielding a settling velocity \( w_{s,m} = 6.6 \times 10^{-6} \) m/s which is equivalent to a Stokes diameter of 2.5 \( \mu \)m.

According to Krone (1962), the settling velocity in Eq. (11.5.5) is the value measured at \( \tau_b = 0 \) (zero flow). The value of \( w_{s,m} = 6.6 \times 10^{-4} \) m/s (equivalent to a Stokes diameter of 2.5 \( \mu \)m) indicates that flocculation is of minor importance (Stokes diameter is about equal to that of the individual particles). Flocculation increased significantly when a grid was placed in the flume to create extra turbulence and hence flocculation (more particle collisions).

The \( \alpha \)-coefficient of Eq. (11.5.5) was found to be

\[
\alpha = (1 - \frac{\tau_b}{\tau_{d,\text{full}}})
\]  
(11.5.8)

with \( \alpha = 0 \) for \( \tau_b = \tau_{d,\text{full}} \) and \( \alpha = 1 \) for \( \tau_b = 0 \)

in which:
\( \tau_b \) = applied bed-shear stress
\( \tau_{d,\text{full}} \) = critical bed-shear for full deposition

The term \( \alpha \ w_{s,m} \) represents the apparent settling velocity in flowing water (see Eq. (11.5.2)). Based on the experimental results of Krone (1962) for low concentrations \((c < 0.3 \) kg/m³), the following processes can be distinguished:

\[
\text{no deposition} : \quad D = 0 \quad \text{for} \quad \tau_b \geq \tau_{d,\text{full}}
\]  
(11.5.9)

\[
\text{full deposition} : \quad D = \left(1 - \frac{\tau_b}{\tau_{d,\text{full}}}\right) c \ w_{s,m} \quad \text{for} \quad \tau_b < \tau_{d,\text{full}}
\]  
(11.5.10)
According to Eq. (11.5.7), all sediment particles will eventually be deposited yielding full deposition. The range of hindered or partial deposition was not observed by Krone. Probably for low concentrations there is a sharper transition from full deposition to no deposition because the floc sizes will be small (infrequent particle collisions) yielding a relatively uniform floc size and floc strength distribution. In a high-concentration suspension the floc size and floc strength distributions will be rather wide resulting in partial deposition in a certain bed-shear stress range.

11.5.5 Critical bed-shear stresses for deposition

Based on the experimental work of Mehta and Partheniades (1975), two critical bed-shear stresses for deposition can be distinguished: \( \tau_{d,\text{full}} \) and \( \tau_{d,\text{part}} \). The minimum bed-shear stress for full deposition \( (\tau_{d,\text{full}}) \) is defined as the bed-shear stress below which full deposition of the sediment flocs will occur. The maximum bed-shear stress for deposition \( (\tau_{d,\text{part}}) \) is defined as the bed-shear stress above which no deposition of the sediment flocs will occur. For a bed-shear stress in the range between \( \tau_{d,\text{full}} \) and \( \tau_{d,\text{part}} \) partial deposition may occur (see Fig. 11.5.6).

Experimental values of \( (\tau_{d,\text{full}}) \) and \( (\tau_{d,\text{part}}) \) have been presented by Mehta (1984) and by Winterwerp et al. (1991).

Mehta (1984) used three sediments: a mud from Maracaibo Estuary (Venezuela) and San Francisco Bay mud (USA), both in 35 ppt salt water and kaolinite in distilled water. The muds initially suspended at high velocities in a flume were allowed to deposit at a lower velocity. The initial concentrations were in the range of 1 to 5 kg/m³ (see Fig. 11.5.4). Winterwerp et al (1991) tested various natural muds from lakes, rivers and estuaries in The Netherlands in a circular flume using native fluids. The initial concentrations were in the range of 0.1 to 1 kg/m³. The method of Mehta and Partheniades (1975) was used to determine the \( (\tau_{d,\text{full}}) \)-values.

The results of Mehta (1984) and Winterwerp et al. (1991) are given in Table 11.3.

<table>
<thead>
<tr>
<th>Mud type</th>
<th>Sand (%)</th>
<th>Organics (%)</th>
<th>Critical deposition bed-shear stress (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \tau_{d,\text{full}} )</td>
</tr>
<tr>
<td>kaolinite (saline)</td>
<td>-</td>
<td>-</td>
<td>0.05</td>
</tr>
<tr>
<td>kaolinite (distilled)</td>
<td>-</td>
<td>-</td>
<td>0.15</td>
</tr>
<tr>
<td>Hollands Diep 1 (lake)</td>
<td>9</td>
<td>10</td>
<td>0.10</td>
</tr>
<tr>
<td>Hollands Diep 2 (lake)</td>
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<td>9</td>
<td>0.08</td>
</tr>
<tr>
<td>Ketelmeer (lake)</td>
<td>7</td>
<td>12</td>
<td>0.08</td>
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<tr>
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<td>8</td>
<td>8</td>
<td>0.04</td>
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<tr>
<td>Maas (river)</td>
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<td>0.06</td>
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<tr>
<td>Ijssel Noord (estuary)</td>
<td>60</td>
<td>?</td>
<td>0.08</td>
</tr>
<tr>
<td>San Francisco Bay</td>
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<td>-</td>
<td>0.10</td>
</tr>
<tr>
<td>Maracaibo estuary</td>
<td>-</td>
<td>-</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 11.3 Critical bed-shear stresses for deposition
11.5.6 Deposition rates

The maximum deposition rate \( D_{\text{max}} = c \, w_{s,m} \) will occur at low flow conditions as present during slack tide. Figure 11.5.7 shows the (maximum) deposition rate as a function of concentration, based on a settling velocity in the range of \( 10^6 \) to \( 10^3 \) m/s. As can be observed, the deposition rate is largest for a concentration of about \( 10 \) kg/m\(^3\). The variation of the deposition rate is largest in the flocculation range and smallest in the hindered settling range.

At higher (tidal) flow velocities the deposition rates will be reduced due to the effect of turbulence resulting in a smaller apparent settling velocity \( (w_{s,m}^* = \alpha \, w_{s,m} \text{ with } \alpha < 1) \).

Figure 11.5.7 Deposition rate as a function of concentration

11.6 Consolidation

The increase or decrease of the fluid-bed interface is equal to the rate of deposition (from the suspension) or erosion minus the rate of consolidation (of the bed). Consolidation is a process of floc compaction under the influence of gravity forces with a simultaneous expulsion of pore water and a gain in strength of the bed material.
Generally, three consolidation stages are distinguished (see also Fig. 11.6.1):

- **initial stage (days)**: The process consists of hindered settling and consolidation. The flocs in a freshly deposited layer are grouped in an open structure with a large pore volume. The weakest bonds are broken down first and the network gradually collapses. The bed surface sinks with time $t$.

- **secondary stage (weeks)**: The pore volume between the flocs is further reduced. Small thin vertical pipes (drains) are formed allowing the pore water to escape. The bed surface sinks with $t^{0.5}$ or $\log(t)$.

- **final stage (years)**: The pore volume inside the flocs is further reduced and the flocs are broken down. The bed surface sinks with $\log(t)$.

The consolidation process is strongly affected by the:

- initial thickness of the mud layer ($h_o$),
- initial concentration of the mud layer ($c_o$),
- permeability ($k$) of the mud layer (sediment composition and size, content of organic material, salinity, water temperature).

Gibson et al (1967) formulated an equation to describe the consolidation process. The consolidation equation expresses the void ratio ($e$) as a function of the vertical coordinate ($z$) and time ($t$). The void ratio ($e$) is the ratio of the void volume and the solid volume. The consolidation equation represents: the vertical equilibrium of the soil, the flow (Law of Darcy) and continuity of the pore fluid. The soil skeleton is assumed to be homogeneous. A review has been given by Kuijper (1992).

Based on experimental results, the functional relationship of the consolidation time scale ($T_c$) was found to be:

$$T_c = f\left(\frac{h_o^2}{k}\right)$$

(11.6.1)

Figure 11.6.3 shows the consolidation process in a layer of pure kaolinite material in saline water (31%). The initial height of the uniform suspension was $h_o = 0.1$ m and $h_o = 0.3$ m. The initial sediment concentrations (or dry densities) were 25, 50, 100, 175, 250 and 500 kg/m³ (Van Rijn, 1985). The tests consisted of measuring the height ($h$) of the kaolinite surface above the bottom of the consolidation column as a function of time. The various consolidation phases can be clearly observed. A thin layer ($h_o = 0.1$ m) of mud consolidates faster than a thick layer ($h_o = 0.3$ m) with the same initial concentration ($c_o$) because the pore water must travel over a longer distance to the mud surface. In a thick layer the kaolinite material is however more compacted (higher density) after the same consolidation period (7 days) due to the larger overburden (weight), as shown in Figure 11.6.3. Owen (1975) found opposite results for natural mud. A thicker final bed layer resulted in a lower dry density, probably due to the presence of organic materials.

A higher initial concentration will of course give a larger final bed thickness and thus a larger consolidation time. The final density also increases with increasing initial concentrations, probably because more dense flocs are formed (see Figure 11.6.3).

The consolidation behaviour of a natural mud is different (depending on the size ranges and the content of organic material) from that of pure kaolinite material.

Figure 11.6.2 shows the consolidation process of a very fine natural mud (without sand) from the Bangkok Bar Channel in saline water (Nedeco, 1985). Different initial suspension heights ($h_o$) and wet densities ($\rho_o$) were tested.

Similar to the consolidation of pure kaolinite, the consolidation process of the Bangkok mud proceeds relatively fast in a thin layer and relatively slow in a thick layer. The mud samples with a low initial density tend to have a somewhat lower final density than the samples with the higher initial density (similar to the pure kaolinite case).
**Figure 11.6.1** Consolidation phases (Mignot, 1989)

**Figure 11.6.2** Wet sediment density as a function of time
Bangkok mud (Nedeco, 1985)
A. CONSOLIDATION TEST, $h_o=0.1\text{ m}$

B. CONSOLIDATION TEST, $h_o=0.3\text{ m}$

**Figure 11.6.3** Mud layer thickness as a function of time
Kaolinite in saline water (Van Rijn, 1985)
In one experiment the consolidated material was stirred at regular intervals to simulate wave-induced and current-induced motions as present in nature, yielding a retardation of the consolidation process. The consolidation time scale of the Bangkok mud is much larger than that of pure kaolinite. A kaolinite layer with a thickness of 0.1 m obtains its final density after about 7 days, whereas the Bangkok mud shows a time scale of about 1 month. This latter behaviour is probably related to the presence of organic materials (with a dry density of about 1200 to 1500 kg/m³) which cause biochemical reactions resulting in gas production. Gas bubbles may escape during the initial consolidation phase when the material strength is low, thereby retarding the consolidation process. During later stages (or deeper layers) with increasing material strength, the gas bubbles may remain in the bed and affect the consolidation process, depending on the permeability of the bed material.

The influence of the sediment size and composition of natural muds on the consolidation process can be clearly observed from Figure 11.6.4, showing consolidation curves of natural muds (with different sand percentages) from lakes and estuaries in The Netherlands. The initial layer thickness and concentration were about the same for all tests (h₀ = 0.3 m, c₀ = 50 kg/m³). The consolidation curve of pure kaolinite in saline water is also shown. During the initial phase the consolidation of the natural muds proceeds much faster than that of pure kaolinite material due to the presence of high percentages of sand particles. During the final stage the consolidation of the natural muds proceeds considerably slower probably due to the presence of organic materials. The two mud samples with the highest content of organic material have the highest final bed thickness.

Information of the vertical distribution of the sediment concentration (dry density) of consolidated mud layers is given in Figure 11.6.5 based on data of Owen (1970), Thorn and Parson (1980) and Winterwerp et al (1991). The latter used an initial consolidation height of 0.3 m, whereas the former used values of 4.5 to 10 m. Thin mud layers show higher densities in the upper layers (z > 0.5 h) than thick mud layers. The relative densities in the top surface layer are about the same for all muds. Salinity had no marked influence on the density distribution (Owen, 1970).

Absolute values of the dry sediment density in top and bottom layer are given in Table 11.4. The top surface layer is defined as the surface layer with a thickness smaller than about 10 mm. The samples with a very high percentage of sand ( > 50 %) showed a top layer of mud on a lower layer of sand due to differential settling velocities. The dry densities in the top layer are in the range of 50 to 250 kg/m³ depending on the sediment composition and consolidation time. The dry densities in the bottom layer are in the range of 400 to 1500 kg/m³ depending on the percentage of sand.

The density profile of natural mud can be represented by

\[ c_z = \alpha \bar{c} \left( \frac{z}{h} \right)^{-\beta} \quad \text{for} \quad 0.1 \leq z/h \leq 0.9 \]  

(11.6.2)

where:

- \( c_z \) = dry sediment density at height \( z \) above "hard" bottom
- \( \bar{c} \) = average dry sediment density of layer
- \( h \) = thickness of mud layer
- \( \alpha, \beta \) = coefficients (\( \alpha + \beta = 1 \))

The \( \beta \) coefficient is about 0.3 to 0.6 for natural muds with low sand percentages ( < 5 %). Equation (11.6.2) is not accurate in the upper and lower layers.
Figure 11.6.4  Mud layer thickness as a function of time
Kaolinite and natural muds (Winterwerp et al, 1991)

Figure 11.6.5  Vertical profile of dry sediment density
Kaolinite and natural muds
Finally, values of the wet and dry sediment densities \( \rho_{\text{wet}} = \rho + \rho_{\text{dry}} (\rho_s - \rho) / \rho_s \) in successive consolidation stages in estuarine conditions are given in Table 11.5. Various consolidation stages are distinguished. Mud with a wet density of about 1200 kg/m\(^3\) is still sailable for ships. The nautical depth in the harbour of Rotterdam is defined at a wet density of 1200 kg/m\(^3\). The transition from the fluid to the solid phase occurs at a wet density of about 1300 kg/m\(^3\).

<table>
<thead>
<tr>
<th>Mud type</th>
<th>Sand (%)</th>
<th>Organic (%)</th>
<th>Thickness bed layer (m)</th>
<th>Thickness top layer (mm)</th>
<th>Dry sediment density (kg/m(^3))</th>
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<td></td>
<td></td>
<td>Top layer</td>
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<td>Bottom layer</td>
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<td></td>
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<td>≥ 7 days</td>
<td>1 day</td>
<td>≥ 7 days</td>
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</tr>
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<td>Grangemouth (estuary)</td>
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<td>0.3-0.4</td>
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<td>50-100</td>
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<td>350-500</td>
</tr>
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<td>Helawan (estuary)</td>
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<td>0.3-0.4</td>
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<td>50-100</td>
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<td>350-500</td>
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**Table 11.4 Densities of consolidated mud**

<table>
<thead>
<tr>
<th>Consolidation stage</th>
<th>Rheological behaviour</th>
<th>Wet sediment density (kg/m(^3))</th>
<th>Dry sediment density (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>freshly consolidated (1 day)</td>
<td>dilute fluid mud</td>
<td>1000-1050</td>
<td>0-100</td>
</tr>
<tr>
<td>weakly consolidated (1 week)</td>
<td>fluid mud (Bingham)</td>
<td>1050-1150</td>
<td>100-250</td>
</tr>
<tr>
<td>medium consolidated (1 month)</td>
<td>dense fluid mud (Bingham)</td>
<td>1150-1250</td>
<td>250-400</td>
</tr>
<tr>
<td>highly consolidated (1 year)</td>
<td>fluid-solid</td>
<td>1250-1350</td>
<td>400-550</td>
</tr>
<tr>
<td>stiff mud (10 years)</td>
<td>solid</td>
<td>1350-1400</td>
<td>550-650</td>
</tr>
<tr>
<td>hard mud (100 years)</td>
<td>solid</td>
<td>&gt; 1400</td>
<td>&gt; 650</td>
</tr>
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</table>

**Table 11.5 Density ranges of consolidated mud**

11.7 Erosion

11.7.1 Introduction

Sediment particles, flocs or lumps of the bed surface (including fluid mud layers) will be eroded when the applied current-induced or wave-induced bed-shear stress \( \tau_b \) exceeds a critical value for erosion \( \tau_e \), which depends on the bed material characteristics (mineral composition, organic materials, salinity, density etc.) and bed structure. Experimental results
show that the critical bed-shear stress for erosion is strongly dependent on the deposition and consolidation history. The critical bed-shear stress for erosion was (by many researchers) found to be larger than the critical bed-shear stress for full deposition \( \tau_e > \tau_{d,full} \). Mehta and Partheniades (1975) observed partial deposition (and no erosion) of mud material for (high) bed-shear stresses up to 1.5 N/m² in steady flows, because the bed consisted of strong dense flocs deposited during high-shear conditions in which the weak flocs could not be deposited. Thus, \( \tau_e \) may be very large when the top layer of the bed consists of strong flocs deposited at relatively high velocities \( \tau_{e > \tau_{d,part}} \).

In tidal conditions with decreasing velocities (near slack tide) the weak flocs will be deposited on top of the strong flocs deposited earlier at higher velocities. At increasing velocities of the next tidal cycle, these weak flocs can be eroded rather easily at low velocities. Summarizing, erosion will occur, when:

\[
\tau_e \geq \tau_{d,full} \quad \text{(11.7.1)}
\]

\[
\tau_e \geq \tau_{d,part} \text{ or } \tau_e > \tau_{d,part} \quad \text{depending on deposition history} \quad \text{(11.7.2)}
\]

Various types of erosion are distinguished in the literature: 1. particle or floc erosion (surface erosion) which is the one by one removal of particles and/or aggregates, 2. mass erosion which is the erosion of clusters or lumps of aggregates due to failure within the bed. Many attempts have been made to relate the \( \tau_e \)-values to basic parameters as plasticity index, void ratio, water content, yield stress and others.

Generally accepted relationships, however, are not available. Determination of the \( \tau_e \)-value must, therefore, be based on laboratory tests using natural mud or on in-situ field tests. Young and Southard (1978) used an in-situ flume to determine \( \tau_e \)-values. The observed values were found to be considerably smaller than those for corresponding laboratory flumes using the same material. The difference was ascribed to a change in sediment cohesion caused by natural bioturbation (biologically aggregated sediments) yielding lower densities.

### 11.7.2 Consolidated hard deposits

All information is based on the analysis of scour data of small natural channels. Lane (1955) proposed the following threshold velocities for erosion (Table 11.6):

<table>
<thead>
<tr>
<th>Material</th>
<th>Loose</th>
<th>Moderate compacted</th>
<th>Compact</th>
</tr>
</thead>
<tbody>
<tr>
<td>sandy clay</td>
<td>0.45 m/s</td>
<td>0.9 m/s</td>
<td>1.3 m/s</td>
</tr>
<tr>
<td>clay</td>
<td>0.35 m/s</td>
<td>0.8 m/s</td>
<td>1.2 m/s</td>
</tr>
<tr>
<td>lean clayey soil</td>
<td>0.3 m/s</td>
<td>0.7 m/s</td>
<td>1.0 m/s</td>
</tr>
</tbody>
</table>

*Table 11.6 Critical depth-averaged erosion velocities*

Hughes (1980) examined the scouring process in small natural channels (at 150 locations) in the southwestern USA by measuring and statistically analyzing velocities associated with initiation of scour. Three types of soils were distinguished: sandy silt, silty clay (firm compacted) and clay (compact). The results are shown in Fig. 11.7.1. The 1% threshold velocity is the maximum velocity which will not give channel erosion in 99 cases out of 100 and for practical purposes is equivalent to the maximum non-eroding velocity.

Comparison with other data showed good agreement when the 1% threshold value was used. Kamphuis (1982) found that a fluid containing sand particles will cause erosion of a consolidated cohesive bed at much lower fluid velocities (or shear stresses) than if the fluid was free of sand. The average erosion rates were of the order of \( 2 \times 10^{-3} \) m/hour for velocities in the range of 1 to 2 m/s.
Figure 11.7.1 Critical erosion velocities of consolidated hard deposits. Hughes (1980)
11.7.3 Consolidated soft deposits

Migniot (1968), Cornault (1971), Thorn (1981), Parchure and Mehta (1985), Winterwerp et al (1991) and others have observed that the critical bed-shear stress for erosion ($\tau_e$) is related to the sediment concentration (dry density).

The annular flume experiments of Parchure and Mehta (1985) clearly demonstrate that the $\tau_e$-value increases with depth in the bed layer, because the dry sediment density increases with depth. For a given constant current-related bed-shear stress an equilibrium concentration is established after some time (Fig. 11.7.2). A further increase of the current velocity and hence the bed-shear stress ($\tau_b$) will result (after some time) in a new but larger equilibrium concentration (Fig. 11.7.2).

This sequence can be interpreted as erosion of the top layer down to a level at which the bed strength ($\tau_b$) is equal to the applied bed-shear stress ($\tau_e$). Then, the erosion is stopped. This interpretation is confirmed by the results of a special experiment in which the fluid-sediment suspension above the bed was slowly replaced by clear water showing no further erosion (replaced fluid remained clear). Thus, the establishment of an equilibrium concentration is caused by an erosion stop ($\tau_b = \tau_e$) and not by saturation of the suspension. Parchure and Mehta (1985) determined $\tau_e$-values as a function of the dry sediment density. The tests were performed in an annular flume. Suspension concentrations were measured. Samples were taken from the bed by using a metal corer (after freezing). The samples were divided in segments to determine the dry densities. The thickness of the eroded bed layers at each stage (with constant equilibrium concentration) was determined from the increase of the suspension concentration. Since, the applied current-induced bed-shear stress ($\tau_e$) is always known, it follows that $\tau_e = \tau_b$ when there is no more erosion, while the dry sediment density of the top bed surface can be determined from the observed increase of the suspension concentration (corresponding to an eroded layer thickness). Figure 11.7.2 shows the results for kaolinite in saline water. Other experiments of Parchure and Mehta showed a clear increase of the $\tau_e$-value with increasing salinity (see Fig. 11.7.3). The effects were largest for salinities in the range of 0 to 2%. Further increase of the salinity only had a marginal effect. A larger consolidation time resulted in a larger $\tau_e$-value. No further increase was observed after a consolidation period of 10 days. The $\tau_e$-value at the top bed surface ($z=0$) was not dependent on consolidation time.

The results of Winterwerp et al (1991) for natural muds from The Netherlands give information of $\tau_e$-values related to surface erosion (particle or floc erosion) and $\tau_e$-values related to mass erosion (bed failure), see Table 11.7.

Two types of mud had a high percentage of sand particles, which were deposited in the bottom layers. In most tests a marked top layer consisting of mud with a thickness of about 5 mm was observed, which was eroded by surface erosion at bed-shear stresses in the range of about 0.15-0.30 N/m². Thereafter, the lower layers were eroded, first by surface erosion and after that by mass erosion (bed failure) at bed-shear stresses larger than about 0.5 N/m².

The critical bed-shear stress for erosion can be related to the dry sediment density ($\rho_{dry}$ in kg/m³), as follows:

$$\tau_e = \alpha (\rho_{dry})^\beta$$

(11.7.3)

The $\beta$-coefficient was found to be in the range from 1 to 2.5. Thorn and Parsons (1980) found $\beta = 2.3$ for mud from the Brisbane river (Australia), Grangemouth estuary (Scotland) and Belawan (Indonesia). Burt (1990) found $\beta = 1.5$ for mud from Cardiff Bay (England). The muds from The Netherlands suggest $\beta = 1.5$ (see Table 11.8).
Figure 11.7.2 Erosion tests of kaolinite (saline water) after Parchure and Mehta (1985)

Figure 11.7.3 Influence of salinity on critical bed-shear stress for erosion Lake Francis mud, Parchure and Mehta (1985)
Table 11.8 shows values of the critical bed-shear stresses for kaolinite and natural muds from The Netherlands, Australia, Scotland, Indonesia, France and England.

Migniot (1968) and Otsubo-Muraoko (1988) studied the relationship between the critical bed-shear stress for erosion ($\tau_\text{e}$) and the yield stress ($\tau_\text{y}$).

Migniot (1968) found $\tau_\text{e} = 2.5 \tau_\text{y}^{0.5}$ for $\tau_\text{y} < 1.5 \text{ N/m}^2$ and $\tau_\text{e} = 2 \tau_\text{y}$ for $\tau_\text{y} > 1.5 \text{ N/m}^2$.

Otsubo and Muraoko (1988) performed many tests to determine the critical shear stresses of natural muds. Based on their experimental results, the critical shear stress for surface erosion ($\tau_\text{e,1}$) and mass erosion ($\tau_\text{e,2}$) was related to the yield stress ($\tau_\text{y}$), as follows: $\tau_\text{e,1} = 0.27 \tau_\text{y}^{0.56}$ and $\tau_\text{e,2} = 0.79 \tau_\text{y}$.

A roto viscometer was used to determine the $\tau_\text{e,1}$-value. The $\tau_\text{e,1}$-values were in the range of 0.05-0.5 N/m$^2$, and $\tau_\text{e,2}$ in the range of 0.1-1 N/m$^2$.

<table>
<thead>
<tr>
<th>Mud type</th>
<th>Sand (%)</th>
<th>Organic (%)</th>
<th>$\tau_\text{e}$ = critical bed-shear stress erosion (N/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Surface erosion of top layer</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 day</td>
</tr>
<tr>
<td>kaolinite (saline water)</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
</tr>
<tr>
<td>Hollands Diep 1 (lake)</td>
<td>9</td>
<td>10</td>
<td>0.20</td>
</tr>
<tr>
<td>Hollands Diep 2 (lake)</td>
<td>23</td>
<td>9</td>
<td>0.20</td>
</tr>
<tr>
<td>Ketelmeer (lake)</td>
<td>7</td>
<td>12</td>
<td>0.15</td>
</tr>
<tr>
<td>Biesbosch (lake)</td>
<td>8</td>
<td>8</td>
<td>0.15</td>
</tr>
<tr>
<td>Maas (river)</td>
<td>36</td>
<td>8</td>
<td>0.20</td>
</tr>
<tr>
<td>Breskens Harbour (estuary)</td>
<td>77</td>
<td>5</td>
<td>0.20</td>
</tr>
<tr>
<td>Delfzijl Harbour (estuary)</td>
<td>60</td>
<td>2</td>
<td>0.10</td>
</tr>
<tr>
<td>Loswal Noord (sea)</td>
<td>69</td>
<td>2</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 11.7 Critical bed-shear stress for surface and mass erosion for consolidation periods of 1 and 7 days

<table>
<thead>
<tr>
<th>Mud type</th>
<th>Sand (%)</th>
<th>Organic (%)</th>
<th>$\tau_\text{e}$ = critical bed-shear stress erosion (N/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$c = 100$ kg/m$^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 day</td>
</tr>
<tr>
<td>kaolinite (saline water)</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>kaolinite (distilled water)</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Hollands Diep 1 (lake)</td>
<td>9</td>
<td>10</td>
<td>0.15-0.25</td>
</tr>
<tr>
<td>Hollands Diep 2 (lake)</td>
<td>23</td>
<td>9</td>
<td>0.15-0.25</td>
</tr>
<tr>
<td>Ketelmeer (lake)</td>
<td>7</td>
<td>12</td>
<td>0.10</td>
</tr>
<tr>
<td>Biesbosch (lake)</td>
<td>8</td>
<td>8</td>
<td>0.15-0.25</td>
</tr>
<tr>
<td>Maas (river)</td>
<td>36</td>
<td>8</td>
<td>0.15-0.30</td>
</tr>
<tr>
<td>Breskens Harbour (estuary)</td>
<td>27</td>
<td>5</td>
<td>0.15-0.25</td>
</tr>
<tr>
<td>Delfzijl Harbour (estuary)</td>
<td>60</td>
<td>2</td>
<td>0.05-0.15</td>
</tr>
<tr>
<td>Loswal Noord (sea)</td>
<td>69</td>
<td>2</td>
<td>0.20-0.30</td>
</tr>
<tr>
<td>Brisbane, Grangemouth, Belawan</td>
<td>0</td>
<td>-</td>
<td>0.20-0.30</td>
</tr>
<tr>
<td>Loire</td>
<td>-</td>
<td>-</td>
<td>0.10-0.15</td>
</tr>
<tr>
<td>Cardiff Bay</td>
<td>-</td>
<td>-</td>
<td>0.20-0.30</td>
</tr>
</tbody>
</table>

Table 11.8 Critical bed-shear stress for erosion for different sediment concentrations (dry density)
Finally, the influence of the various physical-chemical parameters is summarized. Experimental results (see Winterwerp, 1989) indicate that the critical bed-shear stress $\tau_e$:

- decreases for increasing temperature (weaker bonds),
- decreases for increasing pH-values (weaker bonds),
- decreases for increasing sand concentration (weaker bonds),
- increases for increasing clay concentration (stronger bonds),
- increases for increasing organic content (stronger bonds),
- increases for increasing salinity (stronger bonds),
- increases for increasing SAR (stronger bonds),
- increases for increasing CEC (stronger bonds).

### 11.7.4 Erosion rates

The erosion rates for a bed of constant density ($\tau_\infty = \text{constant}$) is, generally, expressed as (Partheniades, 1963):

$$ E = \frac{dm}{dt} = M \left[ \frac{\tau_b - \tau_\infty}{\tau_\infty} \right] \quad \text{for } \tau_b > \tau_\infty $$

(11.7.4)

The $M$-coefficient (in mass per unit area and time) is a material "constant" depending on mineral composition, organic material, salinity etc. Reported values are in the range of $M = 0.00001$-0.0005 kg/sm² for soft natural muds.

Parchure and Mehta (1985) found for a bed of increasing density ($\tau_\infty = \text{not constant}$) a relationship of the form:

$$ E - E_0 = \exp\left[\alpha(\tau_b - \tau_\infty)^{0.5}\right] \quad \text{for } \tau_b > \tau_\infty(z) $$

(11.7.5)

The $E_0$-value (in kg/sm²) is defined as the value for $\tau_b = \tau_\infty$ (at the surface $z = 0$) and can be determined by extrapolation from a plot of $E$ against $\tau_b - \tau_\infty$. The $E_0$-values are in the range of 0.00001 to 0.001 (kg/sm²) for kaolinite and natural mud in saline water. The $\alpha$-coefficient is in the range of 5 to 30 (m/N⁰.⁵).

### 11.7.5 Bed forms and roughness

When the critical shear stress for erosion is exceeded ($\tau_b > \tau_\infty$), various types of bed forms may be generated, as follows:

- longitudinal grooves and ridges with a typical spacing in the range of 0.01 m to 1 m, which are probably generated by spiral fluid motions. Meandering grooves have also been observed.
- flute marks which are spoon-shaped depressions in the bed surface probably generated by the highest velocities of turbulent eddies.
- small-scale isolated ripples with a length of 0.1 m and a height of 0.005 m have been observed in flumes (velocity = 0.3 m/s, $\tau_b = 0.2$ N/m²).

The micro topography of a muddy bed in low-energy regions may also be controlled by biological activity.

Although locally, small-scale bed forms are generated, the flow over a cohesive bed surface generally is found to be hydraulically smooth with relatively large Chézy-coefficients in the range of 60 to 100 m⁰.⁵/s. The effective bed roughness height is in the range of 0.1 to 1 mm.
11.8 Transport of mud by currents

11.8.1 (Quasi) steady flow

1. Wash load

The wash load of a river consists of the finest-grained portion of the total sediment load. The wash load shows an almost uniform concentration profile (small vertical gradients). The wash load typically is controlled by the upstream supply rate and not by the local river bed characteristics. Generally, the wash load bears no relationship to the discharge of the river. Concentrations tend to be large when there is a ready source of fine sediments in the drainage basin (soil erosion). Extremely large concentrations (hyperconcentrations up to 50% sediment by weight in the fluid) are a common occurrence during flash floods in semi-arid environments. Generally, this may occur in small rivers. The Yellow river is a dramatic exception with hyperconcentrations in the range of 500 to 1000 kg/m³ due to presence of extensive loess deposits in the upper river basin. About 10% of this material consists of clay (< 5 μm). The remaining material consists of silt and fine sand (25 to 50 μm), (Ning et al., 1986). Hyperconcentrations may enhance the total bed material transport because the suspended sand particles have a reduced settling velocity and remain longer in suspension before they are deposited again on the bed.

2. Density currents in reservoir

A density current is defined as the movement of a fluid-sediment mixture by gravity under, through or over another mixture fluid with a lower density. Density currents consisting of a high fraction of fine silt and clay material have been observed in reservoirs built in heavy silt-laden rivers in China (Jiahua, 1986, 1991) and the USA.

A density current will be generated at the upstream end of a reservoir when the internal density-related Froude number exceeds 0.8 (Jiahua, 1986). Thus:

\[
\frac{\bar{u}_o}{\left(\frac{\Delta \rho}{\rho} g h_o \right)^{0.5}} \geq 0.8
\]

(11.8.1)

in which:
- \( \bar{u}_o \) = mean velocity at initial location
- \( h_o \) = depth at initial location
- \( \Delta \rho \) = density difference
- \( \rho \) = density of incoming fluid-sediment mixture
- \( \Delta \rho = \rho - \rho_o \) = density difference
- \( \rho_o \) = density of surrounding fluid (clear water)

Equation (11.8.1) is valid for concentrations of the incoming mixture smaller than 100 kg/m³.

An indication for the generation of a density current at the upstream end of a reservoir is the plunging of the muddy river flow into the deeper reservoir. At the plunge point reversed flow and vortices have been observed at the water surface. Figure 11.8.1 shows the behaviour of a density current during the period of 16 to 18 August 1961 in the Sanmenxia Reservoir in China. Velocity and concentration profiles were measured along the reservoir. The density current had a thickness of approximately 3 to 5 m. A clear interface can be observed. Velocities inside the density current were in the range of 0.4 to 0.6 m/s. The density current traveled over the full length of the reservoir. Part of the density current material was deposited on the bed.
A density current can be reflected (partly) against an obstacle resulting in an increase of the layer thickness and a decrease of the velocity. This will enhance deposition.

The generation of a density current into a reservoir usually is a short period phenomenon (days), because the incoming flood discharge in arid regions increases and decreases abruptly. During a flood the silt and clay concentration (due to land surface erosion) increases enormously.

![Diagram of density current]

*Figure 11.8.1 Density current in Sanmenxia Reservoir, China*

11.8.2 Non-steady (tidal) flow

Field observations in tidal flow show the presence of a cyclic process of erosion, transport, settling, deposition and consolidation.

Generally, a three-layer system can be distinguished in vertical direction (see Fig. 11.8.2), as follows:

- consolidated mud layer at the bottom with concentrations larger than about 300 kg/m³. The flocs and particles are supported by the internal floc framework. The mud interface is detectable by echosounding instruments (30 kHz).
- fluid mud suspension layer with concentrations in the range of 10 to 300 kg/m³. The layer thickness is of the order of 0.1 to 1 m. The flocs and particles are supported by fluid drag forces exerted by the escaping fluid (hindered settling effect). A clear interface (lutocline) can be observed from echosounder recordings or from nuclear density recordings. Transport of fluid mud may take place by tide-induced and wave-induced forces and gravity forces (sloping bottom). Figure 11.8.3 shows a fluid mud layer in the Banjarmasin Channel (Kalimantan, Indonesia).
• dilute mud suspension with concentrations in the range of 0 to 10 kg/m³, which are detectable by optical methods and mechanical sampling (see Fig. 11.8.2). Flocculation is dominant. The flocs and particles are supported by turbulence-induced fluid forces and transported by tide-driven and wind-driven currents.

Vertical layers of different densities are influenced by gravity processes which oppose the mixing processes. The stability of the system can be characterized by the gradient Richardson number (R\textsubscript{g}). For R\textsubscript{g} larger than unity (based on experimental data) a stable system will be present (interfacial instabilities will die out). Winterwerp et al (1991) found R\textsubscript{g}-values upto 0.5 for the fluid mud layer in an annular flume experiment expressing the presence of interfacial instabilities (generation of internal waves).

![Figure 11.8.2 Vertical distribution of mud concentrations](image1)

*Figure 11.8.2 Vertical distribution of mud concentrations*

![Figure 11.8.3 Fluid mud layer, Kalimantan, Indonesia](image2)

*Figure 11.8.3 Fluid mud layer, Kalimantan, Indonesia*
Figure 11.8.4 Lutocline layer, Severn estuary, England (Kirby and Parker, 1980, 1983)

Erosion at the consolidated mud interface or at the fluid mud interface will occur in accelerating flow when the applied bed-shear stress exceeds the critical value for erosion ($\tau_b > \tau_c$), see Fig. 11.8.5. Similarly, deposition will take place in decelerating flow when the applied bed-shear stress falls below the critical value for deposition ($\tau_b < \tau_{d,par}$). Deposition is maximum around slack water because the floc destruction due to turbulent shear stresses is minimum and floc growth due to differential settling is maximum (large flocs "fall" on small flocs yielding new larger flocs). As a result of these processes the mud concentrations vary in time and space, see Fig. 11.8.5. The concentrations decrease near the slack water period (deposition) and increase towards maximum flow (erosion). Maximum concentrations generally occur at a certain (lag) period after maximum flow because it takes time to transport the particles or flocs to the upper layers, see Fig. 11.8.5. The lag period is relatively large near the water surface and relatively small near the bottom. The small settling velocities of the individual particles prevent settling of all particles during slack water. Thus, the concentration remains always larger than zero (background concentration). Figure 11.8.6 shows (natural) mud concentrations as a function of bed-shear stress for a tidal flow experiment in a circular flume (Winterwerp et al., 1991). The tidal flow introduces a lag effect (hysteresis effect).

Figure 11.8.4 shows typical velocity and mud concentration profiles from low water slack to high water slack in the Severn estuary, England (mean spring range = 12.3 m, mean neap range = 6.5 m, maximum spring flood velocity = 2 m/s). The mobile suspension system consists of three layers. The transition (middle) layer is called the lutocline layer. As the velocity increases towards maximum flow, the mud is mixed upwards by turbulence and the lutocline layer moves upwards. As the velocity decreases towards slack water, the sediments begin to settle and the lutocline layer moves downwards.
Figure 11.8.5 Variation of mud concentration in tidal flow

Figure 11.8.6 Variation of (natural) mud concentrations as a function of bed-shear stress in tidal flow (Winterwerp et al., 1991)
As the tidal energy decays towards neap tides, there is insufficient turbulence available resulting in a strong reduction of the thickness of the lutocline layer. Deposition and consolidation prevails and the fluid mud layer remains stable over several tidal periods.

Thus, the spring neap cycle may have a marked influence on the vertical structure of the mud suspension:

**springtide**: nearly uniform (well-mixed) concentration distribution at maximum flood and ebb velocities, formation of fluid mud interfaces (lutoclines) near the bottom during slack water, lutocline is moving downward, re-entrainment of mud during the next tide,

**neaptide**: tidal velocities are generally too small to cause erosion at the near-bed fluid mud layer, the fluid mud layer may survive several tidal cycles as a stationary layer.

Mathematically, the transport of cohesive sediments in a well-mixed estuary (no vertical salinity stratification) can be described by the convection-diffusion equation. Since the vertical concentration distribution above the fluid mud layer is nearly uniform, the convection-diffusion equation can be averaged over the depth, yielding:

\[
\frac{\partial \bar{c}}{\partial t} + \bar{u} \frac{\partial \bar{c}}{\partial x} + \bar{v} \frac{\partial \bar{c}}{\partial y} - \frac{1}{\bar{h}} \frac{\partial}{\partial x} \left[ hK_x \frac{\partial \bar{c}}{\partial x} \right] - \frac{1}{\bar{h}} \frac{\partial}{\partial y} \left[ hK_y \frac{\partial \bar{c}}{\partial y} \right] - \frac{S}{\bar{h}} = 0
\]  

(11.8.2)

in which:

\( \bar{c} \) = depth-averaged concentration

\( \bar{u}, \bar{v} \) = depth-averaged current velocities in x, y directions

\( K_x, K_y \) = dispersion coefficient in x, y directions

\( h \) = water depth

\( S \) = source-sink term representing erosion and deposition depending on the hydraulic conditions, \( S = E \) for \( \tau_b > \tau_e \) and \( S = D \) for \( \tau_b < \tau_d \) and \( S = 0 \) for \( \tau_d \leq \tau_b \leq \tau_e \).

The boundary conditions, erosion and deposition functions, are given in sections 11.5 and 11.7.

The bed layer should be represented as a number of sublayers, each with its own thickness, density, shear strength and consolidation stage. When deposition occurs, the thickness of the bed layer increases. When erosion occurs, the bed layer thickness decreases. Consolidation causes a continuous decrease of the bed-layer thickness.

In (partially) stratified estuaries the maximum silt and mud concentrations (turbidity maximum) are usually found in the area where the salt wedge is migrating during the tidal cycle. Figure 11.8.7 schematically shows the ebb tide flow of water and mud. Heavy sedimentation will occur in the salt wedge area resulting in the formation of soft mud layers (fluid mud) on the channel bottom.

During the ebb tide the river flow erodes the landward end of the mud layer. Near the toe of the wedge the fresh river water is lifted from the bottom and flows seaward over the heavy saline water. Intensive mixing will occur at the interface between the fresh and saline water. In the salt wedge the bottom current is landward. This saline water meets the fresh water near the toe of the wedge where it is carried upwards.

The mud carried by the river enters the area of the salt wedge together with the mud eroded from the landward end of the mud layer. This mud is mixed (near the interface) with mud already suspended in the saline water resulting in additional flocculation and increased settling velocities. Settling will occur of the sharp salt wedge interface where the mixing is reduced. The mud particles fall towards the bottom where they are transported landward again by relatively strong bottom currents of saline water (see Fig. 11.8.7). Summarizing, there are
relatively strong landward bottom-currents (Fig. 11.8.8) with high mud concentrations during flood tide and relatively weak seaward bottom-currents (Fig. 11.8.8) with low mud concentrations during ebb tide resulting in a net transport of mud in landward direction. Mathematically, the transport of silt and mud in a stratified estuary can be described by the time-dependent three-dimensional convection-diffusion equation. The erosion of bed material is represented by an erosion function specifying the erosion rate at the bed. The settling effect is represented by the settling velocity.

The convection-diffusion equation reads, as:

\[
\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) + \frac{\partial}{\partial z}(w-w_s)c - \frac{\partial}{\partial x}(\epsilon_{s,x} \frac{\partial c}{\partial x}) - \frac{\partial}{\partial y}(\epsilon_{s,y} \frac{\partial c}{\partial y}) - \frac{\partial}{\partial z}(\epsilon_{s,z} \frac{\partial c}{\partial z}) = 0
\]  

(11.8.3)

in which:
- \(c\) = mud concentration
- \(u,v,w\) = flow velocities in \(x,y,z\) directions
- \(w_s\) = settling velocity
- \(\epsilon_{s,x}, \epsilon_{s,y}, \epsilon_{s,z}\) = sediment mixing coefficients in \(x,y,z\) directions

\[\text{Figure 11.8.7} \quad \text{Flow of water and silt/mud near the salt wedge in a stratified estuary}\]
Figure 11.8.8 Characteristic velocity profiles during ebb and flood tide in case of a horizontal landward salinity (density) gradient

11.9 Transport of mud by waves

Field observations have shown a significant increase of the mud concentrations near the bed during and after periods with large waves (Van Rijn and Louisse, 1987; Kendrick and Clifford, 1981). Flume experiments with waves over a consolidated mud bed have shown that large waves can easily fluidize the top layer of the mud bed and generate a thin fluid mud layer with concentrations larger than 100 kg/m³ (Van Rijn and Louisse, 1987; Maa and Mehta, 1986). Fluidization of the top mud layer is initiated by wave-induced pressure variations at the bed, which lead to an increase of the water pressure in the pores and hence to a reduction of the internal soil shear strength. Various types of sediment mixtures in saline water were tested by Van Rijn and Louisse (1985, 1987). In four experiments the bed consisted of pure (100%) kaolinite with different densities. The dry density of the top layer (thickness of about 0.01 m) was less than about 200 kg/m³ in these tests. In two other experiments the bed consisted of a mixture of fine sand (= 100 μm) and kaolinite. Each experiment consisted of executing a sequence of tests with increasing wave heights. At each constant wave height, an equilibrium concentration was established after a few hours.

Optically and mechanically-collected samples were used to measure the sediment concentrations. The results of test T4 with a bed of kaolinite (ρ_{dry} = 640 kg/m³), a wave period of 1.5 s and a water depth of 0.25 m are shown in Fig. 11.9.1. The following features were observed:

wave height = 0.022 m : general sediment movement, generation of a thin suspension layer (= 0.01 m) with concentrations of about 0.1 kg/m³.

wave height = 0.03 m : suspension layer with a thickness of 0.05 m and concentrations up to 0.5 kg/m³ (Fig. 11.9.1A), a vague interface was visible.
Figure 11.9.1 Concentration profiles in waves Kaolinite bed, Van Rijn (1985)
Wave height $= 0.047 \text{ m}$: initially, the concentrations were reduced due to orbital mixing with clear water from higher levels, after 30 minutes the concentrations increased rapidly. Equilibrium concentrations ranging from $2.5 \text{ kg/m}^3$ near the bed to $0.5 \text{ kg/m}^3$ near the surface were established after 200 minutes (Fig. 11.9.1B). After this period, the wave generator was stopped and the suspension was allowed to settle and consolidate for 43 hours resulting in a clear fluid with locally small scour holes (depth $= 0.02 \text{ m}$) on the bed.

Wave height $= 0.07 \text{ m}$: generation of a fluid mud layer ($= 0.025 \text{ m}$) with concentrations of $100 \text{ kg/m}^3$, the concentrations above the fluid mud layer ranged from $20 \text{ kg/m}^3$ near the bed to $4 \text{ kg/m}^3$ near the surface (Fig. 11.9.1C). Dye injections showed that the fluid mud layer was slowly ($= 0.02 \text{ m/s}$) moving in the wave direction by wave induced velocities (drift). The generation of a fluid mud layer has a stabilizing effect on the erosion process because velocity gradients and hence bed-shear stresses remain relatively small.

Experiments with mixtures of fine sand ($= 100 \mu\text{m}$) and kaolinite showed that a consolidated bed of $75\%$ kaolinite and $25\%$ fine sand had a similar erosional behaviour as a pure ($100\%$) kaolinite bed. A bed consisting of $25\%$ kaolinite and $75\%$ fine sand had a completely different behaviour. Fluidization of the top layer did not occur. The kaolinite concentrations were not larger than about $0.3 \text{ kg/m}^3$, because only the top layer of the bed was washed out. The sand concentrations were also quite small (factor 30 smaller than in case of $100\%$ sand bed) due to a strong suppression of sand ripples (height $= 0.002 \text{ m}$).

Winterwerp et al (1993) tested natural mud ($70\%$ clay and silt, $20\%$ sand, $10\%$ organics) in a wave flume and found a clear influence of the dry density in the top layer of the bed on the fluidization process. Two types of bed (bed 1 and 2) were tested, see Fig. 11.9.2.

![Figure 11.9.2 Dry sediment density profiles of bed 1 and bed 2](image-url)
Bed 1 was obtained by placing the collected mud as one thick layer into the flume. The gas content was in the range of 0.3 to 1.8%. Bed 2 was obtained by deposition from a diluted suspension (120 kg/m³) using a pump system. This resulted in a rather low density (smaller than 200 kg/m³) of the top layer due to stratification effects; the larger particles were deposited in the lower layers whereas the finer particles and the organic materials were deposited in the upper layer (thickness of about 0.04 m). This density profile was obtained after about 1 day, but no change was observed after longer consolidation periods. Deposition in natural conditions is believed to consist of successive deposition events over a long period. Each event will occur during slack tide (low velocities) and result in a rather thin layer (< 1 mm) of deposited mud. Consolidation of a thin layer can proceed rapidly resulting in a relatively high density as represented by bed 1.

Regular and irregular waves with peak bed-shear stresses between 0.1 and 1 N/m² were generated over bed 1. Erosion started at a bed-shear stress of 0.3 N/m². The surface erosion was rather small, even at the highest waves. The eroded layer thickness was of the order of 0.1 mm. The mud concentrations over the water depth (= 0.25 m) were about 0.2 to 0.3 kg/m³. Gas bubbles did not affect the erosion process, but they had enough time to escape from the bed surface. Fluidization of the bed surface did not occur in any of the tests. Similar tests over bed 2 resulted in fluidization of the top layer, which grew in thickness with increasing wave height. The maximum thickness was about 0.05 m at the end of the test. The maximum mud concentration over the water depth was about 2 kg/m³. Wave damping of about 20% due to energy dissipation in the fluidization layer was observed.

Based on these results, it is concluded that a top layer with a dry density less than about 200 kg/m³ can be easily fluidized by wave action resulting in a thin near-bed layer with high concentrations (200 to 400 kg/m³).
REFERENCES


11.46
REFERENCES (continued)


REFERENCES (continued)


12. MATHEMATICAL MODELS OF SEDIMENT TRANSPORT

12.1 Introduction

The selection and application of morphological models are strongly related to the type and scale of the problem studied. When the natural system is largely disturbed as caused by the closure of a channel, the construction of a barrage or a harbour or the reclamation of new land, the morphological consequences should be studied on the basis of model predictions. Not only the impact of anthropogenic factors (human interference) but also the autonomous coastal development due to natural processes may be studied and simulated by morphological modelling.

Generally, the first stage of a study consists of the analysis of the existing data (flow patterns, bed material composition, development of depth contours). During the second stage of the study, flow models and wave propagation models are applied to determine the hydraulic conditions in the existing and the future situation. The results of these models are then used as input data for the morphological model. Two types of models can be distinguished:

- initial or sediment transport models (Fig. 1.1) which compute the sediment transport rates and the bed level changes for one time step or for one tidal cycle, resulting in a short-term prediction;
- dynamic morphological models (Fig. 1.1) which compute the flow velocities, the wave heights, the sediment transport rates, the bed level changes and again the new flow velocities, etc. in a continuous sequence (loop) resulting in long-term predictions.

![Structure of initial and dynamic morphological models](image)

**Figure 12.1.1 Structure of initial and dynamic morphological models**

Operational dynamic models are available for one-dimensional (1D), two-dimensional vertical (2DV) and horizontal (2DH) simulations. The application of dynamic models for three-dimensional (3D) situations is not yet feasible because of excessive computer cost.

The selection of a model, initial or dynamic, 1D or 2D, depends on:

- the available input data (flow velocities, wave heights, bed material composition),
- the available calibration data (sediment transport rates, erosion and deposition rates),
- the physical reliability (degree of physical schematization),
• the scale of the problem,
• the required accuracy, and
• the available budget.

Especially, the quality of the input data is very important. There is no point in selecting a sophisticated mathematical model when the input data are of poor quality. Accurate predictions require a detailed calibration of the model aimed at reproducing the morphological development in the present (existing) situation. The most relevant parameters (particle diameter, fall velocity, bed roughness, sediment transport) of the model should be varied within their validity ranges to evaluate their effect on the morphological results (sensitivity computations).

The calibrated model is then used to compute the morphological development for the new (future) situation.

Usually, three conditions are considered:
• the worst-storm condition (24 hours) involving the 1 in 100 years significant wave height combined with maximum occurring current velocity,
• the medium-term (1 year) prediction,
• the long-term prediction (10 to 50 years).

Much experience is necessary to interpret the results of morphological models because of the relatively poor schematization of the physical processes (lack of knowledge).

General background information of flow models, wave-propagation models, sediment transport models and morphological models is presented in sections 12.2, 12.3 and 12.4. State of the art reviews are presented by Van Rijn (1989) and by O’Connor (1991).

12.2 Flow models

12.2.1 Introduction

It has been a common practice to divide the mathematical flow models into different classes according to the dimensionality of the phenomena involved. Thus, for example, a flow in which the motion is predominantly confined to one direction as may occur in a pipe, a flume or a straight channel, is said to be: one-dimensional flow. Consequently, the continuum equations that describe this motion in a mathematical model are formulated in terms of one independent space variable.

Similarly, the flow in a well-mixed shallow estuary is predominantly two-dimensional, whereas the flow near structures (with separation and reattachment) essentially is three-dimensional.

Sometimes, for reasons of simplicity, a flow model makes use of the assumption that the pressure distribution follows the hydrostatic law. This restricts the range of applications of the model because it cannot be used to compute the flow in the vertical direction. Such a model is then not, strictly speaking, a three-dimensional model, but it is more a two-dimensional model, even though it is described in terms of three independent space variables.

Three-dimensional flow (3D) simulations are of particular interest in situations where the flow field shows significant variations in vertical direction. Examples are salt intrusion in estuaries, fresh water discharges in bays, thermal stratifications in lakes and seas, wind-driven circulations in lakes, seas and oceans, flow near structures etc.

Two-dimensional depth-averaged flow (2DH) simulations are of particular interest in situations where the flow field shows no significant variations in vertical direction and where
the fluid density is constant. Examples are tidal flow in well-mixed estuaries and in seas, wind-driven circulation in shallow lakes.

Two dimensional flow simulations in the vertical plane (2DV) are of interest in situations where the flow is uniform in one horizontal (lateral) direction, but with significant variations in vertical direction. Examples are the flow across a trench or navigation channel, wind-driven circulation perpendicular to the coast, flow over long-crested sand dunes.

One-dimensional flow (1D) simulations are of interest in situations where the flow field shows little variation over the cross section. Examples are river flow and flow in network systems.

12.2.2 Three-dimensional flow models (3D)

1. Equations

The most general hydrodynamic model to describe the flow field in a certain space domain is a three-dimensional time-dependent model. The various processes are commonly described in terms of "balances" such as those of fluid masses, momentum, energy and vorticity. Other aspects of the fluid behaviour are described in terms of more or less empirical equations such as those connecting the fluid density to the temperature and the salinity of the fluid. Usually, the effects of small-scale turbulent motions on the time-averaged flow are also represented by empirical equations connecting shear stresses to velocity gradients (eddy viscosity concept).

The basic equations for time-averaged non-homogeneous flow are the Reynolds' equations, which read as follows:

Momentum balance

\[ \begin{align*}
\frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial x} (\rho uu) + \frac{\partial}{\partial y} (\rho uv) + \frac{\partial}{\partial z} (\rho uw) + \frac{\partial}{\partial x} (\rho \frac{p}{\rho}) + \\
- \frac{\partial}{\partial x} \left( \frac{\sigma_{xx}}{\rho} \right) - \frac{\partial}{\partial y} \left( \frac{\tau_{xy}}{\rho} \right) - \frac{\partial}{\partial z} \left( \frac{\tau_{xz}}{\rho} \right) - \sum \frac{F_x}{\rho} = 0
\end{align*} \] (12.2.1)

\[ \begin{align*}
\frac{\partial}{\partial t} (\rho v) + \frac{\partial}{\partial x} (\rho vu) + \frac{\partial}{\partial y} (\rho vv) + \frac{\partial}{\partial z} (\rho vw) + \frac{\partial}{\partial y} (\rho \frac{p}{\rho}) + \\
- \frac{\partial}{\partial x} \left( \frac{\sigma_{yx}}{\rho} \right) - \frac{\partial}{\partial y} \left( \frac{\tau_{yx}}{\rho} \right) - \frac{\partial}{\partial z} \left( \frac{\tau_{yz}}{\rho} \right) - \sum \frac{F_y}{\rho} = 0
\end{align*} \] (12.2.2)

\[ \begin{align*}
\frac{\partial}{\partial t} (\rho w) + \frac{\partial}{\partial x} (\rho wu) + \frac{\partial}{\partial y} (\rho vw) + \frac{\partial}{\partial z} (\rho ww) + \frac{\partial}{\partial z} (\rho \frac{p}{\rho}) + \\
- \frac{\partial}{\partial x} \left( \frac{\sigma_{zx}}{\rho} \right) - \frac{\partial}{\partial y} \left( \frac{\tau_{zx}}{\rho} \right) - \frac{\partial}{\partial z} \left( \frac{\tau_{zz}}{\rho} \right) - \sum \frac{F_z}{\rho} = 0
\end{align*} \] (12.2.3)

Mass balance

\[ \frac{\partial}{\partial t} (\rho) + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0 \] (12.2.4)

12.3
in which:
\[ u, v, w = \text{fluid velocities in } x, y, z \text{-directions} \]
\[ p = \text{fluid pressure} \]
\[ \rho = \text{fluid density} \]
\[ t = \text{time} \]
\[ \sigma = \text{Reynolds’ pressure} \]
\[ \tau = \text{Reynolds’ stress} \]
\[ F = \text{external driving force (wind, waves, Coriolis)} \]

The fluid density is a known function of the salt concentration and the fluid temperature. Thus:

\[ \rho = F(s,T) \quad (12.2.5) \]

The Reynolds’ stresses usually are represented through the eddy viscosity concept (first order turbulence model), yielding:

\[ \sigma_{xx} = \rho \varepsilon_x \frac{\partial u}{\partial x} \quad \sigma_{yy} = \rho \varepsilon_y \frac{\partial v}{\partial y} \quad \sigma_{zz} = \rho \varepsilon_z \frac{\partial w}{\partial z} \]

\[ \tau_{xy} = \rho \varepsilon_y \frac{\partial u}{\partial y} \quad \tau_{yx} = \rho \varepsilon_x \frac{\partial u}{\partial x} \]

\[ \tau_{yz} = \rho \varepsilon_z \frac{\partial v}{\partial z} \quad \tau_{zy} = \rho \varepsilon_y \frac{\partial v}{\partial y} \]

\[ \tau_{xz} = \rho \varepsilon_x \frac{\partial w}{\partial x} \quad \tau_{zx} = \rho \varepsilon_z \frac{\partial w}{\partial z} \quad (12.2.6) \]

Generally, the \( \varepsilon_x \)- and \( \varepsilon_y \)-values are assumed to be constant, whereas the \( \varepsilon_z \)-value is represented as a parabolic function over the water depth (for gradually varying flow conditions).

The variation of the salt concentration (salinity) and the fluid temperature and hence the fluid density variation are described by mass balance equations, as follows:

\[ \frac{\partial s}{\partial t} + \frac{\partial (us)}{\partial x} + \frac{\partial (vs)}{\partial y} + \frac{\partial (ws)}{\partial z} - \frac{\partial}{\partial x} \left( \varepsilon_x \frac{\partial s}{\partial x} \right) - \frac{\partial}{\partial y} \left( \varepsilon_y \frac{\partial s}{\partial y} \right) - \frac{\partial}{\partial z} \left( \varepsilon_z \frac{\partial s}{\partial z} \right) = 0 \quad (12.2.7) \]

in which:
\[ s = \text{salt concentration} \]
\[ \varepsilon_x, \varepsilon_y, \varepsilon_z = \text{eddy viscosity coefficients (function of local Richardson number)} \]

A similar equation is used to describe the fluid temperature.

When the water depth is small compared to the horizontal scale of the computation domain (shallow water approximation), it is allowed to neglect the vertical fluid accelerations with respect to the gravity acceleration resulting in a hydrostatic pressure distribution. The momentum equation in the vertical (z) direction reduces to:

\[ p = p_s + g \int_{z}^{z} \rho \, dz \quad (12.2.8) \]

12.4
in which:
\( p_0 \) = atmospheric pressure at the water surface
\( z_s \) = water surface level above a reference datum.

The shallow water approximation imposes an important restriction to the application of the model: vertical accelerations due to sudden changes of the bottom topography and due to buoyancy effects (density differences) are not represented. Thus, the detailed phenomena near the salinity wedge cannot be represented.

A major problem encountered in 3D-modelling is the determination of suitable and realistic initial and boundary conditions. This often implies the necessity of a very extensive and correspondingly expensive field-measurement program. Experience so far shows that 3D-models are among the most sensitive and least successful of all numerical models. It will probably take another decade before they will generally be accepted in the engineering practice as a standard tool.

2. Applications

Herein, some details of the TRISULA-model of DeJitt Hydraulics (1991b) are given (see also Figure 12.2.1):

- the horizontal computation domain is represented by a (finite difference) curvilinear orthogonal grid of variable size to enable the schematization of irregular boundaries and to increase the computational efficiency (smaller grid sizes at locations with large velocity gradients),
- a fixed number of layers (variable size) in vertical direction (sigma transformation),
- the equations of motion are solved to compute the horizontal \((u, v)\) velocities of each layer, while the vertical velocity \((w)\) and the free surface level are derived from the continuity equation for each layer and the full depth,
- the grid is staggered (by half a grid size) in horizontal direction which means that the velocities and water levels are computed in different points (see Fig. 12.2.1, top right),
- the numerical solution method is a combination of an alternating Direction Implicit method for the horizontal directions and a fully implicit scheme for the vertical direction.

The TRISULA-model was used to simulate the flow in the North Sea, based on the following specifications:

- grid consisting of 70 x 70 points,
- grid size North: \(1/6^\circ = 18.5\) km,
- grid size East: \(1/4^\circ = 14.5\) to \(18.5\) km,
- six layers over the depth,
- tidal flow in combination with wind (7 m/s).

Some results are presented in Figures 12.2.2 to 12.2.5.
Figure 12.2.1 Schematization of TRISULA-model (Delft Hydraulics, 1991b)
Figure 12.2.2 Computation domain (Delft Hydraulics, 1991b)

Figure 12.2.3 Velocity vectors in three layers (Delft Hydraulics, 1991b)
Figure 12.2.5 Velocity vectors in cross section B-B (Delft Hydraulics, 1991b)

12.2.3 Two-dimensional horizontal flow models for estuaries and seas

1. Equations

Sofar, experience shows that the 3D-models are sensitive, expensive and not always succesful which implies a relatively large cost-benefit ratio. Therefore, it has been tried to reduce the dimensionality of the model by integrating over the water depth. As a result the sensitivity of the model is reduced markedly, because each integration process can be seen as some kind of smoothing or filtering process tending to average-out inherent perturbations in the system. The basic equations for homogeneous (constant fluid density) flow in shallow water are:
Momentum balance

\[
\begin{align*}
\text{x: } & \quad \frac{\partial}{\partial t} (h \bar{u}) + \frac{\partial}{\partial x} (h \bar{u} \bar{u}) + \frac{\partial}{\partial y} (h \bar{u} \bar{v}) + \frac{\partial}{\partial x} (h \bar{v} \bar{u}) + gh \frac{\partial}{\partial x} (h \bar{z}_b) + \\
& - k_x h \left( \frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} \right) - \frac{\tau_{bx}}{\rho} - \sum F_x = 0 \\
\text{y: } & \quad \frac{\partial}{\partial t} (h \bar{v}) + \frac{\partial}{\partial x} (h \bar{u} \bar{v}) + \frac{\partial}{\partial y} (h \bar{v} \bar{u}) + \frac{\partial}{\partial y} (h \bar{z}_b) + \\
& - k_y h \left( \frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} \right) - \frac{\tau_{by}}{\rho} - \sum F_y = 0
\end{align*}
\]  
(12.2.9)

Mass balance

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} (h \bar{u}) + \frac{\partial}{\partial y} (h \bar{v}) = 0
\]  
(12.2.10)

in which:
- \(\bar{u}, \bar{v}\) = depth-averaged fluid flow velocities in x, y directions
- \(h\) = local water depth
- \(t\) = time
- \(\rho\) = fluid density
- \(\tau_b\) = bed-shear stress
- \(F \) = external forces (wind, waves, Coriolis)
- \(z_b\) = bed level above reference datum
- \(k_x, k_y\) = effective dispersion coefficients representing the integration effects

The bed-shear stresses usually are related to the depth-averaged velocities, as follows:

\[
\begin{align*}
\tau_{bx} &= \rho g (\bar{u} \bar{v} \bar{v})/C^2 \\
\tau_{by} &= \rho g (\bar{v} \bar{v} \bar{v})/C^2
\end{align*}
\]  
(12.2.12)

in which:
- \(\bar{v} = [\bar{u}^2 + \bar{v}^2]^{0.5}\) = magnitude of velocity vector
- \(C\) = Chezy-coefficient

2. Applications

The TRISULA-model (see section 12.2.2) can also be used as a one-layer model to represent 2-dimensional depth-averaged flow. The model was used to compute the flow field in Morecambe Bay, north of Liverpool, England (Delft Hydraulics, 1991a).

Morecambe Bay is a tidal flat area in which a shipping channel to Barrow-in-Furness was dredged. A curvilinear boundary fitted coordinate system was used with about 8000 active grid points. The smallest grid size was 240 x 290 m²; the largest grid size was 1122 x 1054 m².
Figure 12.2.6 shows the detailed results of the flow field between the mainland and Walney Island based on the application of a nested model with grid sizes of 25 x 25 m² in areas where large velocity gradients did occur.

![Map of flow field between mainland and Walney Island](image)

**Figure 12.2.6** Maximum flood velocities during springtide between Walney Island and mainland; nested model results (Delft Hydraulics, 1991)

12.2.4 Two-dimensional vertical flow model

1. Basic equations

This type of model describes the flow in the vertical (x, z) plane assuming uniform conditions in the lateral (y) direction. Examples are the flow across a channel and the flow across a long-crested sand dune. The basic equations for homogeneous conditions (constant fluid density) are:
Momentum balance

\[
\begin{align*}
\chi: & \quad \frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho uu)}{\partial x} + \frac{\partial (\rho uw)}{\partial z} + \frac{\partial}{\partial x} \left( \frac{P}{\rho} \right) - \frac{\partial}{\partial x} \left( \frac{\sigma_{xx}}{\rho} \right) - \frac{\partial}{\partial z} \left( \frac{\tau_{xz}}{\rho} \right) = 0 \\
\zeta: & \quad \frac{\partial (\rho w)}{\partial t} + \frac{\partial (\rho uw)}{\partial x} + \frac{\partial (\rho ww)}{\partial z} + \frac{\partial}{\partial x} \left( \frac{P}{\rho} \right) - \frac{\partial}{\partial x} \left( \frac{\sigma_{xz}}{\rho} \right) - \frac{\partial}{\partial z} \left( \frac{\tau_{zz}}{\rho} \right) = 0
\end{align*}
\]  
(12.2.13)

(12.2.14)

Mass balance

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]  
(12.2.15)

in which:
\begin{itemize}
\item \text{u} = horizontal fluid velocity at height \text{z} above datum
\item \text{w} = vertical fluid velocity at height \text{z} above datum
\item \text{p} = fluid pressure
\item \text{\sigma} = Reynolds' pressure
\item \text{\tau} = Reynolds' stress
\item \text{x, z} = coordinates
\item \text{t} = time
\end{itemize}

Modelling of the Reynolds' stresses, known as the turbulence closure problem, can be done by applying the eddy viscosity concept (first order closure) or by applying the k-epsilon model (second order closure), (Rodi, 1980).

The assumption of a hydrostatic fluid pressure distribution and the application of the eddy viscosity concept result in boundary layer flow for gradually varied flow conditions (no flow separation).

2. Applications

Figure 12.2.7 shows computed velocities, turbulence energy values and shear stresses in a steep-side trench based on a second order turbulence closure (k-epsilon approach) according to Alfrink and Van Rijn (1983).

12.2.5 One-dimensional flow model for rivers or estuaries

1. Equations

In rivers the cross-section often consists of a main channel through which the discharge takes place and shallow flood plains that are mainly functioning as storage areas, particularly when groynes are present for channel regulation. In that case the discharge \text{Q} represents the discharge in the main channel. The extra resistance exerted by the flood plains on the main channel is modeled by an additional resistance coefficient \text{\eta}. 
A. FLOW VELOCITY PROFILES

B. TURBULENCE ENERGY PROFILES

C. SHEAR STRESS PROFILES

Figure 12.2.7  Velocity, turbulence energy and shear stress profiles in a trench (Alfrink and Van Rijn, 1983)

The basic equations read as:

**Momentum balance**

\[
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \alpha \frac{Q^2}{A_s^2} \right) + g A_t \frac{\partial H}{\partial x} + \frac{g Q |Q|}{R A_t C^2} + g A_t \eta = 0
\]  
(12.2.16)

**Mass balance**

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - q_L = 0
\]  
(12.2.17)
in which:
\[ Q = \text{discharge} \]
\[ A = \text{total area of cross-section} \]
\[ A_f = \text{area of cross-section of main channel (flow section)} \]
\[ t = \text{time} \]
\[ H = \text{water surface level above reference datum} \]
\[ R = \text{hydraulic radius} \]
\[ C = \text{Chézy coefficient} = 18 \log(12R/k_s) \]
\[ k_s = \text{bed roughness height} \]
\[ g = \text{acceleration of gravity} \]
\[ q_L = \text{lateral discharge per unit length} \]
\[ \eta = \text{friction coefficient} \]
\[ \alpha = \text{coefficient (≈ 1) representing influence of velocity distribution} \]

2. Applications

Herein, some details of the flow model of Delft Hydraulics (1989) are given:
- a system of channel branches connected by nodes (functions) can be modeled,
- cross-sections of arbitrary shape anywhere along each branch can be represented,
- structures such as sluices, weirs, spillways etc. can be modeled,
- application of an implicit finite-difference solution method on a staggered grid.

Figure 12.2.8 shows computed and observed maximum water levels of a flood wave passing the Rio Jucar in Spain (Delft Hydraulics, 1987).

![Figure 12.2.8 Computed and observed water levels in the Rio Jucar, Spain during flood conditions (Delft Hydraulics, 1987)](image-url)
12.3 Wave models

12.3.1 Introduction

Wave propagation in shallow water is affected by the following phenomena:
- refraction: deflection of the wave direction because of changes in the wave propagation velocity along the wave crest (due to changes in water depth and current velocity);
- diffraction: transfer of wave energy due to spatial differences in wave height (mainly perpendicular to propagation direction);
- reflection: change in wave height and direction due to presence of an obstacle;
- shoaling: change in wave height as a results of changes in water depth;
- dissipation: decrease of wave height due to bottom friction and breaking of the waves;
- production: increase of wave height by wind forces at the water surface.

An essential property of (monochromatic) surface waves is that they propagate in the horizontal space (x,y).

12.3.2 Basic equations

Introducing a velocity potential $\phi$ defined as $u = \partial \phi / \partial x$, $v = \partial \phi / \partial y$ and $w = \partial \phi / \partial z$, the basic equations for irrotational surface waves on an incompressible fluid are given by the mass-balance equation for the fluid (see Dingemans, 1985):

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$  \hspace{1cm} (12.3.1)

and the boundary condition:

$$\frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \eta}{\partial y} = \frac{\partial \phi}{\partial z}$$ \hspace{1cm} \text{at } z = \eta(x,y,t) \hspace{1cm} (12.3.2)

$$\frac{\partial \phi}{\partial x} \frac{\partial h}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial h}{\partial y} + \frac{\partial \phi}{\partial z} = 0$$ \hspace{1cm} \text{at } z = -h(x,y) \hspace{1cm} (12.3.3)

$$\frac{\partial \phi}{\partial t} + g \eta + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial z} \right)^2 + \frac{P}{\rho} = 0$$ \hspace{1cm} \text{at } z = \eta(x,y,t) \hspace{1cm} (12.3.4)

in which:

- $\eta$ = water surface elevation with respect to mean surface level
- $\phi$ = velocity potential ($u = \partial \phi / \partial x$, $v = \partial \phi / \partial y$, $w = \partial \phi / \partial z$)
- $z$ = vertical coordinate (positive upwards)
- $x,y$ = horizontal coordinates
- $h$ = water depth (bottom to still surface level)
- $P$ = fluid pressure
- $\rho$ = fluid density

Equation (12.3.4) is known as the unsteady Bernoulli-equation. The wave models presented herein are based on linearized wave motion. Linearization means that the terms which are non-linear in $\phi$ and $\eta$ are neglected and that the free surface conditions are specified at $z = 0$ (instead of at $z = \eta$).

12.14
The linearized equations are:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{12.3.5}
\]

\[
\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial z} = 0 \quad \text{at } z = h(x,y) \tag{12.3.6}
\]

\[
\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \quad \text{at } z = 0 \tag{12.3.7}
\]

\[
\frac{\partial \phi}{\partial t} + g \eta = 0 \quad \text{at } z = 0 \tag{12.3.8}
\]

For the case of a horizontal bottom the boundary condition at \( z = -h \) becomes \( \partial \phi / \partial z = 0 \) and a linear solution for one harmonic wave is given by:

\[
\eta(x,y,t) = \frac{H}{2} \cos(ka_x x + ka_y y - \omega t) \tag{12.3.9}
\]

\[
\phi(x,y,z,t) = \frac{gH}{2\omega} \frac{\cosh[k(h+z)]}{\cosh[kh]} \sin(ka_x x + ka_y y - \omega t) \tag{12.3.10}
\]

in which:
- \( H \) = wave height
- \( k = 2\pi/L \) = wave number
- \( \omega = 2\pi/ T \) = frequency (\( \omega^2 = gk \tanh(kh) \))
- \( L \) = wave length
- \( T \) = wave period
- \( ka_x \) = \( k \cos(\alpha) \) = component of \( k \) in \( x \)-direction
- \( ka_y \) = \( k \sin(\alpha) \) = component of \( k \) in \( y \)-direction
- \( \alpha \) = angle between wave direction and \( x \)-coordinate (see Fig. 12.3.1)

![Figure 12.3.1 Definition sketch of wave number vector](image)

Figure 12.3.1 Definition sketch of wave number vector
The wave number \( k \) is represented as a vector \( \mathbf{k} = (k_x, k_y) = (k \cos \alpha, k \sin \alpha) \) with
\[
k = |\mathbf{k}| = (k_x^2 + k_y^2)^{1/2}.
\]

Introducing the coordinate vector \( \mathbf{r} = (x,y) \), Equation (12.3.9) can be represented as (see Fig. 12.3.1):
\[
\eta(r,t) = \frac{H}{2} \cos(k \cdot \mathbf{r} - \omega t)
\]  
(12.3.11)

Introducing the local coordinate system \( s = s(\alpha) \) along the wave ray vector, Equation (12.3.9) can also be represented as:
\[
\eta(s,t) = \frac{H}{2} \cos(ks - \omega t)
\]  
(12.3.12)

12.3.3 Two-dimensional horizontal models for combined refraction, diffraction, shoaling and dissipation

1. Mild slope equation

Basic equations

The mild slope equation for monochromatic waves, first presented by Berkhoff (1972), can be derived from the variation principle of Luke.

Suppose that the wave velocity potential \( \phi(x,y,z,t) \) can be expressed locally, as:
\[
\phi(x,y,z,t) = f(z,h) \phi(x,y,t)
\]  
(12.3.13)

with \( f(z,h) = \frac{\cosh[k(z+h)]}{\cosh[kh]} \)  
(12.3.14)

Equation (12.3.13) is substituted in the unsteady Bernoulli-equation and integrated over the depth, as follows:
\[
L = - \int_{-h}^{\eta} \left[ \frac{\partial \phi}{\partial t} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial z} \right)^2 + gz \right] dz
\]  
(12.3.15)

Applying the variation principle of Luke (\( \partial L/\partial \eta = 0 \) and \( \partial L/\partial \phi = 0 \)) and neglecting the terms with \( \partial \mathbf{f}/\partial x \) and \( \partial \mathbf{f}/\partial y \), yields (see Dingemans, 1985):
\[
-\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial x} \left( c \ c_g \ \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( c \ c_g \ \frac{\partial \phi}{\partial y} \right) - (\omega^2 - k^2 \ c \ c_g) \ \phi = 0
\]  
(12.3.16)

A complex wave velocity potential is introduced according to:
\[
\phi(x,y,t) = \text{Re} \left[ \psi(x,y) \ e^{-i\omega t} \right]
\]  
(12.3.17)

12.16
Substitution of Eq. (12.3.17) into Eq. (12.3.16) yields the mild-slope equation:

\[
\frac{\partial}{\partial x} \left( c \ c_g \ \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left( c \ c_g \ \frac{\partial \psi}{\partial y} \right) + k^2 \ c \ c_g \ \psi = 0 \quad (12.3.18)
\]

in which:
- \( c = \omega/k \) = wave phase velocity (scalar)
- \( c_g = n \ c \) = wave group velocity (scalar)
- \( \psi \) = two-dimensional complex wave velocity potential

The mild-slope equation which represents combined refraction and diffraction effects, is based on the assumption that Eq. (12.3.14) may be applied for a mild-sloping bottom because terms with \( \partial f/\partial x \) and \( \partial f/\partial y \) are neglected. Thus, for a mild-sloping bottom Eq. (12.3.14) is supposed to hold locally. The diffraction effect representing the spreading of wave energy related to spatial differences in wave height is expressed by the \( \partial^2 \psi/\partial x^2 \) and \( \partial^2 \psi/\partial y^2 \) terms. Originally, Eq. (12.3.14) is derived for a horizontal bottom.

The wave height \( H \) can be obtained from the amplitude parameter \( \psi \); \( \psi \approx H \).

Equation (12.3.18) is an elliptic type of equation, requiring a boundary condition along all boundaries of the solution domain. Possible boundary conditions are:
- partial or full reflection at fixed boundaries;
- known incident wave field and/or a radiation condition (no reflection) at open boundaries.

A numerical solution (PHAROS model) of the full mild-slope equation based on a finite-element method was given by Kostense et al., 1986.

Numerical solution of the full mild-slope equation for large areas requires considerable computational effort. Often, a simplified mild-slope equation is used which is called the parabolic mild-slope equation. The parabolic approximation is based on the assumption of wave propagation (without reflection) in one main direction and diffraction only in a direction perpendicular to the main propagation (s) direction (\( \partial^2 \psi/\partial s^2 \) is neglected). The field equation for the parabolic approximation reads as (Berkhoff et al., 1982):

\[
\frac{\partial \psi}{\partial s} - \left( ik - \frac{1}{2k \ c \ c_g} \ \frac{\partial}{\partial s} \left( k \ c \ c_g \right) \right) \psi - \frac{i}{2k \ c \ c_g} \ \frac{\partial}{\partial n} \left( c \ c_g \ \frac{\partial \psi}{\partial n} \right) = 0 \quad (12.3.19)
\]

in which:
- \( i \) = imaginary unit \((i^2 = -1)\)
- \( s \) = main direction of wave propagation \((s > 0)\)
- \( n \) = direction perpendicular to \( s \)-direction

The parabolic approximation generally yields good results as long as the local wave direction does not differ too much (within 45\(^\circ\)) from the initially chosen direction. A numerical solution (CREDIS-model) of the parabolic mild-slope equation based on a finite-difference method was given by Berkhoff et al., (1982), Dingemans et al (1984), Dingemans (1986).

**Influence of current refraction**

In shallow water near river outlets the currents may be rather strong and hence the current-refraction effect cannot be neglected. Assuming uniform velocities \((U, V)\) over the depth as derived from the shallow-water equations, the mild-slope equation including current refraction reads, as (see Dingemans, 1985):
\[
\frac{\partial}{\partial x} \left( c c_g \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left( c c_g \frac{\partial \psi}{\partial y} \right) + 2i\omega \left( U \frac{\partial \psi}{\partial x} + V \frac{\partial \psi}{\partial y} \right) + \\
- \left( \omega_r^2 - \omega^2 - \omega^2 n - i\omega \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) \right) \psi = 0
\] (12.3.20)

in which:
\[
\begin{align*}
U, V &= \text{current velocities} \\
\omega_r &= [gk \tanh(kh)]^{0.5} = \text{wave frequency relative to current} \\
\omega &= \omega - (k_x U + k_y V) = \omega - |\hat{k}| |\hat{V}| \cos \gamma \\
\gamma &= \text{angle between wave and current direction} \\
i &= \text{imaginary unit} (i^2 = -1)
\end{align*}
\]

**Wave energy dissipation**

In shallow water where energy dissipation is important, additional terms are introduced in the mild-slope equation.

For example, for waves alone this yields (see Dingemans, 1985):

\[
\frac{\partial}{\partial x} \left( c c_g \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left( c c_g \frac{\partial \psi}{\partial y} \right) + c c_g \left( k^2 + i k \frac{W}{c_g} \right) = 0
\] (12.3.21)

with \( W \) = wave energy dissipation coefficient (bottom friction).

**Applications**

Numerical solutions of the mild-slope equation were compared with detailed wave height measurements in a hydraulic model for the case of an elliptic shoal situated on a bottom slope of 1 to 50 (\( \tan \beta = 0.02 \)), as shown in Fig. 12.3.2. Low amplitude monochromatic waves with a period of 1 sec. were generated. The water depth in the deep water area was 0.45 m. Figure 12.3.3 shows curves of equal relative wave amplitudes based on the measured data (Berkhoff et al., 1982).

The results of a finite-difference solution of the parabolic approximation of the mild-slope equation (CREDIZ-model) for Section 5 and for Section 6 near the shoal are shown in Fig. 12.3.4 (Dingemans, 1985). The Sections 5 and 6 are shown in Fig. 12.3.2. Dissipation effects were not taken into account. Computed results with and without non-linear effects are shown. When the non-linear effects occurring in shallow water above and behind the shoal are represented in the numerical model, a much better agreement between computed and measured wave heights is obtained. The non-linear effects in shallow water were represented in an approximate way by using \( h = h_s + \frac{1}{2} H \) as the effective water depth in the dispersion equation \( \omega_r = [gk \tanh(kh)]^{0.4} \), with \( h_s \) = water depth to still water surface level, \( H \) = wave height.

The results of a finite-element solution of the full mild-slope equation (PHAROS-model) for Sections 5 and 6 near the shoal are also shown in Fig. 12.3.4. Dissipation effects (bottom friction) and non-linear effects in shallow water were not taken into account.

The parabolic approximation and the full mild-slope equation, both without non-linear effects, produce similar results. The numerical solution of the full mild-slope equation shows a wavy pattern of the wave height along Section 6, which expresses minor numerical instabilities related to the modelling of the diffraction effect in the wave propagation direction.
Figure 12.3.2 Plan view of wave basin with elliptic shoal (Berkhoff et al., 1982)

Figure 12.3.3 Curves of equal relative measured wave amplitude (Berkhoff et al., 1982)
Figure 12.3.4 Measured and computed wave height in Section 5 and 6 of shoal experiment
12.3.4 Two-dimensional models for combined refraction, shoaling and dissipation

1. Wave ray models

Basic equation

The refraction approximation can be obtained from the mild-slope equation by introducing \( \psi = B \exp(iS) \) with \( B = gH/(2\omega_r) \) = amplitude function and \( S = \vec{k} \cdot \vec{r} - \omega_r t = \) phase function. Neglecting the diffraction effects (x and y direction) in the mild-slope equation, it can be derived that (Dingemans, 1985):

\[
\left( \frac{\partial S}{\partial x} \right)^2 + \left( \frac{\partial S}{\partial y} \right)^2 - k^2 = 0 \tag{12.3.22}
\]

\[
\frac{\partial}{\partial x} \left( c_{g,x} + U \right) \frac{E}{\omega_r} + \frac{\partial}{\partial y} \left( c_{g,y} + V \right) \frac{E}{\omega_r} + \frac{D}{\omega_r} = 0 \tag{12.3.23}
\]

in which:
- \( S = k_x x + k_y y - \omega_r t \) = phase function, constant along each wave front
- \( E = (1/8) \rho g H^2 \) = wave energy
- \( H \) = wave height
- \( \omega_r = \omega - k \cdot \vec{v} \) = relative wave frequency
- \( D \) = wave energy dissipation per unit area (bottom friction and wave breaking).

Equation (12.3.22), which is known as the eikonal equation, can also be obtained from:

\[
(k_x)^2 + (k_y)^2 = k^2 \quad \text{with} \quad \partial S/\partial x = k_x \quad \text{and} \quad \partial S/\partial y = k_y \tag{12.3.24}
\]

The paths of the wave rays can be determined by a step wise integration (method of characteristics) of the ray Equation (12.3.22), (Lagrangian approach). Equation (12.3.23), which is known as the wave action balance, can be used to determine the wave energy \( E \). Defining a local coordinate \( s \) along which the vector \( \vec{c}_g \) is directed (see Fig. 12.3.5), the wave action balance can be represented as:

\[
\frac{d}{ds} \left[ \left( c_g + U_s \right) \frac{bE}{\omega_r} \right] + \frac{bD}{\omega_r} = 0 \tag{12.3.25}
\]

with \( U_s \) = current velocity component in wave direction, \( b \) = width between two wave rays.

Another approach is the application of a wave ray equation in terms of the angle \( \alpha \) between the wave ray and the positive x-axis (Dean and Dalrymple, 1984).

By definition \( \nabla \times \vec{w}_s = 0 \), yielding \( \nabla \times \vec{k} = 0 \) or

\[
\frac{\partial}{\partial x} (k_x) - \frac{\partial}{\partial y} (k_y) = 0, \quad \text{or} \tag{12.3.26}
\]

\[
\frac{\partial}{\partial x} (k \sin \alpha) - \frac{\partial}{\partial y} (k \cos \alpha) = 0 \tag{12.3.27}
\]
In a local coordinate system Eq. (12.3.27) can be expressed as:

\[
\frac{\partial \alpha}{\partial s} = -\frac{1}{c} \frac{\partial c}{\partial n} \tag{12.3.28}
\]

with \( c = \) local wave propagation velocity.

The wave path can be determined by solving the angle \( \alpha \) from Eq. (12.3.27) or Eq. (12.3.28).

Neglecting current refraction and wave energy dissipation, the distance between the wave rays can be determined from geometrical considerations, yielding:

\[
\frac{d^2 \beta}{ds^2} + p \frac{d\beta}{ds} + q \beta = 0 \tag{12.3.29}
\]

with \( \beta = \frac{b}{b_0} = \) dimensionless width, \( b_0 = \) initial width; \( p \) and \( q \) are functions of \( \alpha \) and \( c \).

The wave energy and hence the wave height can be obtained from \( b \ E \ c_s = \) constant between two wave rays, see Eq. (12.3.25) for \( U_s = 0 \) and \( D = 0 \).

It is noted that this approach yields unrealistic results in regions with a very irregular bathymetry resulting in converging and crossing wave rays (see Fig. 12.3.7).

In case of a straight uniform coast with depth contours parallel to the coastline (x-axis normal to coast), the wave ray direction follows from Eq. (12.3.7), yielding:

\[
\frac{\partial}{\partial x} (k \sin \alpha) = 0 \quad \text{or} \quad k \sin \alpha - \text{constant} \quad \text{or} \quad \frac{\sin \alpha}{c} - \text{constant} \tag{12.3.30}
\]

Equation (12.3.30) is known as the Law of Snel.

The wave action balance reduces to:

\[
\frac{d}{dx} \left( c_{g,x} + U \right) \frac{F}{\omega_r} + \frac{D}{\omega_r} = 0 \tag{12.3.31}
\]

in which: \( U = \) current velocity component in x-direction

*Wave energy averaging method*

Wave ray paths are very sensitive to small variations in the water depth. Small local irregularities of the bottom topography have permanent influence on the wave paths of those wave rays passing that region.

Once, the wave ray is deflected there is no mechanism available to turn it backward to more or less its original course. Therefore, all non-relevant bottom variations should be smoothed out.

In situations where crossing wave rays are present (caustic points), an averaging method is required to obtain relevant information of the wave height (Bouws and Battjes, 1982).

Neglecting dissipation effects, the incoming energy flux \( F_\omega \) along each wave ray is constant. The wave energy at a certain point \( s \) of the wave ray is given by \( E_s = F_\omega / c_\rho \).
The amount of wave energy over length \( s_2 - s_1 \) of the wave ray is (Fig. 12.3.6):

\[
E_{s_2-s_1} = \int_{s_1}^{s_2} (F_J/c_g) \, ds = F_0 \int_{s_1}^{s_2} (1/c_g) \, ds
\]  

(12.3.32)

For a certain region \( G \) with length \( L_1 \) and width \( L_2 \) intersected by \( m \) rays the available energy is:

\[
E_G = F_0 \sum_{i=1}^{m} \int_{s_{i1}}^{s_{i2}} (1/c_g) \, ds
\]  

(12.3.33)

The mean energy per unit area (\( \bar{E} \)) can be obtained as \( \bar{E} = E_G/(L_1 L_2) \). Using this method, the available energy in a certain region is smoothed out over the region considered. When dissipation effects are important, these effects can be taken into account.

**Applications**

Figure 12.3.8 shows the computed wave ray pattern for the shoal experiment (see also Fig. 12.3.2). Behind the shoal a caustic point (wave ray crossings) can be observed.

Figure 12.3.4 shows the computed wave height distribution in Section 5 and 6 based on the averaging method of Bouws and Battjes (1982). The agreement between measured and computed wave height is reasonably good up to a distance of 6 m behind the shoal (see Section 6). The transverse distribution of the computed wave height in Section 5 shows considerable deviations compared with the measured values.

2. **Wave action model (Eulerian approach)**

The wave ray models are based on a Lagrangian solution method in the sense that the wave changes are determined while travelling with the waves along the rays. This approach is numerically inefficient when non-linear dissipation is to be taken into account. In that case an Eulerian approach presented by Holthuijsen et al. (1989) is more favourable.

The basic equation of Holthuijsen et al (1989) reads as:

\[
\frac{\partial}{\partial t} \left( \frac{E}{\omega_r} \right) + \frac{\partial}{\partial x} \left( \frac{c_g x E}{\omega_r} \right) + \frac{\partial}{\partial y} \left( \frac{c_g y E}{\omega_r} \right) + \frac{\partial}{\partial \alpha} \left( \frac{c_\alpha E}{\omega_r} \right) + \frac{\partial}{\partial \omega} \left( \frac{c_\omega E}{\omega_r} \right) = T
\]  

(12.3.34)

in which:

- \( E_{\omega,x,y,\alpha,\beta} = (1/8) \rho g H^2 \) = wave energy
- \( \omega_r = \omega - (k_x U + k_y V) = \omega - \vec{k} \cdot \vec{V} \) = relative wave frequency
- \( \vec{k} \) = wave number vector making an angle \( \alpha \) with x-axis
- \( \vec{V} \) = current velocity vector making an angle \( \beta \) with x-axis
- \( H \) = wave height
- \( T \) = source or sink term

Equation (12.3.34) is known as the wave action balance equation \( A = E/\omega_r \) = wave action). The diffraction effect is not taken into account.
Figure 12.3.5 Definition sketch of wave rays

Figure 12.3.6 Averaging of wave energy

Figure 12.3.7 Computed wave rays

Figure 12.3.8 Computed wave rays (shoal experiment)
The first term on the left-hand side of Eq. (12.3.34) represents the local rate of change of wave action density. The second and the third term represent propagation at a given location \(x, y\) in direction \(\alpha\). However, in general the direction of wave energy propagation is not constant because the wave rays are deflected by refraction in shallow water. A curving wave ray implies that the wave propagation direction changes while travelling along the ray. Thus, the energy flux continuously changes its direction while travelling through the \(x, y\)-space. This can be conceived as energy propagation not only through the \(x, y\)-space but also (and simultaneously) through the \(\alpha\)-space. The propagation velocity in the \(\alpha\)-space is the rate at which the direction changes as propagating with the group velocity along the curving ray. This illustrates that the fourth term on the left-hand side of Eq. (12.3.34) represents the refraction effect. The effect of current refraction is taken into account by including the current velocity and direction in the expressions for \(c_{g_{x}}\), \(c_{g_{y}}\), and \(c_{\alpha}\). The fifth term on the left-hand side represents the change in wave action in the frequency domain induced by time variations of the water depth, current velocity and direction.

The term \(T\) on the right-hand side of Eq. (12.3.34) represents a source-sink term for wave energy generation (wind input) and wave dissipation (bottom friction and wave breaking).

Assuming a stationary situation and neglecting the fifth term, Eq. (12.3.34) reduces to:

\[
\frac{\partial}{\partial x} \left( \frac{c_{g_{x}} E}{\omega_{i}} \right) + \frac{\partial}{\partial y} \left( \frac{c_{g_{y}} E}{\omega_{i}} \right) + \frac{\partial}{\partial \alpha} \left( \frac{c_{\alpha} E}{\omega_{i}} \right) = T \tag{12.3.35}
\]

The wave group velocity (linear wave theory) is defined as:

\[
c_{g} = \frac{1}{k} \frac{\partial \omega_{i}}{\partial k} \left( \hat{k} + \hat{V} \right) \tag{12.3.36}
\]

The wave phase velocity \(c_{\alpha}\) is defined as:

\[
c_{\alpha} = \frac{1}{k} \frac{\partial \omega_{i}}{\partial h} \frac{\partial h}{\partial \hat{n}} - \frac{\hat{k}}{k} \frac{\partial \hat{V}}{\partial \hat{n}} \tag{12.3.37}
\]

where \(h = \) local water depth and \(n = \) coordinate normal to wave propagation direction.

Holthuijsen et al. (1989) presented a numerical solution of Eq. (12.3.35) on a fixed grid for irregular waves in terms of a directional wave spectrum with one discrete frequency. As the full numerical solution of Eq. (12.3.35) is rather complicated, Eq. (12.3.35) is simplified by introducing a parameterization method. To represent the directional effects of the wave spectrum, two directional wave functions are introduced:

- the directional wave action spectrum \(A = A_{\alpha}(\alpha)\), where the function \(A_{\alpha}(\alpha)\) represents the directional distribution of the wave action integrated over the frequencies,
- the mean wave frequency as a function of the directional spectrum, \(\omega = \omega_{\alpha}(\alpha)\).

The propagation terms and the source term of Eq. (12.3.35) are also parameterized. Based on the computed results of \(A\) and \(E\), the significant wave height \(H_{s}\), the mean wave period \(T\), the mean wave direction \(\bar{\alpha}\) and the directional spreading \(\sigma_{\alpha}\) are determined.
12.4 Sediment transport and morphological models

12.4.1 Introduction

The physical understanding and mathematical modelling of the water and sediment motion in rivers, estuaries and coastal waters have made much progress in recent years by the efforts of various research institutes. This has led to a number of more or less ready to use numerical model systems, but it has also raised many new research questions. According to the number of space dimensions and spatial orientation, four classes of models are considered: (quasi-) three-dimensional (3D), two-dimensional vertical (2DV), two-dimensional (2DH) and one-dimensional models (1D).

The number of dimensions to be modelled is strongly related to the scale of the process involved and the problem studied.

Bed load transport in non-uniform and non-steady conditions can be modelled by a formula-type of approach because the adjustment of the transport process to the new hydraulic conditions proceeds rapidly. These types of models are also called Box models. Based on this, a two-dimensional horizontal or a one-dimensional approach is the most efficient approach, provided that the magnitude and direction of the bed-shear stress can be predicted with sufficient accuracy. If necessary, a (quasi-) three-dimensional flow model can be applied to compute the bed-shear stress in complicated situations such as the flow in bends; tide, wind and wave-induced flow in coastal seas.

Suspended load transport does not adjust rapidly to the new hydraulic conditions because it takes time and space to transport the particles upward and downward over the depth, which makes it necessary to model the convective and diffusive processes. These types of models are also called continuum models. The modelling of the small-scale and short-term processes requires a (quasi-) three-dimensional or a two-dimensional vertical approach; for example, scour and deposition near structures and harbours. One-dimensional and two-dimensional horizontal depth-integrated models can be used for large-scale and long-term modelling of transport in rivers and shallow tidal waters.

12.4.2 Basic equations of sediment transport

1. Mass-balance equation

Applied to instantaneous variables, the mass-balance equations of a unit volume are given by:

\[
\begin{align*}
\text{Fluid} & : \frac{\partial}{\partial t} (\rho (1-C)) + \frac{\partial}{\partial x_1} (\rho (1-C) U_{f1}) = 0 \\
\text{Sediment} & : \frac{\partial}{\partial t} (\rho_s C) + \frac{\partial}{\partial x_1} (\rho_s C U_{s1}) = 0
\end{align*}
\]

in which:
- \(C\) = local volume concentration (-)
- \(U_f\) = local fluid velocity (m/s)
- \(U_s\) = local sediment velocity (m/s)
- \(\rho\) = fluid density (kg/m\(^3\))
- \(\rho_s\) = sediment density (kg/m\(^3\))
- \(x\) = coordinate (m)
- \(t\) = time (s)
The effect of turbulence can be introduced by applying the well-known Reynolds’-procedure in which the variables are represented as the sum of a time-averaged (overbar) component and a fluctuation (prime) component, as follows:

\[ U = u + u' \tag{12.4.3} \]
\[ C = c + c' \tag{12.4.4} \]

Substitution of Eqs. (12.4.3) and (12.4.4) in Eqs. (12.4.1) and (12.4.2), averaging over time and assuming the fluid and sediment density to be constant, results in:

**Fluid**

\[ \frac{\partial}{\partial t} (1-c) + \frac{\partial}{\partial x_i} ((1-c) u_{t,i} - \overline{c'u'_{t,i}}) = 0 \tag{12.4.5} \]

**Sediment**

\[ \frac{\partial}{\partial t} c + \frac{\partial}{\partial x_i} (c u_{s,i} + \overline{c'u'_{s,i}}) = 0 \tag{12.4.6} \]

The fluid and sediment particles are assumed to behave as a fluid-sediment mixture. The sediment velocity is assumed to be equal to the fluid velocity (= mixture velocity) with exception of the vertical direction where a constant slip velocity equal to particle fall velocity \( w_s \) is assumed. Thus:

\[ u_{s,i} = u_{t,i} - w_s \delta_i \quad (\delta_x = \delta_y = 0, \delta_z = 1) \tag{12.4.7} \]

The eddy viscosity (diffusion) concept is applied to represent the turbulence-induced transport components. The coefficients expressing the transfer of fluid momentum and sediment mass are herein shortly called fluid and sediment mixing coefficient.

Applying the eddy-viscosity concept, the turbulence-related components are:

\[ \overline{c'u'_{t,i}} = -\varepsilon_f \frac{\partial c}{\partial x_i} \tag{12.4.8} \]
\[ \overline{c'u'_{s,i}} = -\varepsilon_s \frac{\partial c}{\partial x_i} \tag{12.4.9} \]

in which:
- \( \varepsilon_f = \) fluid mixing coefficient \((\text{m}^2/\text{s})\)
- \( \varepsilon_s = \) sediment mixing coefficient \((\text{m}^2/\text{s})\)

Substitution of Eqs. (12.4.7), (12.4.8) and (12.4.9) in Eqs. (12.4.5) and (12.4.6) results in:

**Fluid**

\[ \frac{\partial}{\partial t} (1-c) + \frac{\partial}{\partial x_i} ((1-c) u_{t,i} + \varepsilon_f \frac{\partial c}{\partial x_i}) = 0 \tag{12.4.10} \]

**Sediment**

\[ \frac{\partial}{\partial t} c + \frac{\partial}{\partial x_i} (c u_{s,i} - w_s \delta_i) - \varepsilon_s \frac{\partial c}{\partial x_i} = 0 \tag{12.4.11} \]
In the longitudinal-vertical (x-z) plane the mass-balance equations read, as follows:

**Fluid**  
\[
\frac{\partial}{\partial t} (1-c) + \frac{\partial}{\partial x} \left((1-c) \ u + \varepsilon_f \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial z} \left((1-c) \ w + \varepsilon_f \frac{\partial c}{\partial z}\right) = 0
\]  
(12.4.12)

**Sediment**  
\[
\frac{\partial}{\partial t} (c) + \frac{\partial}{\partial x} \left(c \ u - \varepsilon_s \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial z} \left(c \ (w-w_s) - \varepsilon_s \frac{\partial c}{\partial z}\right) = 0
\]  
(12.4.13)

in which:
- \(x\) = longitudinal coordinate
- \(z\) = vertical coordinate
- \(u\) = fluid velocity in x-direction
- \(w\) = fluid velocity in z-direction

For a steady and uniform flow \(\partial u/\partial t = 0\), \(\partial c/\partial t = 0\), \(\partial u/\partial x = 0\), \(\partial c/\partial x = 0\), the equations reduce to:

**Fluid**  
\[
(1-c) \ w + \varepsilon_f \frac{dc}{dz} = 0
\]  
(12.4.14)

**Sediment**  
\[
c \ (w-w_s) - \varepsilon_s \frac{dc}{dz} = 0
\]  
(12.4.15)

Elimination of the vertical fluid velocity \((w)\) yields:

\[
(1-c) \ c \ w_s + (\varepsilon_s + c \ (\varepsilon_f - \varepsilon_s)) \frac{dc}{dz} = 0
\]  
(12.4.16)

Equation (12.4.16), first presented by Hallbron (1949) and Hunt (1954), represents the sediment concentration profile for equilibrium conditions. By the introduction of the mass-balance equation, the vertical return flow due to the fluid displaced by the falling particles has been taken into account.

Assuming that the fluid and sediment mixing coefficients are approximately equal \((\varepsilon_f = \varepsilon_s)\), it follows that:

\[
(1-c) \ c \ w_s + \varepsilon_s \frac{dc}{dz} = 0
\]  
(12.4.17)

Equation (12.4.17) can also be expressed, as:

\[
c w_{s,m} + \varepsilon_s \frac{dc}{dz} = 0
\]  
(12.4.18)

in which:
- \(w_{s,m} = (1-c)w_s\) = particle fall velocity in fluid-sediment mixture.

Experimental research by Richardson and Zaki (1954) has shown that the fall velocity is not only affected by the return flow due to the displaced fluid but also by additional effects such as: particle collisions, particle-induced turbulence and modified drag coefficients. The overall effect can be represented by:
\[ w_{sm} = (1 - c)^{1 - c} w_s \]  
\[(12.4.19)\]

in which:
\[ w_s = \text{particle fall velocity in a clear, still fluid and} \]
\[ \alpha = \text{coefficient (= 4 to 5 for particles in the range of 50 to 500 \(\mu\)m).} \]

The influence of the sediment particles on the turbulence characteristics resulting in a damping of the turbulence and hence a reduction of the effective mixing coefficient \(\varepsilon_L\) should also be taken into account.

For small concentrations \((1 - c = 1)\) Equation \((12.4.18)\) reduces to the following expression:

\[ c \ w_s + \varepsilon_s \frac{dc}{dz} = 0 \]  
\[(12.4.20)\]

2. Momentum balance for fluid-sediment mixture

To represent the modification of the fluid mixing coefficients and hence the fluid velocity profiles due to the presence of the suspended sediment particles, the momentum balance for the fluid-sediment mixture should be solved. In the present analysis it is assumed that the fluid and sediment velocity (= mixture velocity) are equal in the horizontal directions, while there is a constant slip velocity equal to the particle fall velocity in the vertical direction.

Applied to instantaneous variables, the momentum balance for the fluid-sediment mixture is given by:

\[ \frac{\partial}{\partial t} (\rho \ U_{m,i}) + \frac{\partial}{\partial x_i} (\rho \ U_{m,i} U_{m,i}) = - \frac{\partial}{\partial x_i} (P_m) + \frac{\partial}{\partial x_i} (\tau_{v,ij}) + \rho_m g \delta_i \]  
\[(12.4.21)\]

in which:
\[ \rho_m = (1 - C) \rho + C \psi_s = \text{density of fluid-sediment mixture (kg/m}^3) \]
\[ U_{m,i} = U_{li} - w_s \delta_i = \text{local velocity of fluid-sediment mixture (m/s)} \]
\[ P_m = \text{local pressure (N/m}^2) \]
\[ g = \text{acceleration of gravity (m/s}^2) \]
\[ \tau_{v,ij} = \text{local viscous shear stress (N/m}^2) \]

The local instantaneous fluid velocity of the mixture may also be represented as:
\[ U_{m,i} = (1 - C) \ U_{li} - C \ w_s \delta_i. \]

The viscous shear stress due to the shearing of the intergrain fluid and the "shearing" by sediment particle-fluid interactions can be represented by:

\[ \tau_{v,ij} = \mu_m \frac{\partial U_{m,i}}{\partial x_j} \]  
\[(12.4.22)\]

in which:
\[ \mu_m = \mu_o \ (1 + \alpha_1 C)^{a_2} = \text{dynamic viscosity coefficient of the fluid as modified by the presence of the particles (kg/sm)} \]
\[ \mu_o = \text{dynamic viscosity coefficient of the fluid phase (kg/sm)} \]
\[ \alpha_1, \alpha_2 = \text{coefficients} \]
Usually, the viscous shear stress is neglected in the momentum balance equation for a clear flow. For a two-phase flow the sediment concentrations may, however, be relatively large near the bed. Consequently, the viscous shear stresses near the bed may become important because of the increased viscosity.

The effects of turbulence are introduced by applying the Reynolds'-procedure, as follows:

\[ U_{m,i} = u_{m,i} + u'_{m,i} \]  
\[ P_m = p_m + p'_m \]  
\[ \rho_m = \bar{\rho}_m + \rho'_m \]  

Substitution of Eqs. (12.4.22), (12.4.23), (12.4.24) and (12.4.25) in Eq. (12.4.21) and averaging over time, yields:

\[ \frac{\partial}{\partial t} (\bar{\rho}_m u_{m,i}) + \frac{\partial}{\partial x_j} (\bar{\rho}_m u_{m,i} u_{m,j}) = - \frac{\partial}{\partial x_i} (p_m) + \frac{\partial}{\partial x_j} (\bar{\tau}^\nu_{m,ij} + \bar{\tau}^t_{m,ij}) + \bar{\rho}_mg \delta_i \]  

in which:

\[ \bar{\tau}^\nu_{m,ij} = \bar{\rho}_m \left( \frac{\partial u_{m,i}}{\partial x_j} + \frac{\partial u_{m,j}}{\partial x_i} \right) \text{ is viscous shear stress} \]

\[ \bar{\tau}^t_{m,ij} = - \bar{\rho}_m u'_{m,i} u'_{m,j} = \text{ turbulence-induced shear stress} \]

\[ \bar{\rho}_m = \rho (1 - c) + \rho_s c \text{ = mean density of mixture} \]

\[ \bar{\rho}_m = \mu_0 (1 + \alpha_1 c)^{2} = \text{ dynamic viscosity of mixture} \]

\[ \rho_m = (\rho_s - \rho) c' = \text{ density fluctuation of mixture} \]

To derive Equation (12.4.26), the following terms have been neglected:

- time dependent fluctuations, \( \frac{\partial(\rho_m u_{m,i})}{\partial t} = 0 \)
- fluctuation terms due to viscous shear stress
- triple fluctuation terms of the turbulent shear stress, \( \bar{\tau}'_{m,ij} u'_{m,i} u'_{m,j} = 0 \)
- turbulence-induced pressure, \( \rho u'_{m,i} u'_{m,j} = 0 \)

The turbulence-induced shear stress \( \bar{\tau}^t_{m,ij} \) can be represented by \( i \neq j \):

\[ \bar{\tau}^t_{m,ij} = - (\rho - \rho c + \rho_s c) \varepsilon_f \frac{\partial u_{m,i}}{\partial x_j} - (\rho_s - \rho) \varepsilon_s u_{m,i} \frac{\partial c}{\partial x_j} \]  

In the x-z plane with velocities u and w Equation (12.4.27) reads, as follows:

\[ \frac{\partial}{\partial t} (\bar{\rho}_m u) + \frac{\partial}{\partial x} (\bar{\rho}_m u^2) + \frac{\partial}{\partial z} (\bar{\rho}_m u w) = - \frac{\partial p_m}{\partial x} + \frac{\partial}{\partial z} (\bar{\tau}^\nu_{m,xz} + \bar{\tau}^t_{m,xz}) \]  

\[ \frac{\partial}{\partial t} (\bar{\rho}_m w) + \frac{\partial}{\partial x} (\bar{\rho}_m u w) + \frac{\partial}{\partial z} (\bar{\rho}_m w^2) = - \frac{\partial p_m}{\partial z} + \frac{\partial}{\partial x} (\bar{\tau}^\nu_{m,xz} + \bar{\tau}^t_{m,xz}) + \bar{\rho}_mg \]
in which:

\[
\tau_{m,xx}^m = \frac{\tau_{m,xx}}{\tau_{m,zz}} = \mu_m \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \tag{12.4.30}
\]

\[
\tau_{m,xx}^t = \tau_{m,xx}^t = - (\rho - \rho C + \rho g) \left( \varepsilon \right) \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial z} \right) - (\rho_s - \rho) \left( \varepsilon \right) \left( u \frac{\partial c}{\partial z} + w \frac{\partial c}{\partial x} \right) \tag{12.4.31}
\]

Equations (12.4.12), (12.4.13), (12.4.28) and (12.4.29) define a system of 4 equations with 5 unknown variables, being \( c \), \( u \), \( w \), \( p \), and \( \varepsilon \). By relating the mixing coefficient to local flow variables applying a simple mixing length model or a more sophisticated \( k \)-\( \varepsilon \) model (including terms for the solid phase), the system can be closed and solved using appropriate boundary conditions. The \( k \)-\( \varepsilon \) model which is based on the application of transport equations for the kinetic energy (k) of the turbulence and its dissipation rate (\( \varepsilon \)), offers the most promising representation because the damping of the turbulence energy and hence the reduction of the mixing coefficients can be represented in a straight-forward way.

3. **Horizontal and vertical mixing of fluid and sediment**

**General**

Sediment particles are carried (convected) by the mean flow \((u, v, w)\) in horizontal and vertical directions, while simultaneously the sediment particles are carried (mixed) by the turbulent eddy motions \((u', v', w')\) in all directions and are carried downwards by gravity forces \((w_s)\). The convection and mixing processes are illustrated in Figure 12.4.1 for a line injection of dye \((w_s = 0)\) after release at \( t = 0 \).

Mixing processes in fluids are related to molecular diffusion, turbulence diffusion of small-scale random fluid motions (eddies) and to large-scale circulation cells (near obstacles and structures).

Usually, the eddy-viscosity concept is applied to represent the transfer of fluid momentum and sediment mass. In analogy with the shearing in a laminar fluid flow \((\tau_{ij} = \rho v \partial u_i / \partial x_j)\), the shearing in a turbulent fluid flow is represented as: \( \tau_{ij} = \rho \varepsilon \partial u_i / \partial x_j \) with \( \varepsilon \) = eddy viscosity or fluid mixing coefficient.

Basically, the fluid mixing coefficient is defined as:

\[
\varepsilon_f = \frac{u'_i}{L_j} \tag{12.4.32}
\]

in which:

- \( u'_i \) = fluid velocity fluctuation and
- \( L_j \) = mixing length

Herein, two types of shear flows are distinguished: wall-bounded shear flows and free shear flows.

For wall-bounded shear flows with a constant shear stress in the near-wall region, the eddy viscosity coefficient can be derived by assuming that the mixing length varies in proportion to the distance \((z)\) from the wall and that the velocity fluctuations are of the order of the current-related shear velocity \((\bar{u}_w)\), yielding:

\[
\varepsilon_f = \kappa \bar{u}_w z \tag{12.4.33}
\]
in which: \( \kappa = \) proportionality constant (= 0.4). The depth-averaged \( \varepsilon_f \)-value is of the order of \( \bar{\varepsilon}_f \approx 0.1 \ u_* \ h \).

For free-shear flows the eddy viscosity coefficient usually is related to the local thickness of the mixing layer and the velocity difference across this layer, yielding:

\[
\varepsilon_f = \alpha \ b \ \Delta u
\]  
(12.4.34)

in which:

\( \Delta u = \) velocity difference

\( b = \) mixing layer thickness and

\( \alpha = \) proportionality constant (= 0.01)

In shallow water the wall-shear and the free-shear effect interact, but generally the free-shear effect is dominating when:

\[
b \gg 10 \ \frac{u_*}{\Delta u} \ h
\]

Assuming \( u_*/\Delta u \approx 0.1 \), this means \( b \gg h \).

The sediment mixing coefficient \( (\varepsilon_s) \) generally is related to the fluid mixing coefficient \( (\varepsilon_f) \) as follows:

\[
\varepsilon_s = \beta \ \phi \ \varepsilon_f
\]  
(12.4.35)

in which:

\( \beta = \) factor related to the mixing of sediment mass and fluid momentum (see Chapter 7)

\( \phi = \) turbulence damping factor (see Chapter 7)

**Horizontal mixing**

Horizontal mixing generally is most important near circulation cells because it governs the transfer of fluid momentum and sediment mass from the main flow to the circulation cell. Herein, the horizontal mixing coefficient is assumed to be constant over the depth. Most information from the literature is related to the dispersion coefficient \( (\kappa) \) which is different from the eddy viscosity coefficient \( (\varepsilon_f) \). Usually, the dispersion coefficient is determined from a depth-integrated mathematical model applied to simulate a dye-release experiment by tuning the dispersion coefficient. Applying this approach, a dispersion coefficient is introduced which becomes a tuning parameter reflecting all possible effects such as convective transport by secondary currents and convective effects related to the depth-averaging procedure. Generally, the dispersion coefficient \( \kappa \) is much larger than the mixing coefficient \( \varepsilon_{es} \). When this latter parameter is neglected, the dispersion coefficient also represents turbulence-related mixing processes. Thus, the price to be paid for simplification (integration over the depth) is the introduction of a dispersion coefficient which is mainly related to the flow system considered. The magnitude of the dispersion coefficient can only be determined from mathematical simulation of dye-release experiments in the field. An order of magnitude estimate can be obtained from formulae and graphs presented in the literature. Some information of the dispersion coefficient is given by Okubo (1971), who reexamined data related to the horizontal spreading of instantaneous dye releases in the upper mixed layers of marine and estuarine flows. No difference was made between transverse and longitudinal mixing because of the extremely irregular dye distribution patterns. The dispersion length scale was arbitrarily defined as the diameter of an equivalent circle in which 95% of the dye
was enclosed at the time of observation. Okubo presents the effective horizontal dispersion coefficient as a function of a dispersion length scale (L). Some typical values are (see Figure 12.4.2):

\[
\begin{align*}
L &= 10^2 \text{ m} & K &= 10^3 \text{ m}^2/\text{s} \\
L &= 10^3 \text{ m} & K &= 10^6 \text{ m}^2/\text{s} \\
L &= 10^4 \text{ m} & K &= 10^8 \text{ m}^2/\text{s} \\
L &= 10^5 \text{ m} & K &= 10^{10} \text{ m}^2/\text{s}
\end{align*}
\]

Information of the longitudinal dispersion coefficient is also given by Taylor (1954) and Elder (1959). For fully developed boundary layer flow in a straight channel they derived theoretically:

\[K_x = \alpha_x u_\ast h\]  \hspace{1cm} (12.4.36)

with \(\alpha_x = 10\)

Much larger \(\alpha_x\)-values have been found for large-scale meandering rivers (Sayre, 1975). Equation (12.4.36) yields values which are much larger than the turbulence-related mixing coefficient. Recall, for example, that the depth-averaged eddy-viscosity coefficient is \(\bar{\varepsilon}_f = 0.1 \ u_\ast h\).

Information of the transverse dispersion coefficient in fully developed boundary layer flow in field conditions is given by Sayre (1975) and by Prych (1970). Based on analysis of experimental results, it was found that:

\[K_y = \alpha_y u_\ast h\]  \hspace{1cm} (12.4.37)

with \(\alpha_y = 0.1\) to 0.2 for small-scale straight channels and \(\alpha_y = 0.2\) to 2 for large-scale meandering rivers. Detailed analysis of the experimental results in straight channels shows some (unclear) influence of the channel width or better the width-depth ratio because this parameter affects the size of vertical secondary circulation cells causing transverse mixing. Generally-accepted relationships representing the width-effect are not available. Natural rivers have the following typical features intensifying the transverse mixing process:

- irregular depth,
- meandering planform, and
- presence of structures and obstacles (groins, spurdikes, bridge piers etc.).

Transverse mixing is intensified by generation of free-shear layers and by secondary currents. In case of a slowly meandering river and a small side-wall roughness (few groins) the \(\alpha_y\)-value will be in the range of \(\alpha_y = 0.2\) to 1. In case of a sharply curved river with a large side-wall roughness (many groins) the \(\alpha_y\)-value will be in the range of \(\alpha_y = 1\) to 2.

Transverse mixing in free-shear layers can be estimated from Eq. (12.4.34), giving:

\[\varepsilon_{xy} = 0.01 \ b \ \Delta u\]  \hspace{1cm} (12.4.38)

Detailed information of the influence of the horizontal mixing characteristics on the computed depth-averaged velocity field in and near recirculating flows is given by Flokstra et al (1986) applying a depth-averaged flow model. Various values of the horizontal dispersion coefficient were used in the computations (K = 10. 1. 0.1 m²/s). The computer results show a relatively large influence of the K-value on the velocity field near recirculation zones when the K-value is in the range of 10 to 1 m²/s. The influence is small for K in the range of 1 to 0.1 m²/s. Flokstra et al also found a relatively large influence of the local bathymetry on the velocities near the recirculation zone. Applying Eq. (12.4.38) with \(b = 100\) m and \(\Delta u = 1\) m/s, which
are typical values for the geometry considered by Flokstra et al, the K-value is found to be about 1 m²/s. The values of Okubo (1971) yield \( K = 0.1 \) m²/s. Thus, typical K-values for shallow water flow are in the range of 0.1 to 1 m²/s. The exact value is not of essential importance because its effect on the computed velocities is relatively small for K in this range.

Although the K-values may be less important for a good representation of the recirculation zone, a good estimate of the K-value is still important for the diffusive sediment transport, because it directly affects the magnitude of the transverse diffusive transport of sediments from the main flow to the recirculation zone.

**Vertical mixing**

Assuming quasi-equilibrium boundary flow, the vertical current-related mixing coefficient usually can be represented by a parabolic distribution (see Chapter 7), as follows:

\[
e_{s, c} = e_{s, \text{max}} - e_{s, \text{max}} \left(1 - \frac{2z}{h}\right)^2
\]  

(12.4.39)

\[
e_{s, \text{max}} = 0.25 \beta \kappa u_* h
\]  

(12.4.40)

in which:

- \( e_{s, c} \) = current-related vertical sediment mixing coefficient
- \( u_* \) = bed-shear velocity
- \( h \) = water depth
- \( \kappa \) = Von Karman constant (= 0.4)
- \( \beta \) = ratio of sediment and fluid mixing (= 1)

The maximum \( e_{s, z} \)-value is equal to \( 0.1 u_* h \) for \( \kappa = 0.4 \) and \( \beta = 1 \). The depth-averaged value is \( e_{s, z} = 0.067 u_* h \).

A parabolic-constant mixing coefficient distribution may also be applied (see Chapter 7), as follows (see Figure 12.4.3):

\[
e_{s, c} = e_{s, \text{max}} - e_{s, \text{max}} \left(1 - \frac{2z}{h}\right)^2 \]  

for \( z/h < 0.5 \)  

(12.4.41)

\[
e_{s, c} = e_{s, \text{max}} = 0.25 \beta \kappa u_* h \]  

for \( z/h \geq 0.5 \)  

(12.4.42)

Waves are capable of suspending sediment particles in the near-bed layers (see Chapter 8). This behaviour can be represented by introducing an effective mixing concept. The wave-related mixing over the depth was found to be (see Chapter 8), (see Figure 12.4.3):

\[
e_{s, w} = e_{s, w, \text{bed}} \]  

for \( z \leq \delta_s \)  

(12.4.43)

\[
e_{s, w} = e_{s, w, \text{max}} \]  

for \( z \geq 0.5 h \)  

(12.4.44)

\[
e_{s, w} = e_{s, w, \text{bed}} + (e_{s, w, \text{max}} - e_{s, w, \text{bed}}) \left(\frac{z - \delta_s}{0.5h - \delta_s}\right) \]  

for \( \delta_s < z \leq 0.5 h \)  

(12.4.45)
in which:

\[ \varepsilon_{s,w} = \text{wave-related mixing coefficient} \]
\[ \varepsilon_{s,w,\text{bed}} = 0.004 \ D_s \ \delta_s \ \hat{U}_\delta \ = \text{wave-related mixing coefficient near the bed} \]
\[ \varepsilon_{s,w,\text{max}} = 0.035 \ h \ H_s / T_p \ = \text{wave-related mixing coefficient in the upper layer} \]
\[ H_s \ = \text{significant wave height} \]
\[ T_p \ = \text{peak period of wave energy spectrum} \]
\[ h \ = \text{water depth} \]
\[ \hat{U}_\delta \ = \text{peak value of orbital velocity at edge of wave boundary layer} \]
\[ \delta_s \ = \text{thickness of near-bed mixing layer} \]
\[ D_s \ = \text{dimensionless particle parameter} \]

In combined current and wave conditions the vertical distribution of the mixing coefficient can be obtained from (see Chapter 9):

\[ \varepsilon_{s,cw} = \left[ (\varepsilon_{s,c})^2 + (\varepsilon_{s,w})^2 \right]^{1/2} \quad (12.4.46) \]

**Influence of vertical density gradient**

Vertical density gradients caused by the presence of temperature, salinity and sediment concentration variations have a damping (stabilizing) effect on the vertical mixing process because additional energy is needed to mix a fluid parcel of a larger density in a lower layer with a fluid parcel of a lesser density in an upper layer against the action of gravity. This additional energy is extracted from the turbulent kinetic energy resulting in smaller turbulent velocity fluctuations and eddy-viscosity coefficients. This effect generally is modelled by introduction of the Richardson number, as follows:

\[ \varepsilon_f = \varepsilon_{f,0} (1 + \alpha \ \text{Ri})^\beta \quad (12.4.47) \]

in which:

\[ \varepsilon_{f,0} = \text{fluid mixing coefficient in the absence of vertical density gradient} \]
\[ \text{Ri} = \frac{g(\partial \rho / \partial z)/(\rho (\partial u / \partial z)^2)}{\alpha, \beta = \text{coefficients}} \]

Silt and mud suspensions have a small vertical density gradient (except close to the bottom, fluid mud) and thus these suspensions are expected to have little effect upon the velocity profile and the fluid mixing coefficient distribution. Fine sand suspensions have a relatively large vertical concentration gradient near the bed yielding a considerable damping of turbulence.

**4. Scale analysis**

Information of the order of magnitude of the convective and diffusive transport terms can be obtained from scale analysis of the sediment mass-balance equation (Eq. 12.4.13).

Assuming small concentrations (dilute suspension), this equation reads as:

\[ \frac{\partial c}{\partial t} + \frac{\partial}{\partial x} (uc) - \frac{\partial}{\partial x} (\varepsilon_s \frac{\partial c}{\partial x}) + \frac{\partial}{\partial z} (c(w-w_d)) - \frac{\partial}{\partial z} (\varepsilon_s \frac{\partial c}{\partial z}) = 0 \quad (12.4.48) \]
Figure 12.4.1 Mixing process of dye in case of a line injection

Figure 12.4.2 Dispersion coefficients according to Okubo (1971)

Figure 12.4.3 Vertical distribution of current and wave-related mixing
Applying the fluid mass balance \( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \), Eq. (12.4.48) can be simplified to:

\[
\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + w \frac{\partial c}{\partial z} - w_s \frac{\partial c}{\partial z} - \frac{\partial}{\partial x} \left( \varepsilon_s \frac{\partial c}{\partial x} \right) - \frac{\partial}{\partial z} \left( \varepsilon_s \frac{\partial c}{\partial z} \right) = 0
\]  

(12.4.49)

The magnitude of each single term can be estimated by using a normalizing method. Each term is represented as the product of a constant scale factor and a dimensionless variable of the order of unity, \( O(1) \). The importance of each term is then indicated by the relative magnitude of its scale factor.

Applying \( t = T' \), \( c = C c' \), \( u = U u' \), \( w = W w' \), \( x = L x' \), \( z = H z' \) and \( \varepsilon_s = E \varepsilon_s' \), Equation (12.4.49) can be expressed as:

\[
\frac{C}{T} \left[ \frac{\partial c'}{\partial t'} \right] + \frac{U C}{L} \left[ \frac{u'}{\partial x'} \right] + \frac{W C}{H} \left[ \frac{w'}{\partial z'} \right] - \frac{w_s C}{H} \left[ \frac{\partial c'}{\partial z'} \right] +
\]

\[
- \frac{E C}{L^2} \left[ \frac{\partial}{\partial x'} \left( \varepsilon_s' \frac{\partial c'}{\partial x'} \right) \right] - \frac{E C}{H^2} \left[ \frac{\partial}{\partial z'} \left( \varepsilon_s' \frac{\partial c'}{\partial z'} \right) \right] = 0
\]  

(12.4.50)

The settling term \( w_s \frac{\partial c}{\partial z} \) is taken as the reference term and each term is multiplied by \( H/(w_s C) \), giving:

\[
\frac{H}{w_s T} \left[ \frac{\partial c'}{\partial t'} \right] + \frac{U H}{w_s L} \left[ \frac{u'}{\partial x'} \right] + \frac{W}{w_s} \left[ \frac{w'}{\partial z'} \right] - \left[ \frac{\partial c'}{\partial z'} \right] +
\]

\[
- \frac{E H}{w_s L^2} \left[ \frac{\partial}{\partial x'} \left( \varepsilon_s' \frac{\partial c'}{\partial x'} \right) \right] - \frac{E}{w_s H} \left[ \frac{\partial}{\partial z'} \left( \varepsilon_s' \frac{\partial c'}{\partial z'} \right) \right] = 0
\]  

(12.4.51)

As an example the case of trench dredged in a tidal channel is considered. The trench has a depth scale of \( H = 10 \) m, a length scale of \( L = 100 \) m. The velocity scales are \( U = 1 \) m/s, \( W = 0.1 \) m/s. The mixing coefficient scale is \( E = 0.1 \) m²/s. The fall velocity is \( w_s = 0.01 \) m/s. The tidal time scale is \( T = 10000 \) s.

The scale factors of Eq. (12.4.51) become:

\[
\frac{H}{w_s T} = O(10^{-1})
\]

\[
\frac{U H}{w_s L} = O(10)
\]

\[
\frac{W}{w_s} = O(10)
\]

\[
\frac{E H}{w_s L^2} = O(10^{-2})
\]

\[
\frac{E}{w_s H} = O(10^0)
\]

Based on these results, the longitudinal diffusive transport \( \varepsilon_s \frac{\partial c}{\partial x} \) is found to be negligible small. The time-dependent concentration term \( \frac{\partial c}{\partial t} \) is also relatively small, which means
that the time lag between the concentrations and flow velocities remains small in case of a depth of 10 m and a fall velocity of 0.01 m/s. The time lag will increase for a larger depth and a smaller fall velocity (silts and muds).

12.4.3 Three dimensional models (3D)

1. Introduction

For computations at wide horizontal scales (estuaries, coastal zones, seas), usually a depth-averaged flow model in combination with a simple equilibrium sediment transport formula is used. Such an approach is only valid when the horizontal length scale of the adjustment process of the local suspended sediment transport to the local equilibrium transport is smaller than the maximum allowable horizontal grid size of the applied mathematical model. If this latter requirement is not satisfied, it is of essential importance to represent the horizontal and vertical adjustment process of the suspended sediment transport in the model. Basically, two types of modelling can be used: (1) the depth-integrated approach as introduced by Galappatti and Vreugdenhil (1985) and (2) the three-dimensional approach as applied by Sheng and Butler (1982), O'Connor and Nicholson (1988), Miller (1983), Wang and Adeff (1986), Van Rijn and Meijer (1988) and others. The application of the depth-integrated approach is limited to situations where the difference between the local true suspended sediment transport and the local equilibrium transport is relatively small (Wang, 1989), otherwise the results are not accurate. An advantage of the depth-integrated approach is the relatively low computer cost compared with that of three-dimensional models. A good example of these latter developments is the full unsteady three-dimensional model for fluid velocities and sediment concentrations of Wang and Adeff (1986) and McAnally et al (1986).

Usually, the 3D-models are only applied to predict the initial rate of sedimentation and erosion in a given situation for reasons of limited computational power. The initial models provide good insight into the short-term effects of a proposed structure (new harbour, closure of a channel etc.), but they are of limited value for the long-term morphological evolution, at least without interpretation of the results. This interpretation should be based on experience in similar situations and basic knowledge of morphological processes.

2. Equations

The 3D-mass balance equation for the suspended sediment reads as:

\[
\frac{\partial c}{\partial t} + \frac{\partial}{\partial x} (uc) + \frac{\partial}{\partial y} (vc) + \frac{\partial}{\partial z} ((w-w_s) c) - \frac{\partial}{\partial x} \left( \varepsilon_{sx} \frac{\partial c}{\partial x} \right) + \\
- \frac{\partial}{\partial y} \left( \varepsilon_{sy} \frac{\partial c}{\partial y} \right) - \frac{\partial}{\partial z} \left( \varepsilon_{sz} \frac{\partial c}{\partial z} \right) = 0
\]

(12.4.52)

where:

- \( c \) = sediment concentration
- \( u, v, w \) = fluid velocity components in x, y, z directions
- \( \varepsilon_{s} \) = sediment mixing coefficient
- \( w_{s} \) = particle fall velocity
- \( t \) = time

To operate a 3D-model, the flow velocities, wave heights and mixing coefficients must be known a priori (see sections 12.2 and 12.3).
To represent the flow and mixing field, two approaches are generally applied:
- Full 3D-solution of the 3D-Reynolds’ equations in case of a complicated geometry with flow separation (Toro et al., 1989; Cham, 1987; Wang and Adeff, 1986) and
- Quasi 3D-solution in case of a simple geometry (De Vriend, 1987; Van Rijn-Meijer, 1988; Toro et al., 1989).

The most simple quasi-3D approach is the application of a depth-averaged flow model (see section 12.2) in combination with logarithmic velocity profiles, as follows:

\[
\begin{align*}
\bar{u} &= \frac{u}{z/h - 1 + \ln(h/z_o)} \ln(z/z_o) \\
\bar{v} &= \frac{v}{z/h - 1 + \ln(h/z_o)} \ln(z/z_o)
\end{align*}
\] (12.4.53)

The vertical velocity can be obtained from the continuity equation, yielding:

\[
w = w_h + \int_{z}^{h} \left( \frac{\partial u}{\partial x} \right) dz + \int_{z}^{h} \left( \frac{\partial v}{\partial y} \right) dz
\] (12.4.55)

in which:
\( u, v, w \) = local flow velocity components in \( z, y, z \)-direction
\( \bar{u}, \bar{v} \) = depth-averaged velocities
\( h \) = water depth
\( z_o \) = zero-velocity level
\( w_h \) = vertical flow velocity at water surface

The vertical mixing coefficients in a quasi-3D approach can be obtained from Eqs. (12.4.39) to (12.4.46). The horizontal mixing coefficients are assumed to be constant.

A more detailed quasi-3D modelling of the flow field is presented by De Vriend (1987). Starting from the full 3D-Reynolds equations (integrated over the short wave period) and applying a gradient-type turbulence closure (scalar mixing coefficient), vertical similarity hypotheses have been made for all dependent variables. In this way the primary flow in the direction of the depth-averaged velocity can be distinguished from the secondary flow. Wave-induced and wind-induced effects are included.

Bed level changes can be obtained from the depth-integrated mass-balance equation, yielding:

\[
(1-p) \frac{\partial z_b}{\partial t} + \frac{\partial}{\partial x} (q_{b,x} + q_{a,x}) + \frac{\partial}{\partial y} (q_{b,y} + q_{a,y}) = 0
\] (12.4.56)

in which:
\( z_b \) = bed level with respect to a horizontal datum
\( p \) = porosity factor
\( q_{b,x}, q_{b,y} \) = volumetric bed load transport rates
\( q_{a,x}, q_{a,y} \) = volumetric suspended load transport rates
The suspended load transport rates are given by:

\[
q_{s,x} = \int_{a}^{h} \left( uc \cdot e_{x} \cdot \frac{\partial c}{\partial x} \right) dz \quad (12.4.57)
\]
\[
q_{s,y} = \int_{a}^{h} \left( vc \cdot e_{y} \cdot \frac{\partial c}{\partial y} \right) dz \quad (12.4.58)
\]

3. **Boundary conditions**

To specify the boundary conditions of the transport model, information of the bathymetry, water depths and sediment characteristics (size, fall velocity, density etc.) is required. The most fundamental boundary condition is the process that controls the exchange of sediment particles at the bed.

Three options are available to describe the exchange process at the bed:

- The convective flux is assumed to be equal to the equilibrium flux and specified as a known function of near-bed hydraulic and sediment parameters:

  \[
w_{s} \cdot c_{a,e} = F(\tau_{b}, d_{50}, w_{s}, ...)
  \quad (12.4.59)
  \]

  For sandy environments Van Rijn (1984) has proposed the following expression:

  \[
c_{a,e} = 0.015 \frac{d_{s0}}{a} \frac{T^{1.5}}{D_{*}^{0.3}}
  \quad (12.4.60)
  \]

- The diffusive flux is assumed to be equal to the equilibrium flux and specified as a known function:

  \[
  \left( e_{s} \frac{\partial c}{\partial z} \right)_{z=a} = F(\tau_{b}, d_{50}, w_{s}, ...)
  \quad (12.4.61)
  \]

  This method usually is applied for silty and muddy conditions.

- The net flux is prescribed

  \[
  \left( w_{s} c + e_{s} \frac{\partial c}{\partial z} \right)_{z=a} = -\alpha w_{s} (c_{a} - c_{a,e})
  \quad (12.4.62)
  \]

in which:

- \(c_{a,e}\) = equilibrium concentration at a small height \(a\) above the bed
- \(c_{s}\) = actual concentration at height \(a\) above the bed
- \(w_{s}\) = settling velocity
- \(\tau_{b}\) = time-averaged bed-shear stress due to current and waves
- \(d_{s0}\) = median particle diameter of bed material
- \(T\) = dimensionless bed-shear stress parameter
- \(D_{*}\) = dimensionless particle size parameter

12.40
The other boundary conditions are:

\[
\text{inlet boundary} \quad : \quad c_z = c_{z,\text{in}} \quad \text{or} \quad c_z = \text{given} \quad (12.4.63)
\]

\[
\text{outlet boundary} \quad : \quad \frac{\partial}{\partial n} \left( \varepsilon_s \frac{\partial c}{\partial n} \right) = 0 \quad (12.4.64)
\]

\[
\text{solid boundary} \quad : \quad \frac{\partial}{\partial n} \left( \varepsilon_s \frac{\partial c}{\partial n} \right) = 0 \quad \text{or} \quad w_s c + \varepsilon_s \frac{\partial c}{\partial z} = 0 \quad (12.4.65)
\]

\[
\text{water surface} \quad : \quad w_s c + \varepsilon_s \frac{\partial c}{\partial z} = 0 \quad (12.4.66)
\]

4. Solution methods

To solve the convection-diffusion equation, finite-difference and finite-element methods have been used. Schoellhamer (1988) applied a Lagrangian solution method. Efficient and accurate results require a grid refinement in the bottom region where the concentration gradients are relatively large. The following Table 12.1 provides information of the influence of the number of vertical grid points (applying a logarithmic vertical scale) on the accuracy of the numerically computed concentrations and transport rates in comparison to analytically computed results (Van Rijn, 1987).

The maximum error does occur in the water surface region, where the vertical grid size is maximum. The errors in the depth-integrated transport rate are much smaller because most of the material is transported in the nearbed region where the errors in the concentrations are small. In a sandy environment \((w_s/\bar{u}_* = 0.3)\) the number of grid points should be about 10, while in a muddy environment \((w_s/\bar{u}_* = 0.06)\) about 5 grid points can be used.

<table>
<thead>
<tr>
<th>number of points</th>
<th>maximum error in concentration</th>
<th>error in depth-integrated suspended sediment transport</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(w_s/\bar{u}_* = 0.06)</td>
<td>(w_s/\bar{u}_* = 0.3)</td>
</tr>
<tr>
<td>20</td>
<td>0.2%</td>
<td>5%</td>
</tr>
<tr>
<td>10</td>
<td>1%</td>
<td>20%</td>
</tr>
<tr>
<td>5</td>
<td>5%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 12.1 Number of vertical grid points

5. Applications

O’Connor and Nicholson (1988) applied their 3D-model to compute the concentration field in a laboratory channel, which was partially blocked by a pier against one side-wall. Figure 12.4.4 shows measured and computed concentrations for 28 s after the dump. The sediment material consisted of polystyrene spheres \((w_s = 0.0017 \text{ m/s})\). The concentrations were measured by means of an optical method. As can be observed, there is an acceptable degree of agreement with respect to the sediment cloud and the peak concentrations.

Van Rijn et al (1990) applied their 3D-model to compute the transport rates and initial bed level changes in a tidal area with velocities in the range of 0.6 to 1.5 m/s and depths in the
range of 5 to 15 m. The bed material consisted of sand with \( d_{50} = 250 \mu \text{m} \). The bed roughness was found to be 0.5 m. The velocity field was computed by applying a 2DH-model. Logarithmic velocity profiles were applied to obtain a quasi-3D velocity field. Measured and computed mean velocities and transport rates in 6 locations during maximum ebb flow are reported in the following Table 12.2.

<table>
<thead>
<tr>
<th>Location</th>
<th>velocity (m/s)</th>
<th>transport (kg/sm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>measured</td>
<td>computed</td>
</tr>
<tr>
<td>A</td>
<td>0.94</td>
<td>0.97</td>
</tr>
<tr>
<td>B</td>
<td>1.25</td>
<td>1.26</td>
</tr>
<tr>
<td>C</td>
<td>1.28</td>
<td>1.30</td>
</tr>
<tr>
<td>D</td>
<td>1.07</td>
<td>1.33</td>
</tr>
<tr>
<td>E</td>
<td>1.28</td>
<td>1.42</td>
</tr>
<tr>
<td>F</td>
<td>1.11</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Table 12.2 Measured and computed velocities and transport rates

Figure 12.4.5 shows initial bed level changes during ebb flow. The models of O’Connor-Nicholson (1988) and Van Rijn et al (1990) have also been applied to compute transport rates in muddy conditions.

Shimizu et al (1990) applied a quasi-3D model including bed-load transport and suspended load transport to compute the bed morphology of a river bend.

12.4.4 Two-dimensional vertical models (2DV)

1. Introduction

Early attempts of two-dimensional vertical mathematical modelling for non-uniform conditions were presented by Kalinske (1940) and Dobbins (1943). Later a more general mathematical approach was presented by O’Connor (1971). Smith and O’Connor (1977) presented a two-dimensional vertical model based on the laterally integrated momentum and continuity equations for the fluid and sediment phases, and a "one-equation" turbulence closure model to represent the fluid shear stresses and diffusion coefficients. Celik and Rodi (1984, 1985, 1988) presented a similar model as that of Smith and O’Connor. However, the former applied a "two-equation" turbulence closure model (K-Epsilon model). Bechteler and Schripf (1984) presented a relatively simple two-dimensional model neglecting vertical convection and horizontal diffusion. The fluid field was represented by logarithmic velocity profiles. Markofsky et al (1985) presented a 2DV-model for mud concentrations.

In the Netherlands two-dimensional modelling of suspended sediment was initiated by Kerssens (1974). Kerssens et al (1977 and 1979) presented a two-dimensional vertical model for gradually varying flows neglecting vertical convection and horizontal diffusion. Logarithmic velocity profiles were used to represent the fluid velocity field. The vertical sediment mixing coefficients were represented by a parabolic-constant distribution. A concentration-type boundary condition was applied at the bed, assuming instantaneous adjustment to equilibrium conditions close to the bed.

Later, the flow width was introduced to extend the applicability of the model to gradually varying flows in transverse direction. A method based on the application of flexible profiles (shape functions) was introduced to obtain a better description of the velocity profiles and the sediment mixing coefficients (Van Rijn, 1985, 1986, 1987).
Figure 12.4.4 Measured and computed concentrations (in g/l) at 28 s after a dump of sediment (O’Connor and Nicholson, 1988)

Figure 12.4.5 Initial bed level changes in a tidal channel (Van Rijn et al., 1989)

2. Equations

In case of a sandy environment the mass balance equation of the sediment phase can be simplified to

\[
\frac{\partial}{\partial x} (uc) + \frac{\partial}{\partial y} (vc) + \frac{\partial}{\partial z} ((w-w_s)c) - \frac{\partial}{\partial z} (e_s \frac{\partial c}{\partial z}) = 0
\]  

(12.4.67)

because the time-dependent term \((\partial c/\partial t)\) and the horizontal diffusion terms \((e_s \partial c/\partial x, e_s \partial c/\partial y)\) are small with respect to the other terms (Kerssens et al, 1979).
Assuming the variables to be approximately constant in transverse direction \(y\), Equation (12.4.67) can be integrated in this direction yielding a width-integrated equation which can be applied to model the sediment transport in gradually varying channels (or stream tubes), as follows:

\[
\frac{1}{b} \frac{\partial}{\partial x} (b u_c) + \frac{\partial}{\partial z} (w-w_c)c - \frac{\partial}{\partial z} (e_s \frac{\partial c}{\partial z}) = 0
\] (12.4.68)

Equation (12.4.68) can also be used for tidal conditions by schematizing the tidal cycle to a number of quasi-steady flow periods.

The flow velocity profiles can be determined from various mathematical models depending on the complexity of the bathymetry (see section 12.2). For complicated flows with largely perturbed velocity profiles including flow separation, a refined mathematical approach is of essential importance. The most accurate description can be obtained by applying the 2DV Reynolds' equations in combination with a two-equation (K-Epsilon) turbulence closure. The K-E model consists of transport equations for the turbulence kinetic energy \(k\) and its dissipation. The fluid (and sediment) mixing coefficient can be derived from the K-Epsilon model in case of a complicated bathymetry (flow separation). For long-term computations this model is not yet attractive. Therefore, Van Rijn used the K-Epsilon model to calibrate the more simple profile model (Van Rijn, 1987). The vertical distribution (profile) was represented by a parabolic-constant distribution: parabolic in the lower half and constant in the upper half of the depth. The longitudinal variation of the mixing coefficient was represented by a first order differential equation.

The boundary conditions for 2DV models are similar to those of 3D models, see section 12.4.3. Numerical solution methods are also similar to those of 3D-models.

3. Applications

Two-dimensional vertical models have been applied to predict transport rates, sedimentation and erosion in rivers, estuaries and coastal waters. The applications in rivers usually are related to sedimentation of sand material in pipeline, tunnel trenches and settling traps for irrigation channels. Applications in estuaries and coastal waters require stream tube modelling, which means that the dimensions of the stream tube should be known from field measurements or mathematical flow models. The 2DV-models have been successfully applied for sedimentation predictions of pipeline, tunnel trenches and navigation channels. In case of sandy conditions a quasi-steady approach can be used, which means that the tidal cycle is schematized to quasi-steady flow periods. In case of muddy conditions this approach cannot be applied because the \(\partial q/\partial t\)-term is relatively important.

Çelik and Rodi (1985, 1988) applied a 2DV-model to compute the longitudinal development of sand concentration profiles in a laboratory channel. The flow velocities were derived from the K-Epsilon model. Good agreement between measured and computed concentrations can be observed, as shown in Figure 12.4.6.

Van Rijn (1986, 1987) applied a 2DV-model to compute the sedimentation and migration of a trench in a laboratory flume. The bed material consisted of sand with a median diameter of 160 \(\mu\)m. Three trenches with side slopes of 1:3, 1:7 and 1:10 were considered. Figure 12.4.7 shows computed and measured concentration profiles at various locations. Figure 12.4.8 shows computed and measured bed level profiles after 15 hours. Good agreement can be observed.

The 2DV-model of Van Rijn (1987) was also used to compute the sedimentation rate of a trench in the Western Scheldt, a tidal estuary in The Netherlands. The bed material consisted
of sand with a median diameter of 180 µm. The mean tidal cycle was assumed to represent the neap-spring cycle. The mean tidal cycle was schematized to 4 quasi-steady flow periods of 2 hours each (see Figure 12.4.9). The computed and measured bed levels after 22 and 80 days are shown in Figure 12.4.9.

![Graph showing concentration profiles](image)

**Figure 12.4.6 Computed and measured concentration profiles (Celik and Rodi. 1985)**

Hofmans (1992) developed a 2DV model to compute the scour hole downstream of a rigid bottom. The flow and turbulence field was computed by a model based on the mixing length approach (DUCT-model). Special attention was given to the bed-boundary condition for the sediment; the stochastic approach introduced by Van Rijn (1987) was improved. Figure 12.4.10 shows the computed and measured concentrations and bed levels in a scour hole for a laboratory experiment (d₅₀ = 150 µm).

Markofsky et al (1985) computed the temporal distribution (tidal cycle) of mud concentrations in the Weser Estuary in Germany applying a constant fall velocity and a constant vertical mixing coefficient. Figure 12.4.11 shows concentrations at 5 different heights close to the upstream end of the estuary. The critical bed-shear stresses for erosion and sedimentation were varied to study the effects on the concentration distributions.
Figure 12.4.7 Computed and measured sediment concentrations (Van Rijn, 1987)
Figure 12.4.8 Computed and measured bed level profiles (Van Rijn, 1987)
A. TIDAL CONDITIONS

B. BED LEVELS

Figure 12.4.9 Tidal conditions and bed level profiles in a trench in the Western Scheldt, The Netherlands (Van Rijn, 1987)
Figure 12.4.10 Computed and measured scour hole (Hofmans, 1992)

Figure 12.4.11 Computed mud concentrations (Markofsky et al, 1985)
12.4.5 Two-dimensional horizontal models (2DH)

1. Introduction

Two-dimensional horizontal sediment transport and bed-evolution models are based on the depth-integrated equations of motions in combination with a wave model and a sediment transport model.

2. Equation

The flow field and the wave field equations are presented in sections 12.2 and 12.3. In the case of relatively coarse sandy materials ($d_{50} > 300 \mu m$), the sediment transport can be represented by a formula, as follows:

$$q_t = F(\vec{v}, \hat{U}, h, T_p, \phi, d_{50}, \text{etc})$$  \hspace{1cm} (12.4.69)

in which:
- $\vec{v}$ = current velocity vector
- $\hat{U}$ = peak orbital velocity near the bed
- $h$ = water depth
- $T_p$ = peak wave period
- $\phi$ = angle between wave and current direction

In case of transport of relatively fine material ($d_{50} < 300 \mu m$) with an adjustment length scale larger than the applied grid scale of the model, it is important to represent the vertical distribution of the concentration profiles. An attractive depth-integrated approach was proposed by Galapatti and Vreugdenhil (1985). This method is based on the convection-diffusion equation including boundary conditions at the bed and at the surface. It is assumed that the velocity and concentration profiles have a distribution similar to that of the equilibrium profiles, which restricts the application of the model to gradually varying flow conditions. For example, the complete first-order solution for a one-dimensional channel reads as:

$$c = \alpha \bar{c} + \beta \frac{h}{w_s} \frac{\partial \bar{c}}{\partial t} + \gamma \frac{\bar{u}h}{w_s} \frac{\partial \bar{c}}{\partial x}$$  \hspace{1cm} (12.4.70)

in which:
- $\bar{c}$ = depth-averaged concentration
- $h$ = water depth
- $w_s$ = fall velocity
- $\bar{u}$ = depth-averaged velocity
- $\alpha, \beta, \gamma$ = coefficients representing the vertical distribution effects (profile functions).

The coefficients (functions of the vertical coordinate) can be determined in advance applying equilibrium profiles for the velocity, mixing coefficients and concentrations. Wang (1989) generalized this method to two horizontal dimensions. The boundary conditions for the concentration model are similar to those presented in section 12.4.3.

For mud transport a relatively simple depth-averaged approach can be applied because the concentrations are nearly constant in vertical direction (Ariathurai and Krone, 1976; Cole and Miles, 1983; Teissson and Gritsche, 1988), yielding:
\[
\frac{\partial \vec{c}}{\partial t} + \vec{u} \cdot \frac{\partial \vec{c}}{\partial x} + \vec{v} \cdot \frac{\partial \vec{c}}{\partial y} - \frac{1}{h} \frac{\partial}{\partial x} (hD \frac{\partial \vec{c}}{\partial x}) - \frac{1}{h} \frac{\partial}{\partial y} (hD \frac{\partial \vec{c}}{\partial y}) - \frac{S}{h} = 0
\] (12.4.71)

in which:
\[h = \text{depth}\]
\[D = \text{diffusion coefficient}\]
\[S = \text{source-sink term}\]

The source-sink term accounts for erosion or sedimentation. Generally, the expressions of Krone (1962) and Partheniades (1965) are applied. Proper simulation also requires modelling of the consolidation process (Teisson and Fritsch, 1988), see Chapter 11.

3. Applications

Struiksma et al (1984) have presented a 2D-H-model to compute the bed evolution in a river bend. The sediment transport formula of Engelund and Hansen (1967) was used taking slope effects (\(\partial z / \partial x\)) and secondary flow effects (bed shear stress makes a small angle with the current vector) into account.

Figure 12.4.12 shows measured and computed bed levels along the left bank, the right bank and along the axis for a laboratory experiment. The point bar along the left bank and a deep pit along the right bank are represented quite well.

Figure 12.4.13 shows measured and computed bed levels for a bend in the Waal river, The Netherlands.

![Figure 12.4.12 Computed and measured bed levels in a river bend (Struiksma et al, 1985)](image)

Andersen et al (1988) presented a morphological model for coastal waters. The sediment transport is computed by a formula taking the effect of the waves into account. The application considers a cooling water intake at the coast with a water depth of 4 m. The area is exposed to waves from 2 directions with a wave height \(H_{rms} = 1\) m in both directions. The intake discharge is constant at 100 m³/s. The bed material is sand with a median diameter of 200 µm. Figure 12.4.14 shows the initial wave field, the current field and the bed levels after 18 and 32 months.
Figure 12.4.13  Computed and measured bed levels in a bend of the Waal river, The Netherlands (Struijsma, 1985)
Figure 12.4.14 Bed levels after 18 and 32 months (Andersen et al., 1988)

Figure 12.4.15 Bed levels for a semi-circular bay (Roelvink, 1992)
Roelvink (1992) presented a similar morphological model for coastal waters; flow, waves and sediment transport formula. The application considers a semi-circular bay. The area is exposed to waves which generate a longshore current in the surfzone. The incoming wave height is $H_{rms-o} = 0.6$ m at a water depth of 5.5 m. The peak period is $T_p = 5$ s. The bed material is sand with a median particle diameter of 200 µm. Figure 12.4.15 shows the initial bathymetry and that after 5 days. Deposition can be observed in the Bay area.

Wang (1989) developed a 2DH-model (ESMOR-2DH) based on the depth-integrated approach of Galappatti (see Eq. 12.4.70). The ESMOR-model was used to compute the sediment transport pattern for steady flow in a partially closed channel, as shown in Fig. 12.4.16. The channel has a horizontal bed and a water depth of 6 m, the approach velocity is 0.65 m/s. The bed material is sand with a median particle diameter of 200 µm. The bed roughness is 0.25 m. Figure 12.4.16 shows the sediment transport rates along streamline B.

Van Rijn (1987) used a 2DV-model according to the streamtube approach (Eq. 12.4.68) and a 3D-model to compute the sediment transport rates in the same partially closed channel. The computed transport rates along streamline B are also shown in Fig. 12.4.16. Good agreement between the results of the three models can be observed in the acceleration zone upstream of the dam; the agreement in the deceleration zone is less good.

![Figure 12.4.16 Streamlines (above) and sediment transport along streamline B (below)](image-url)

12.54
12.4.6 One dimensional models

1. Rivers and estuaries

One-dimensional models are most frequently used to simulate the large-scale morphological changes in rivers and estuaries. In the latter case the system of tidal flood and ebb channels is modelled as a network system in which only vertical bed level changes are considered. The sediment transport and the bed roughness can be represented by simple formulas in terms of the mean flow variables and sediment properties. Analytical solutions can be obtained for simple schematized cases (De Vries, 1975). Numerical solutions are required for more realistic cases. If required, the non-uniformity of the bed material can be taken into account by dividing the bed material in a number of size classes (Thomas, 1982; Ribberink, 1987; Armanini-Di Silvio, 1988). Adjustment effects related to the suspended sediment transport process can be taken into account by using the depth-integrated approach of Galappatti and Vreugdenhil (1985). Figure 12.4.17 shows measured and computed longitudinal bed level profiles after 30, 60 and 90 days for a dredged channel in the IJssel river in The Netherlands. The channel length was 200 m, the width was 60 m. The bed material consisted of sand with median particle diameter of 600 μm. The Meyer-Peter Müller formula was used to compute the bed-load transport rates. Figure 12.4.18 shows results of a 1D-model based on the Galappatti-approach compared with measurements and 2DV-model results (Galappatti and Vreugdenhil, 1985).

Detailed information of one-dimensional river models with respect to numerical solution methods and practical applications is given by Cunge et al (1980) and by Jansen et al (1979). A state of the art review is given by De Vries et al (1989).

Figure 12.4.17 Computed and measured longitudinal bed level profiles of dredged channel in the IJssel river, The Netherlands
2. Coastal waters

One-dimensional models are also used to simulate the cross-shore and longshore bed evolution in coastal waters. Applying cross-shore models, problems like beach and dune erosion and beach nourishment can be studied. The basic sub-models of a cross-shore profile model are:

- wave propagation model (including low-frequency effects, energy dissipation by breaking and by bottom friction),
- wave velocity model (asymmetrical orbital velocities),
- current velocity model (wave-induced net velocities),
- sediment transport model (bed-load, suspended load and slope effects),
- bed evolution model.

Examples of cross-shore profile models are those of De Vries and Bailard (1988), Nairn (1988), Roelvink and Stive (1989) and Steetzel (1987). Figure 12.4.19 shows computed cross-shore profiles after 8.2 years at three locations along the Dutch coast (Groenendijk and Roelvink, 1992).

Longshore models can be used to compute coastline changes near structures like groins, breakwaters and harbours. Usually, the coast is schematized as a single line. It is assumed that the shape of the cross-shore profile does not change so that the cross-shore profile can be shifted horizontally as a result of accretion or erosion. The basic equation is the sediment mass balance equation in longshore direction. The longshore sediment transport is represented by a formula specifying the total longshore transport per radian coastline rotation.

![Figure 12.4.18 Computed and measured bed level profiles for a trench](image)

12.56
Figure 12.4.19  Computed cross-shore profiles after 8.2 years at three locations along the Dutch coast, (Groenendijk and Roelvink, 1992)
REFERENCES


12.58
REFERENCES (continued)


REFERENCES (continued)


REFERENCES (continued)


12.61
REFERENCES (continued)


REFERENCES (continued)


13 MEASURING INSTRUMENTS FOR SEDIMENT TRANSPORT, SETTLING VELOCITY AND WET BULK DENSITY

13.1 Introduction

Most morphological systems can be considered to be in a state of dynamic equilibrium between deposition and erosion. The general characteristics may change very slowly with time. Human interference with the governing phenomena in such a delicate equilibrium will have morphological consequences. To predict these consequences for a specific project, it is of essential importance to have detailed knowledge of the local morphological variables such as the bed material size, the settling velocities of the suspended solids and the transport rates. To obtain this information, an extensive field survey should be carried out. An important phase prior to the actual field survey is the selection of the most appropriate instruments, which usually is a rather difficult problem because a wide range of instruments has been developed from simple mechanical samplers to sophisticated optical and acoustical samplers. The selection of instruments is primarily dependent on the variables to be measured, the available facilities (boat, platform, winch) and the required accuracy. Especially, the required accuracy should be considered carefully. For example, a reconnaissance study requires the use of much less sophisticated instruments than a basic research study.

This chapter provides information of all relevant aspects related to sediment transport measurements such as:
• measuring principles and statistics,
• type and accuracy of the instruments,
• selection of the instruments.

The attention is focused on those instruments which have been proven to be reliable and successful in field conditions. Instruments that are in developing stage, are not considered.


13.2 Measuring facilities

Sediment transport measurements in rivers and estuaries usually are performed from a survey boat or vessel. Electric winches should be used when rapid profile measurements are required.

Sediment transport measurements in coastal conditions with combined currents and waves cannot be performed from an anchored boat or vessel.

Other facilities have been designed and used (see Basinski, 1989):
• pier connected to shore,
• platform or towers resting on sea bed,
• tripod resting on sea bed,
• pile founded in sea bed,
• towed sledge or vehicle.

A problem of all facilities near coasts is the wave and current-induced scour near the legs or wheels of the structure or vehicle. The free span of a pier or a platform should be as large as possible to allow measurements between the legs. An example of a poorly designed pier is the Duck pier, North Carolina, USA, with longshore pile spacing of 4.6 m and cross-shore spacing of 12.2 m, which has caused considerable scour in the vicinity of the pier. An exam-
Figure 13.2.1 Measuring facilities in coastal regions
ple of a slender pier with cross-shore pile spacings of 36 m is the Sally Wharf near Bordeaux, France (see Figure 13.2.1).

Stand-alone tripods are placed on the sea bed from a vessel during calm weather. They are vulnerable in high waves (overturning) or in areas with intensive fishing activities. Another problem is the absence of control of the measuring elevation of the instruments in conditions with rapidly changing bed levels (surf zone). The instruments can be easily damaged when placed close to the bed.

Piles driven or washed into the sea bed offer a good solution for hydrodynamic measurements (wind, waves). Measurements of velocities and sediment concentrations usually cannot be performed because of the influence of the vortices and scour generated by the pile construction (see Figure 13.2.1).

Towed sledges or vehicles with a mast above the water surface are used for (continuous) cross-shore bed level soundings and wave height measurements in the surf zone during storm periods. Measurements of velocities and concentrations may not be reliable because of the influence of the sledge on the local velocity and concentration field. Self-moving vehicles on wheels or caterpillars have also been used for sounding purposes.

The instruments most frequently used for hydrodynamic measurements (instantaneous water surface elevation and velocity) are:
- pressure sensor,
- capacity wire,
- wave staff,
- wave rider buoy,
- electro-magnetic current meter (two horizontal directions),
- acoustic-doppler current meter (two horizontal directions).

A review of these instruments based on information supplied by researchers is given by Terwindt et al. (1992).

13.3 Measuring principles

13.3.1 Suspended load transport

1. Introduction

Samplers for suspended sediment transport have been developed according to two different principles: the direct and the indirect measurement of the sediment transport. To explain this, the following expression for the local suspended sediment transport is given:

\[
s = \frac{U_s C_s}{(u_s + u'_s)} (c_s + c'_s) = u_s c_s + u'_s c'_s
\]

where:
- \( s \) = time-averaged suspended sediment transport at height \( z \) above the bed,
- \( U_s \) = instantaneous sediment particle velocity at height \( z \)
- \( C_s \) = instantaneous sediment concentration at height \( z \)
- \( u_s \) = time-averaged sediment particle velocity at height \( z \)
- \( c_s \) = time-averaged sediment concentration at height \( z \)
- \( u'_s \) = fluctuation of sediment particle velocity at height \( z \)
- \( c'_s \) = fluctuation of sediment concentration at height \( z \).
2. Direct method

The direct method is based on the direct measurement of the time-averaged sediment transport \( \overline{U_sC_s} \) in a certain point (point-integrating) or over a certain depth range (depth-integrating). This latter procedure implies vertical movement at a uniform speed of the sampler over a certain depth range. Using a mechanical sampler based on the collection of a water-sediment sample, this procedure is only applicable in shallow streams.

Examples of samplers based on the direct measuring principle are the Delft Bottle and the Acoustic doppler samplers (see Section 13.7).

3. Indirect method

The indirect method is based on the simultaneous but separate measurement of the time-averaged fluid velocity and the time-averaged sediment concentration, which are multiplied to obtain the time-averaged sediment transport. This method implies two assumptions which introduce errors:

1. the \( \overline{U_sC_s} \)-term is assumed to be zero, and
2. the fluid and sediment particle velocity are assumed to be equal.

Some information on the inaccuracy of the indirect method is given by Anderson (1975). Based on his results, the sediment transport according to the direct method is about 10% smaller than that according to the indirect method. Mulder et al. (1985) report a value of 5%, while Soulsby et al. (1985) report values of about 1%.

The time-averaged concentration can be measured in a single point (point-integrating) or over a certain depth range (depth-integrating). Examples of samplers based on the indirect measuring principle are the bottle and trap samplers, the USD-49, the USP-61, the pump-samplers and the optical samplers.

13.3.2 Bed-load transport

1. Direct method

The most widely used method for the sampling of bed load is the direct method by means of so-called trap-type samplers. Many versions of the trap-type sampler have been used with varying amount of success. The problems encountered with the trap-type sampler are the lowering and raising of the sampler to and from the streambed and the efficiency of the sampler in collecting the bed-load particles.

2. Indirect method

The most widely-known indirect measuring methods are:
- bed form migration studies (tracking),
- sediment deposition and erosion studies,
- tracer studies.

Bed form tracking

Bed form tracking implies periodic depth measurements along longitudinal profiles (previously fixed). By comparison of sequential profiles the migration velocity of the bed forms can be determined.
Sediment deposition and erosion

Deposition and erosion studies imply periodic soundings of local bed levels (natural or dredged channel, scour near structure, harbour basin). By comparison of sequential profiles the bed-load transport rate can be obtained and related to the prevailing hydraulic conditions. An accurate (but costly) method to determine the bed-load transport rate in a river is the dredging of a trench across the river. This method yields a reliable time- and space-averaged value.

Tracer studies

Grains that are marked in such a way that their transport characteristics are not changed, are added to the flow in small quantities and their displacements are determined. From their displacements the transport can be computed.

Several types of tracers are used:

- **Fluorescent (luminofores)**
  Marked grains can be detected after sampling under UV-light. Different types can be used simultaneously.

- **Radioactive**
  Natural sand is provided with a coating with radioactive material. Large quantities are necessary to remain above the back-ground level of radioactivity.

- **Activation**
  Particles are marked and radiated after sampling.

Several methods for interpretation are used:

- **Constant injection method**
  A constant amount of tracer material (rate \( \tau \)) is distributed over the profile and injected during a long time-interval. At a downstream cross-section samples are collected and concentration as a function of time is determined. After some time the concentration becomes constant = \( c_0 \). Then the rate of transport can be computed from the relation: \( s = \tau/c_0 \).

- **Point injection method**
  At a certain time an amount of tracer material is injected. At several downstream locations concentration is determined as a function of distance. From the displacement of the centre of gravity of the concentration-distance curves the average transport velocity can be computed. Multiplication with the effective depth of transportation \( \delta \) gives the rate of transport. The effective depth is of the order of half the height of the bed forms and can also be determined by sampling in the bed.

Problems are the length of the measuring interval, the fact that the external conditions have to be constant and the large number of observations. Some of these restrictions can be diminished by applying "dispersion methods". The data are compared with a theoretical dispersion model. A review of existing measuring techniques is given by Jansen et al. (1979).
13.4 Measuring statistics

13.4.1 General aspects

An important aspect of any morphological study is the field survey during which the samples to be analyzed are collected. It is important to be aware of the fact that the quality of the total study can only be as good as the quality of the information gained through sampling. Thus, any errors incurred during sampling will manifest themselves by limiting the accuracy of the study.

The objective of a field survey is to obtain samples from the project area with the purpose of characterizing the area sampled. The sample size should be small enough to be conveniently handled and transported and yet sufficient to meet the requirements of accuracy.

The quality of the sampling process and analysis is dependent upon:
- selecting representative sampling sites in the project area,
- collecting sufficient samples at each sampling site,
- using appropriate sampling methods,
- protecting the samples during the storage period (sample preservation),
- flexibility of the sampling programme.

13.4.2 Sampling site

The selected sites should be well-distributed over the project area and be representative for the (mean annual) prevailing hydraulic and morphologic conditions. Some general requirements are:
- located in a straight reach,
- located in a stable cross section (no erosion or deposition),
- uniform bed topography,
- regular spatial velocity distributions (no converging or diverging streamlines, no vortices, backward flow or dead water zones),
- located normal to the main flow direction,
- uniform wave characteristics,
- sufficiently deep with respect to the dimensions of the sampling equipment,
- accessible and clear of natural and/or artificial obstacles (trees, bridges, piers),
- well-defined geometrical dimensions (local depth, width, position).

In raising the question of representativeness, it is possible to define two populations: one population is the actual population at the sampling point; the second population is the population of the collected samples. Ideally, both populations should be the same. However, it is necessary to be aware of the fact that differences may exist between these two populations because of bias in the sampling programme. Factors that can contribute to the bias are oversampling and equipment limitation.

13.4.3 Number of measurements for suspended load transport

1. General aspects

The total load consists of bed material load and wash load. The bed material can be subdivided in bed load and suspended load transport.

The wash load consists of sediment particles (fines $< 62 \, \mu m$) with particle sizes smaller than those found in appreciable quantities in the bed material. The fine particles usually are uniformly distributed over the entire cross-section. The sediment discharge can simply be
obtained by multiplication of the flow discharge and the concentration. Since the concentration is approximately constant over the cross-section, the number of samples can be limited to a few samples. Usually, the samples are taken in reference points which have been found to be representative for the entire cross-section. This should be checked regularly (dry and wet season; low and high discharge).

The suspended sand discharge (particles > 62 μm) usually is measured by taking a number of samples over the depth and over the width of the river. The cross-section is divided into a number of subsections. The sediment discharge passing through each subsection is determined by taking (point or depth-integrated) samples along one vertical within each subsection.

The accuracy of the suspended sediment discharge depends on:
• the number of points over the depth,
• the number of verticals over the bed-form length,
• the number of verticals over the width (cross-section),
• the number of verticals over time (flood period, ebb period).

Errors related to measurements can be classified as systematic errors and random errors. A typical systematic error is the sediment discharge in the unmeasured zones below the lowest sampling point and above the highest sampling point (related to the size of the sampler). Systematic errors accumulate with increasing number of measurements. Random errors can be eliminated (averaged out) by taking more measurements.

Factors influencing the accuracy are:
• measuring method,
• instruments available (calibrations),
• natural fluctuations of concentration, velocity, transport,
• calculation method (extrapolation unmeasured zones).

2. Number of points in a vertical

Equation

The depth-integrated suspended load transport per unit width can be approximated by (see Section 13.5.1):

\[ s = \sum_{i=1}^{k} (u_i c_i \Delta z_i) + \epsilon \]  \hspace{1cm} (13.4.1)

in which:
\( u \) = time-averaged velocity at height \( z \) (point \( i \)) above the bed,
\( c \) = time-averaged concentration at height \( z \) (point \( i \)) above the bed,
\( \Delta z \) = vertical increment \( i \) \( (\Delta z_i = z_i - z_{i-1}) \),
\( \epsilon \) = interpolation error,
\( k \) = number of points over the depth.

The interpolation error is related to the representation of a smooth curve by a limited number of linear segments \( (\epsilon \rightarrow 0 \text{ for } k \rightarrow \infty) \). The magnitude of this error depends on the number of points over the depth and the distribution of these points over the depth. Since the sediment concentrations are largest near the bed, the interpolation error can be reduced by taking small increments (many points) near the bed.

13.7
Assuming that the variables \( u, c \) and \( \Delta z \) are independent stochastic variables with a normal Gaussian distribution, the relative error of the suspended sediment transport can be estimated by:

\[
\left( \frac{\sigma_s}{s} \right)^2 = \frac{1}{k} \left[ \left( \frac{\sigma_u}{u} \right)^2 + \left( \frac{\sigma_c}{c} \right)^2 + \left( \frac{\sigma_{\Delta z}}{\Delta z} \right)^2 \right] + (\varepsilon_{\text{rel}})^2
\]  

(13.4.2)

in which \( \sigma \) represents the standard deviation and \( \varepsilon_{\text{rel}} = \varepsilon/s \).

**Relative standard deviation**

Taking a time-averaging period of 2 to 3 minutes, the relative standard deviation of the local flow velocities will be about \( \sigma_u/u = 0.1 \).

The relative standard deviation of the sediment concentration will be relatively large, especially close to the bed, say \( \sigma_c/c = 0.3 \). Similarly, \( \sigma_{\Delta z}/\Delta z = 0.3 \).

**Relative interpolation error**

The relative interpolation error \( (\varepsilon_{\text{rel}} = \varepsilon/s) \) was determined from flume and field measurement results. The flume-experiments of Barton and Lin (1955) were used. During Run 20 (depth \( h = 0.19 \) m, mean velocity \( \bar{u} = 0.91 \) m/s, sediment size \( d_{50} = 180 \) \( \mu \)m) the flow velocities and sediment concentrations were measured in 14 points distributed over the depth between \( z = 0.01 \) h and 0.9 h. Based on these values, the depth-integrated suspended sediment transport was computed. The sediment concentrations in the unmeasured zone near the bed were represented by theoretical curves fitted through the lowest three measuring points. The computed transport rate (based on 14 points) was adopted as the true transport rate. Thus, the relative interpolation error is assumed to be zero for 14 points \( (\varepsilon_{\text{rel}} = 0 \) for \( k = 14 \)). These computations were repeated for \( k = 11, 9, 7, 5 \) and 4 points, using two different schemes (A and B) for the distribution of the points over the depth, as follows:

scheme A: \( z = 0.01 \) h, 0.05 h, 0.1 h, \( \ldots, \) 0.9 h

scheme B: \( z = 0.05 \) h, 0.1 h, 0.25 h, \( \ldots, \) 0.9 h.

The transport rates were computed for each case \( (k = 11, 9, 7, 5 \) and 4 points) and compared to the true transport rate \( (k = 14 \) points) to determine the relative error \( (\varepsilon_{\text{rel}}) \), see Table 13.1.

<table>
<thead>
<tr>
<th>Number of vertical points ( k )</th>
<th>Relative interpolation error ( (\varepsilon_{\text{rel}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>scheme A</td>
</tr>
<tr>
<td>14</td>
<td>0 %</td>
</tr>
<tr>
<td>11</td>
<td>+ 0.5 %</td>
</tr>
<tr>
<td>9</td>
<td>+ 1.1 %</td>
</tr>
<tr>
<td>7</td>
<td>+ 1.2 %</td>
</tr>
<tr>
<td>5</td>
<td>+ 4.0 %</td>
</tr>
<tr>
<td>4</td>
<td>+ 11 %</td>
</tr>
</tbody>
</table>

*Table 13.1 Relative interpolation error of depth-integrated transport rate
\(+ = \text{overestimation}, - = \text{underestimation}\)*
A similar analysis for field conditions (water depth = 16 m) was performed by Rijkswaterstaat (1979, 1980). The measurement elevations were in the range of \( z = 0.03 \text{ h} \) to \( z = 0.9 \text{ h} \). The number of points was varied from \( k = 10 \) to \( k = 2 \), using various distribution schemes. The true transport rate was assumed to be obtained for \( k = 10 \). In all 89 cases (profiles) were considered. The results are presented in Table 13.2.

<table>
<thead>
<tr>
<th>Number of vertical points ( k )</th>
<th>Mean relative interpolation error ( \epsilon_{\text{rel}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 %</td>
</tr>
<tr>
<td>9</td>
<td>4.1 %</td>
</tr>
<tr>
<td>8</td>
<td>5.2 %</td>
</tr>
<tr>
<td>7</td>
<td>6.5 %</td>
</tr>
<tr>
<td>6</td>
<td>6.3 %</td>
</tr>
<tr>
<td>5</td>
<td>14.7 %</td>
</tr>
<tr>
<td>4</td>
<td>15.1 %</td>
</tr>
<tr>
<td>3</td>
<td>16.1 %</td>
</tr>
<tr>
<td>2</td>
<td>76.1 %</td>
</tr>
</tbody>
</table>

**Table 13.2 Relative interpolation error of depth-integrated transport rate**

**Relative error of depth-integrated transport rate**

The relative error of the depth-integrated transport rate can be determined from Eq. (13.4.2) with \( \sigma_{u}/u = 0.1 \), \( \sigma_{c}/c = 0.3 \), \( \sigma_{\Delta z}/\Delta z = 0.3 \) and \( \epsilon_{\text{rel}} = 0.10 \) to 0.15 for \( k = 10 \) to 4. The results are presented in Table 13.3.

<table>
<thead>
<tr>
<th>Number of vertical points ( k )</th>
<th>Relative error of depth-integrated transport rate ( \sigma_{s}/s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>17 %</td>
</tr>
<tr>
<td>9</td>
<td>18 %</td>
</tr>
<tr>
<td>8</td>
<td>19 %</td>
</tr>
<tr>
<td>7</td>
<td>21 %</td>
</tr>
<tr>
<td>6</td>
<td>22 %</td>
</tr>
<tr>
<td>5</td>
<td>24 %</td>
</tr>
<tr>
<td>4</td>
<td>27 %</td>
</tr>
</tbody>
</table>

**Table 13.3 Relative error of depth-integrated transport rate**

These results are also shown in Figure 13.4.1. Based on these results, the maximum relative error of the depth-integrated suspended transport rate will be about 20% for 6 measuring points \( (k = 6) \). The following distribution of the measuring points over the depth is proposed: \( z = 0.01 \text{ h}, 0.05 \text{ h}, 0.1 \text{ h}, 0.25 \text{ h}, 0.5 \text{ h}, 0.9 \text{ h} \) above the bed.

3. **Number of verticals over bed-form length**

When large-scale bed forms such as sand dunes \( (\lambda = 7 \text{ h}) \) are present, the error of the depth-integrated transport rate will also depend on the location of the measurement station with respect to the dune crest. To reduce the error related to this effect, measurements should be performed in at least five verticals distributed equally over the length of the bed form. Quantitative information of the error related to the suspended sediment transport is not available.
4. **Number of verticals over width of cross section**

*Equation*

The cross-section integrated suspended sediment transport rate $S$ can be approximated by:

$$S = \sum_{i=1}^{m} (s_i \Delta b_i) + \delta$$  \hspace{1cm} (13.4.3)

in which:
- $s_i$ = depth-integrated suspended sediment transport rate in subsection $i$,
- $\Delta b_i$ = width of subsection $i$,
- $m$ = number of subsections in lateral direction,
- $\delta$ = interpolation error ($\delta \to 0$ for $m \to \infty$).

The interpolation error depends on the number of verticals and also on the distribution of the verticals over the cross-section. In this analysis equal distances between the verticals have been assumed. Other methods are equal areas or equal discharges per subsection. This latter method is less practical because the lateral velocity distribution must be known (a priori). When only a few verticals are selected in a wide irregular cross-section, it is better to select representative locations (in the deepest channels where the velocities are maximum).

Assuming that the variables $s$ and $\Delta b$ are independent stochastic variables with a normal Gaussian distribution, the relative error of the suspended sediment transport ($S$) can be estimated by:
\[
\left( \frac{\sigma_s}{S} \right)^2 = \frac{1}{m} \left[ \left( \frac{\sigma_s}{s} \right)^2 + \left( \frac{\sigma_{\Delta b}}{\Delta b} \right)^2 \right] + (\delta_{rel})^2
\]  
(13.4.4)

**Relative standard deviations**

According to the results presented in Figure 13.4.1, the relative error of the depth-integrated transport rate per unit width is about \( \sigma_s/s = 0.2 \) for a scheme with 6 points over the water depth.

The relative error of the width of each lateral subsection is estimated to be about \( \sigma_{\Delta b}/\Delta b = 0.05 \).

**Interpolation error**

An estimate of the relative interpolation error \( (\delta_{rel} = \delta/S) \) was obtained by using a formula to compute the transport rate at various verticals in a wide irregular cross-section (consisting of 2 main channels with a depth of \( h = 4 \) m separated by a shallow part with a depth of 2 m, total surface width was 500 m; \( d_{50} = 400 \mu m; \bar{u} = 1 \) m/s). The total number of verticals was varied from \( m = 4 \) to 20 (equal distance between the verticals). The true transport was assumed to be obtained for \( m = 20 \). The relative interpolation errors are given in Table 13.4.

<table>
<thead>
<tr>
<th>Number of verticals</th>
<th>Relative interpolation error of cross-section integrated transport rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>( \delta_{rel} )</td>
</tr>
<tr>
<td>20</td>
<td>0 %</td>
</tr>
<tr>
<td>18</td>
<td>- 1 %</td>
</tr>
<tr>
<td>16</td>
<td>- 2 %</td>
</tr>
<tr>
<td>14</td>
<td>+ 5 %</td>
</tr>
<tr>
<td>12</td>
<td>- 5 %</td>
</tr>
<tr>
<td>10</td>
<td>+11 %</td>
</tr>
<tr>
<td>8</td>
<td>-10 %</td>
</tr>
<tr>
<td>6</td>
<td>+17 %</td>
</tr>
<tr>
<td>4</td>
<td>-20 %</td>
</tr>
</tbody>
</table>

*Table 13.4 Relative interpolation error of cross-section integrated transport rate*  
\(+ = \text{overestimation, } - = \text{underestimation}\)

**Relative error of cross-section-integrated transport rate**

The relative error of the cross-section-integrated suspended transport rate can be determined from Eq. (13.4.4) with \( \sigma_s/s = 0.2 \), \( \sigma_{\Delta b}/\Delta b = 0.05 \) and \( \delta_{rel} = 0.2 \) to 0.05 for \( m = 4 \) to 20. The results are presented in Table 13.5 and in Figure 13.4.2.

Bonacci (1981) used a numerical simulation model to determine the relative error \( \sigma_s/S \) in relation to the number of verticals. The results are presented in Figure 13.4.2. A method given by the US Geological Survey (1988) is shown in Figure 13.4.3. The variability of the sand concentration at different sampling verticals is assumed to be related to the variability of \( V^2/D \) with \( V = \) depth-mean velocity (ft/s) and \( D = \) water depth (ft). In the example of Figure 13.4.3, the acceptable relative standard error is 15%, the bed material sample is 100% sand, the ratio \( V^2/D = 2 \) resulting in a required number of verticals of seven. Table 13.5 and Figure 13.4.2 also yield a number of seven verticals for this example.
<table>
<thead>
<tr>
<th>Number of verticals</th>
<th>Relative error of cross-section integrated transport rate $\sigma_x/S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5 %</td>
</tr>
<tr>
<td>18</td>
<td>7 %</td>
</tr>
<tr>
<td>16</td>
<td>8 %</td>
</tr>
<tr>
<td>14</td>
<td>8 %</td>
</tr>
<tr>
<td>12</td>
<td>9 %</td>
</tr>
<tr>
<td>10</td>
<td>12 %</td>
</tr>
<tr>
<td>8</td>
<td>13 %</td>
</tr>
<tr>
<td>6</td>
<td>17 %</td>
</tr>
<tr>
<td>4</td>
<td>23 %</td>
</tr>
</tbody>
</table>

**Table 13.5 Relative error of cross-section integrated transport rate**

The total number of verticals to be selected should be such that the total required measuring period is relatively small with respect to the flood period of the river discharge (discharge should be approximately constant). In flash flood conditions this may give problems and hence a compromise must be made between accuracy and measuring time available.

**Figure 13.4.2 Number of verticals in cross-section**
5. Number of verticals per tide

Equation

The suspended sediment transport per unit width integrated over half a tidal period (flood tide or ebb tide) can be computed as:

\[ M = \sum_{i=1}^{n} (s_i \Delta t_i) + \gamma \] (13.4.5)

in which:
- \( M \) = tide-integrated suspended sediment transport per unit width,
- \( s_i \) = depth-integrated suspended sediment transport per unit width at time \( i \),
- \( \Delta t_i \) = duration of time period \( i \),
- \( n \) = number of time intervals over half a tidal period (ebb or flood),
- \( \gamma \) = interpolation error.

The interpolation error \( \gamma \) depends not only on the number of time steps over the flood (or ebb) period, but also on the distribution over the period. In this analysis equal time intervals are assumed.

Assuming that the variables \( s \) and \( \Delta t \) are independent stochastic variables with a normal Gaussian distribution, the relative error of the tide-integrated suspended load per unit width can be estimated by:

\[ \left( \frac{\sigma_M}{M} \right)^2 = \frac{1}{n} \left[ \frac{\sigma_s}{s} \right]^2 + \left( \frac{\sigma_{\Delta t}}{\Delta t} \right)^2 + \left( \gamma_{\text{rel}} \right)^2 \] (13.4.6)
Relative standard deviation

According to the results presented in Figure 13.4.1 the relative error of the depth-integrated suspended transport rate is about \( \sigma_s/s = 0.2 \) for a scheme of 6 points. The relative error of the time intervals is estimated to be about \( \sigma_{\Delta t}/\Delta t = 0.05 \).

Interpolation error

An estimate of the relative interpolation error \( (\gamma_{rel} = \gamma/\mathcal{M}) \) was obtained by assuming a sinusoidal tidal velocity curve \( v = \hat{v} \sin(\omega t) \) and a sediment transport formula \( s = av^3 \). The tide-integrated transport was obtained by analytical integration and by numerical integration (trapezoidal rule), giving a relative error:

\[
\gamma_{rel} = \frac{\delta \pi^2}{9n^2} \tag{13.4.7}
\]

in which \( n \) = number of intervals.

The results are presented in Table 13.6. A time interval of 30 minutes yields a 6\% underestimation of the transport.

<table>
<thead>
<tr>
<th>Time interval ( \Delta t ) (min)</th>
<th>Number of intervals during 6 hour tidal period ( n )</th>
<th>Relative interpolation error ( \gamma_{rel} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>37</td>
<td>- 0.7%</td>
</tr>
<tr>
<td>20</td>
<td>19</td>
<td>- 2.5%</td>
</tr>
<tr>
<td>30</td>
<td>12</td>
<td>- 6%</td>
</tr>
<tr>
<td>45</td>
<td>8</td>
<td>- 13%</td>
</tr>
<tr>
<td>60</td>
<td>6</td>
<td>- 24%</td>
</tr>
</tbody>
</table>

Table 13.6 Relative interpolation error of tide-integrated transport rate (- underestimation)

Relative error of the tide-integrated transport rate

The relative error of the tide-integrated (flood or ebb) suspended transport can be determined from Eq. (13.4.6) with \( \sigma_s/s = 0.2 \), \( \sigma_{\Delta t}/\Delta t = 0.05 \) and \( \gamma_{rel} = 0.01 \) to 0.25 for \( n = 37 \) to \( n = 6 \). The results are presented in Table 13.7.

<table>
<thead>
<tr>
<th>Time interval ( \Delta t ) (min)</th>
<th>Number of intervals during 6 hour tidal period ( n )</th>
<th>Relative error of tide-integrated transport ( \sigma_M/\mathcal{M} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>37</td>
<td>3%</td>
</tr>
<tr>
<td>20</td>
<td>19</td>
<td>6%</td>
</tr>
<tr>
<td>30</td>
<td>12</td>
<td>9%</td>
</tr>
<tr>
<td>45</td>
<td>8</td>
<td>15%</td>
</tr>
<tr>
<td>60</td>
<td>6</td>
<td>25%</td>
</tr>
</tbody>
</table>

Table 13.7 Relative error of tide-integrated transport rate

13.14
13.4.4 Number of measurements for bed-load transport

1. General aspects

Typical sampling problems related to the variability of the physical processes involved are (see Carey, 1985; Delft Hydraulics, 1991):
- the sampling duration of each individual measurement,
- number of measurement locations along a bed form,
- number of measurements at each location,
- number of locations over the width of the cross-section.

The sampling duration of each individual measurement is discussed in Section 13.6.2.

2. Number of locations over bed-form length and number of measurements at each location

The variability of the bed-load transport rate at one location on a bed form is so large (factor 10 to 100, see Figure 13.4.4) that the mean transport rate can only be determined accurately by taking a large number of samples. Different locations equally distributed along the bed-form length should be selected and many samples should be taken at each location. The bed-form length should be known a priori (echosoundings).

Quite often the sampling location is fixed because the sampler is operated from a bridge or a non-movable boat. In that case accurate determination of the mean transport rate requires sequential sampling over a period long enough for a (migrating) bed-form to pass the sampling location.

![Graph showing variability of bed-load transport](image)

**Figure 13.4.4 Variability of bed-load transport (Carey, 1985)**

Measurements in the Waal river in the Netherlands (Delft Hydraulics, 1991, 1992) have been used to determine the number of samples which should be taken along a bed-form, to obtain a minimum error in the mean transport rate. Table 13.8 shows the variation coefficient in relation to the number of samples.
<table>
<thead>
<tr>
<th>Number of samples</th>
<th>Variation coefficient of the mean transport rate ( vc )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.30</td>
</tr>
<tr>
<td>20</td>
<td>0.25</td>
</tr>
<tr>
<td>30</td>
<td>0.20</td>
</tr>
<tr>
<td>40</td>
<td>0.15</td>
</tr>
<tr>
<td>50</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Table 13.8 Variation coefficient of mean transport rate**

The variation coefficient is defined as:

\[
vc = \frac{\sigma}{\mu\sqrt{N}}
\]

(13.4.8)

in which:
- \( \sigma \) = standard deviation of measured transport rate (over \( N \) samples),
- \( \mu \) = mean value of measured transport rate (over \( N \) samples),
- \( N \) = number of samples.

It is recommended to select at least 5 locations distributed equally along a bed-form (dune) and to take 10 samples at each location yielding a total number of 50 samples. All measurements should be averaged to obtain the mean transport rate. The error of the mean transport will be about 10% (see Table 13.8). The mean transport rate based on these 50 samples will be within \( \pm 10\% \) of the true mean value.

At each location a velocity profile should be measured to determine the depth-averaged velocity \( (u) \), the bed-shear velocity \( (u_s) \) and the bed roughness height \( (k_s) \).

3. **Number of locations over the width of the cross-section**

The number of locations (equal distance) in the cross-section is primarily related to the interpolation error, see Figures 13.4.2 and 13.4.3.

Accepting a relative interpolation error of 15%, at least seven locations in the cross-section are required. At each location approximately 50 samples should be taken to obtain a relative error of 10% in the local transport rate (see Table 13.8). Thus, about \( 7 \times 50 = 350 \) samples are required to obtain an overall relative error of \((15^2 + 10^2)^{0.5} = 20\%\) of the cross-section-integrated bed-load transport rate.

The number of samples (including changing boat position etc.) which can be collected per day is about 50 resulting in a measuring effort of seven days to complete a cross-section. A significant reduction of this effort can be obtained at the expense of less accurate results.

For example, taking three locations along a bed-form with five samples per location and seven locations in the cross-section will result in \( 3 \times 5 \times 7 = 105 \) samples which can be collected in two days. The relative inaccuracy of the local bed-load transport rate (based on \( 3 \times 5 = 15 \) samples) will be about 30%, see Table 13.8. The overall relative error of the cross-section integrated bed-load transport rate will be about \((30^2 + 15^2)^{0.5} = 35\%\).

4. **Sampling frequency**

The frequency of sampling depends on the frequency of the characteristic hydraulic conditions, the available resources and the size of the project. Seasonal fluctuations are important. A sampling frequency of twice a year will probably be sufficient for most projects. General rules are:
1. Rivers

- during low run-off periods,
- during and after high run-off periods.

2. Estuaries

- daily variations,
- during neap, mean and springtides,
- during summer and winter period,
- during dry and wet period.

3. Coasts

- during calm periods,
- during and after storm periods.

13.5 Computation of sediment transport

13.5.1 Suspended load transport per unit width

When the suspended sediment samples are collected as point-integrated samples, there are two methods to compute the depth-integrated suspended load transport (see Figure 13.5.1). First, there is the partial method which gives the suspended load transport between the bed and the highest sampling point using a linear interpolation between adjacent (measured) values. Second, there is the integral method, which gives the total suspended load transport between the bed and the water surface by fitting a theoretical distribution to the measured flow velocity and concentration profiles. Applying this latter method, the suspended load in the unsampled zone is also estimated.

The transport rate of the suspended silt (< 62 μm) and suspended sand particles (≥ 62 μm) should be computed separately. If necessary, more fractions can be used.

1. Partial method

The suspended load transport per unit width (kg/sm) can be computed as:

\[ s_s = \left[ \frac{1}{2}(u_1c_1z_1) + \sum_{i=1}^{n-1} \frac{1}{2}(u_ic_i + u_{i+1}c_{i+1})(z_{i+1} - z_i) \right] \]  \hspace{1cm} (13.5.1) 

or

\[ s_s = \left[ \frac{1}{2} q_1z_1 + \sum_{i=1}^{n-1} \frac{1}{2} (q_i + q_{i+1})(z_{i+1} - z_i) \right] \]  \hspace{1cm} (13.5.2) 

in which:

- \( n \) = number of measuring points in the vertical,
- \( u_i \) = flow velocity in point \( i \) above the bed (m/s),
- \( c_i \) = concentration in point \( i \) above the bed (kg/m\(^3\)),
- \( z_i \) = height above bed of point \( i \) (m),
- \( q_i \) = suspended sediment transport (direct method) in point \( i \) above the bed (kg/sm\(^3\)).
The first term in Eqs. (13.5.1) and (13.5.2) is an estimate for the transport between the lowest sampling point and the bed.

The depth-averaged flow velocity (m/s) is:

\[
\bar{u} = \frac{1}{z_n} \left[ \frac{1}{2}(u_1z_1) + \sum_{i=1}^{n-1} \frac{1}{2}(u_i + u_{i+1})(z_{i+1} - z_i) \right]
\]  

(13.5.3)

in which:

\(z_n\) = height above bed of highest measuring point (m).

**Figure 13.5.1** Partial and integral methods
2. **Integral method**

**Velocity**

The velocities between the bed and the first measuring point \( z_i \) can be represented by (see Figure 13.5.2):

\[
v = v_1 \left( \frac{z}{z_1} \right)^{0.25} \quad \text{for } 0 < z < z_1
\]  

(13.5.4)

in which:

\( v_1 \) = fluid velocity in first measuring point above the bed,

\( z_1 \) = height above bed of first measuring point.

The velocities between the highest measuring point \( z_L \) and the water surface can be taken equal to the velocity \( v_L \) in the highest measuring point, (see Figure 13.5.2). Thus:

\[
v = v_L \quad \text{for } z_L < z < h
\]  

(13.5.5)

**Concentration**

The sediment concentrations between the highest measuring point and the water surface can be represented by a linear function giving a zero concentration at the surface, as follows (see Figure 13.5.2):

\[
c = \left( \frac{h-z}{h-z_L} \right) c_L \quad \text{for } z_L < z < h
\]  

(13.5.6)

in which:

\( c_L \) = concentration in highest measuring point,

\( z_L \) = height above bed of highest measuring point.

Since the exact distribution of the sediment concentrations in the near-bed zone is not known and considering the relative importance of the concentrations in this zone, three different extrapolation methods can be applied to represent the concentration profile between the bed and the first measuring point. Method 1 is supposed to give an under limit, while method 2 is supposed to give an upper limit.

**method 1**

The sediment concentrations between the bed and the first measuring point \( z - z_i \) are assumed to be equal to the concentration \( c_i \) in the first measuring point (see Figure 13.5.2):

\[
c = c_i \quad \text{for } 0 < z < z_1
\]  

(13.5.7)
Figure 13.5.2 Extrapolation of velocity and concentration profile, method 1

Figure 13.5.3 Extrapolation of concentration profile, method 2

Figure 13.5.4 Extrapolation of concentration profile, method 3
method 2

The sediment concentrations between the bed and the first measuring point are computed by (see Figure 13.5.3):

\[ c = A \ Y^B \]  \hspace{1cm} (13.5.8)

in which:
- \( Y = (h-z)/z \) = dimensionless vertical coordinate,
- \( z \) = vertical coordinate above bed,
- \( h \) = water depth,
- \( A, B \) = coefficients.

The \( A \) and \( B \) coefficients are determined by a regression method applying the measured concentrations of the first three measuring points above the bed, as follows:

- select \( B = 0.1 \),
- compute \( A = \sum_{l=1}^{3} \left( Y_k^B C_k \right) / \sum_{l=1}^{3} \left( Y_k^B Y_k^B \right) \),
- compute \( T = \sum_{l=1}^{3} \left( A \ Y_k^B - c_k \right) \),  \hspace{1cm} (13.5.9)
- select \( B = 0.2 \) (\( B \) is varied over the range 0.1 to 5),
- repeat procedure.

Finally, the \( A \) and \( B \) coefficients corresponding to a minimum \( T \)-value are selected as the "best" coefficients. Applying Eq. (13.5.8), the sediment concentrations are computed in 50 (equidistant) points between the bed (defined at \( z = 2d_{50} \)) and the first measuring point \( (z = z_i) \). The maximum concentration is assumed to be 1600 kg/m³.

method 3

The sediment concentrations between the bed and the first measuring point are represented by (Fig. 13.5.4):

\[ c = e^{Az} + B \]  \hspace{1cm} (13.5.10)

in which
- \( z \) = height above bed
- \( A, B \) = coefficients.

The \( A \) and \( B \) coefficients are determined by a linear regression method applying the measured concentrations of the first three measuring points above the bed, as follows:

\[ A = \frac{3 \sum_{l=1}^{3} \left( z_k \ln c_k \right) - \sum_{l=1}^{3} \left( z_k \right) \sum_{l=1}^{3} \left( \ln c_k \right)}{3 \sum_{l=1}^{3} \left( z_k z_k \right) - \left( \sum_{l=1}^{3} z_k \right)^2} \]  \hspace{1cm} (13.5.11)
\[
B = \frac{\sum_{i=1}^{3} (z_k z_k) \sum_{i=1}^{3} (\text{inc}_k) - \sum_{i=1}^{3} (z_k) \sum_{i=1}^{3} (\text{inc}_k)}{3 \sum_{i=1}^{3} (z_k z_k) - \left( \sum_{i=1}^{3} z_k \right)^2} \tag{13.5.12}
\]

Applying (13.5.10) the sediment concentrations are computed in 50 (equidistant) points between the bed (defined at \( z = 2d_{so} \)) and the first measuring point (\( z = z_1 \)). The maximum concentration is assumed to be 1600 kg/m³.

**Suspended sediment transport**

Numerical computation of the depth integrated suspended sediment transport \((s_s)\) requires the specification of velocities and concentrations at equal elevations above the bed (at equal \( z \)-values). When the \( z \)-values of the velocities and concentrations are not corresponding, linear interpolation should be applied to obtain the required data.

The depth-integrated suspended sediment transport \((s_s)\) is computed as:

\[
s_s = \sum_{i=1}^{N} \frac{1}{2} (v_i c_i + v_{i-1} c_{i-1}) (z_i - z_{i-1}) \tag{13.5.13}
\]

in which:
- \( v_i \) = fluid velocity at height \( z_i \) above the bed,
- \( c_i \) = sediment concentration at height \( z_i \) above the bed,
- \( N \) = total number of points (including extrapolated and interpolated values).

Since three different methods are applied to represent the sediment concentrations in the unmeasured zone near the bed, three different values of the suspended sediment transport are obtained. These values can be averaged to obtain a reliable estimate of the depth-integrated suspended load transport rate.

Finally, it is noted that the values based on extrapolation into the unmeasured zone may be unreliable, especially for relatively large particle sizes. In that case the sediment transport below the lowest measuring point is large compared with that above the lowest measuring point and the extrapolated results will be of limited value only.

**13.5.2 Total load transport per unit width**

The total load transport per unit width is obtained by summation of the bed-load and suspended load transport per unit width.

**13.5.3 Total load transport in cross-section**

The total load transport \((\text{kg/s})\) in a cross-section can be computed as (see Figure 13.5.5):

\[
S = \sum_{k=1}^{n} (s_{b,k} + s_{s,k}) b_k \tag{13.5.14}
\]
in which:
\( s_{b,k} \) = bed-load transport per unit width in subsection \( k \) (kg/sm),
\( s_{s,k} \) = suspended load transport per unit width (kg/sm) in subsection \( k \) according to partial or integral method,
\( b_{k} \) = width of subsection \( k \) (m),
\( n_{k} \) = number of subsections in lateral direction.

The flow discharge (m\(^3\)/s) in the cross-section can be computed as:

\[
Q = \sum_{l}^{n_{k}} \overline{u}_{k} h_{k} b_{k} \tag{13.5.15}
\]

in which:
\( \overline{u}_{k} \) = depth-averaged flow velocity (m/s) in section \( k \) according to partial or integral method,
\( h_{k} \) = mean depth in section \( k \) (m).

The cross-section-averaged flow velocity (m/s) is:

\[
\overline{u} = \frac{Q}{\sum_{l}^{n_{k}} h_{k} b_{k}} \tag{13.5.16}
\]

The discharge-weighted concentration (kg/m\(^3\)) is:

\[
\overline{c} = \frac{S}{Q} \tag{13.5.17}
\]

**Figure 13.5.5 Total load transport in cross-section**
13.5.4 Tide-integrated total load

In case of tidal flow conditions the sediment transport is a function of time. This implies simultaneous measurement of velocity and concentration profiles as a function of time. The time period needed to complete the measurement of each velocity and concentration profile should be as short as possible (< 30 min).

The total amount of water (\(V\)) and sediment (\(M\)) passing a cross-section are (see also Figure 13.5.6):

\[
\begin{align*}
V_{\text{flood}} &= \sum_{i}^{nf} 1/2 Q_{i} (t_{i} - t_{o}) + 1/2(Q_{i+1} + Q_{i})(t_{i+1} - t_{i}) + 1/2 Q_{n} (t_{M} - t_{n}) \quad (13.5.18) \\
V_{\text{ebb}} &= \sum_{i}^{ne} 1/2 Q_{i} (t_{1} - t_{M}) + 1/2(Q_{i+1} + Q_{i})(t_{i+1} - t_{i}) + 1/2 Q_{n} (t_{L} - t_{n}) \quad (13.5.19) \\
V_{R} &= V_{\text{flood}} - V_{\text{ebb}} \quad (13.5.20)
\end{align*}
\]

in which:
- \(V_{R}\) = resulting water volume (m³)
- \(Q_{i}\) = discharge of water (m³/s) passing a cross-section at time \(t\).

\[
\begin{align*}
M_{\text{flood}} &= \sum_{i}^{nf} 1/2 S_{1} (t_{1} - t_{o}) + 1/2(S_{i+1} + S_{i})(t_{i+1} - t_{i}) + 1/2 S_{n} (t_{M} - t_{n}) \quad (13.5.21) \\
M_{\text{ebb}} &= \sum_{i}^{ne} 1/2 S_{1} (t_{1} - t_{M}) + 1/2(S_{i+1} + S_{i})(t_{i+1} - t_{i}) + 1/2 S_{n} (t_{L} - t_{n}) \quad (13.5.22) \\
M_{R} &= M_{\text{flood}} - M_{\text{ebb}} \quad (13.5.23)
\end{align*}
\]
in which:

\( M_r \) = resulting sediment volume (kg),

\( S_t \) = discharge of sediment (kg/s) passing a cross-section at time \( t \), Eq. (13.5.14),

\( n_f \) = number of measurements during flood,

\( n_e \) = number of measurements during ebb.

13.6 Instruments for bed-load transport

13.6.1 Introduction

Two methods are available to measure the bed-load transport rate. Simple mechanical trap-type samplers, that collect the sediment particles transported close to the bed (trap sampling), can be used.

Another possibility is the recording (echo-sounding) of longitudinal bed profiles as a function of time (bed-form tracking).

13.6.2 Trap sampling

1. General aspects

The basic principle of mechanical trap-type bed-load samplers is the interception of the sediment particles which are in transport close to the bed over a small incremental width of the channel bed. Most of the particles close to the bed are transported as bed-load but the sampler will inherently collect a small part of the suspended load (related to vertical size of intake mouth).

The bed-load transport measured by a mechanical sampler is dependent on its efficiency (instrumental errors), on its location with respect to the bed-form geometry (spatial variability) and on the near-bed turbulence structure (temporal variability).

The efficiency of the bed-load sampler depends on the hydraulic coefficient, the percentage of width of the sampler nozzle in contact with the bed during sampling and on sampling disturbances generated at the beginning and the end of the sampling period.

The hydraulic coefficient, defined as the ratio of the inflow velocity and the ambient flow velocity, depends on the geometry and construction of the sampler nozzle, on the position of the nozzle at the bed, on the percentage of filling of the bag with sand particles (volume of catch) and on the percentage of blocking of the bag material by fine sand, silt or clay particles and organic materials depending on the mesh size of the bag (Beschta, 1981).

Laboratory experiments have shown (Emmett, 1980; Delft Hydraulics, 1991) that the sampler (bag or basket) can be filled to about 40% of its capacity without reduction of the hydraulic efficiency and loss of sediment particles during raising of the instrument.

Thus, the problems of bed-load transport measurements are related to the instrumental design and to the physical processes involved.

Typical instrumental problems of a (bag-type) bed-load sampler are:

- the initial effect: sand particles of the bed may be stirred up and trapped when the instrument is placed on the bed (oversampling),
- the gap effect: a gap between the bed and the sampler mouth may be present initially or generated at a later stage under the mouth of the sampler due to migrating ripples or erosion processes (undersampling),
- the blocking effect: blocking of the bag material by sand, silt, clay particles and organic materials will reduce the hydraulic coefficient and thus the sampling efficiency (undersampling),
the scooping effect; the instrument may drift downstream from the survey boat during lowering to the bed and it may be pulled forward (scoop) over the bed when it is raised again so that it acts as grab sampler (oversampling).

Typical sampling problems related to the variability of the physical processes of bed-load transport are (see Carey, 1985; Delft Hydraulics, 1991):
- the number of measurement locations along a bed-form (see Section 13.4.4),
- the number of measurements at each location (see Section 13.4.4),
- the sampling duration of each measurement (see below),
- the number of locations in the cross-section (see Section 13.4.4).

**Sampling duration**

Three effects related to the sampling duration are considered:
- the volume of the nylon bag (maximum sampling period),
- the blocking of the nylon material by sediment particles,
- the presence or generation of a gap under the sampler mouth.

The size of the bag imposes a maximum sampling period which depends on the transport rate and hence on the current velocity. Taking a maximum filling percentage of 50%, the maximum sand catch of a 2 liter bag is about 2000 grams. The bed-load transport formula of Meyer-Peter-Müller has been used to compute the maximum sampling period for bed material of 500 µm and velocities in the range of 0.75 to 2 m/s. The maximum transport rate is assumed to be equal to three times the mean rate \( q_{b,\text{max}} = 3 q_{b,\text{mean}} \). The results are given in the Table 13.9.

<table>
<thead>
<tr>
<th>Depth averaged velocity (m/s)</th>
<th>Maximum sampling period (scc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>450</td>
</tr>
<tr>
<td>1.0</td>
<td>120</td>
</tr>
<tr>
<td>1.25</td>
<td>50</td>
</tr>
<tr>
<td>1.5</td>
<td>30</td>
</tr>
<tr>
<td>1.75</td>
<td>15</td>
</tr>
<tr>
<td>2.0</td>
<td>10</td>
</tr>
</tbody>
</table>

**Table 13.9 Maximum sampling period**

Blocking of standard nylon bags by fine sediment was studied by Beschta (1981) using the Helley-Smith bed-load sampler in flume and field conditions. The 200 µm-bag was found to be rapidly clogged by fine sediment particles reducing the sampling efficiency to 50% after 30 seconds and to only 10% after 300 seconds. Thus, the sampling period must be rather short (< 30 seconds) whenever a standard bag is used.

A special bag consisting of nylon material with a mesh size of 250 µm and a patch (0.10 x 0.15 m²) of 500 µm at the upper side of the bag has been used by Van Rijn and Gaweesh (1992). The application of the special bag yields good results as regards the hydraulic coefficient.

Camera observations (Delft Hydraulics, 1991, 1992) did show the presence of an initial gap or the generation of a gap at a later stage (after 2 or 3 minutes) during some of the sampling periods. Given the stochastic nature of the generation of gaps (migrating ripples), its effect on the average transport rate will be the least when many samples of short duration are taken (gap effect is averaged out). Furthermore, the application of a short sampling period reduces the time available for the generation of a gap when initially no gap is present.

13.26
Based on the above-given considerations, it is advised to use a maximum sampling period of 3 minutes for depth-averaged velocities up to 0.8 m/s. At higher velocities (higher transport rates) the maximum sampling period must be reduced (between 0.5 and 3 minutes) or a larger bag should be used. The largest sand catch should not be larger than 50% of the bag volume.

2. Bed-load Transport Meter Arnhem (BTMA)

The instrument is based on the collection of sediment particles by means of a basket type sampler. The basket consisting of fine wire mesh and mounted in a frame, is pressed (by means of a spring) on the channel bed after lowering of the frame (Figure 13.6.1). The form of the basket causes a pressure reduction behind the instrument so that the water and sediment particles enter the mouth of the basket with the same velocity as that of the ambient flow, provided that the sediment content already in the basket is relatively small. The sampler can collect particles coarser than 0.3 mm (mesh size of basket) but finer than 50 mm (opening height).

A. BED LOAD SAMPLER

![Diagram of Bed-load Transport Meter Arnhem (BTMA)]

C. CALIBRATION CURVES

Figure 13.6.1 Bed-load Transport Meter Arnhem (BTMA)

The bed-load transport (in kg/s/m) can be determined as:

\[ s_b = \frac{\alpha (1-p) \rho_s V_s}{b T}, \quad \text{or} \quad s_b = \frac{\alpha G_s}{b T} \]  

(13.6.1)

in which:
- \( \alpha \) = calibration factor (= 2 for BTMA)
- \( p \) = porosity factor (≈ 0.4)
- \( \rho_s \) = density of sediment particles (≈ 2650 kg/m³)
- \( V_s \) = immersed volume of sediment catch (m³)
- \( G_s \) = dry mass of sediment catch (kg)
- \( b \) = width of intake opening (= 0.085 m for BMTA)
- \( T \) = sampling period (s)
The accuracy of the measured bed-load transport is strongly dependent on the accuracy of the calibration factor $\alpha$, the number of measurements and the sampling procedure. Figure 13.6.1 presents calibration curves for the BTMA showing considerable variations. In practice, a calibration factor equal to 2 is used.

The technical specifications are:

- Intake opening
  - width 0.085 m
  - height 0.050 m
- dimensions
  - frame 0.40 x 0.8 x 1.85 m
  - basket 0.10 x 0.15 x 0.50 m
- Mesh size of basket = 300 $\mu$m (= 0.3 mm)
- Weight of sampler = 32 kg

3. Helley Smith Sampler (HSS)

The Helley-Smith sampler consists of a nozzle, sample bag and frame (Figure 13.6.2), (Helley and Smith, 1971). The sample has a square entrance nozzle (0.076 x 0.076 m$^2$) and a sample bag constructed of 250 $\mu$m-mesh polyester. Several different versions of the sampler have been used for various field conditions. Larger nozzles are generally used to sample larger sediment sizes and heavier samplers become necessary as deeper and faster rivers are sampled. An important advantage of the Helley-Smith sampler is the extensive calibration (based on about 10,000 samples) and its simple operation. The Helley-Smith sampler is the most widely-used bed-load transport measuring instrument.

The bed-load transport (in kg/sm) can be determined as:

$$s_b = \frac{\alpha (1-p) \rho_s V_s}{b T}, \quad \text{or} \quad s_b = \frac{\alpha G_s}{b T} \quad (13.6.2)$$

in which:

- $\alpha$ = calibration factor
  - = 0.5 for particles of 0.25 to 0.50 mm
  - = 1.0 for particles of 0.50 to 16 mm
  - = 1.5 for particles of 16 to 32 mm
- $p$ = porosity factor (~0.4)
- $\rho_s$ = density of sediment particles (~2650 kg/m$^3$)
- $V_s$ = immersed volume of sediment catch (m$^3$)
- $G_s$ = dry mass of sediment catch (kg)
- $b$ = width of intake opening (= 0.0762 m, in m)
- $T$ = sampling period (s)

The technical specifications are:

- Intake opening
  - width 0.0762 m
  - height 0.0762 m
- dimensions
  - frame 0.18 x 0.32 x 1.0 m
- Mesh size of bag = 250 $\mu$m
- Weight of sampler = 30 kg
Figure 13.6.2 Helley Smith bed-load sampler (HSS)

4. Delft Nile Sampler (DNS)

The instrument consists of a bed-load sampler (and a suspended-load sampler) attached to a supporting frame (see Figure 13.6.3). The sampler has a weight of about 60 kg. The suspended-load sampler consists of 7 intake nozzles (inner diameter = 0.003 m) which are connected to plastic hoses and operated by pumps.
Figure 13.6.3 Delft Nile Sampler (DNS)

The bed-load sampler consists of a nozzle (entrance width = 0.096 m, entrance height = 0.055 m, length = 0.085 m, rear width = 0.105 m, rear height = 0.06 m) connected to a bag. The bag consists of nylon material with a mesh size of 150 or 250 μm, depending on the size of bed material. At the upper side of the bag a patch (0.10 x 0.15 m²) consisting of
500 \( \mu \text{m} \) material is present to reduce the blocking effect by fine particles as much as possible (see Figure 13.6.3). The bottom side of the sampler nozzle has a forward-tilting slope of 1 to 10. The bed-load sampler is connected to a swing arm, which can move in the vertical direction (upward and downward) over a distance of about 0.15 m with respect to the supporting frame. In its hanging position during lowering of the instrument, the bed-load sampler is in its most upward position. When the frame of the instrument is placed on the bed, the sampler nozzle makes contact with the bed by relaxing the tension in the cable. When the instrument is raised, first the nozzle of the bed-load sampler is raised over 0.15 m and then the frame is lifted from the bed. The swing-arm construction was designed to reduce the gap effect and the scooping effect as much as possible. The instrument was developed for bed-load measurements in the Nile river.

The hydraulic coefficient of the bed-load sampler, defined as the ratio of the inflow velocity and the ambient flow velocity, was determined by measurements in a flume (see Table 13.10). A small Ott-propeller meter (diameter = 0.02 m) was placed in the entrance of the nozzle. The velocity in the nozzle was compared with the flow velocity measured simultaneously at the same height in a section two metres upstream of the sampler. Ten successive measurements of 30 seconds each were carried out. Three types of bags were used, as follows:
- a 250 \( \mu \text{m} \) - nylon bag and a 500 \( \mu \text{m} \) - nylon patch (0.1 x 0.15 m\(^2\)) at the upper side of the bag (see Figure 13.6.3),
- a 150 \( \mu \text{m} \) - nylon bag and a 500 \( \mu \text{m} \) - nylon patch (0.1 x 0.15 m\(^2\)) at the upper side of the bag,
- an impermeable plastic bag and a 500 \( \mu \text{m} \) - nylon patch (0.1 x 0.15 m\(^2\)) at the upper side of the bag.

This latter bag is supposed to simulate a nylon bag which is almost fully blocked by fine silt and clay material as present in most natural conditions (Beschta, 1981). Four filling percentages of the bag were tested: 0%, 25%, 50% and 75%.

<table>
<thead>
<tr>
<th>Velocity in mouth (m/s)</th>
<th>Mesh size bag (( \mu \text{m} ))</th>
<th>Mesh size patch (( \mu \text{m} ))</th>
<th>Hydraulic coefficient filling percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>250</td>
<td>500</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>500</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>0 (impermeable)</td>
<td>500</td>
<td>0.82</td>
</tr>
<tr>
<td>0.8</td>
<td>250</td>
<td>500</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>500</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>0 (impermeable)</td>
<td>500</td>
<td>0.84</td>
</tr>
</tbody>
</table>

*Table 13.10 Hydraulic coefficients (average values)*

The hydraulic coefficients of the 250 \( \mu \text{m} \)-bag and the 150 \( \mu \text{m} \)-bag, both with a patch of 500 \( \mu \text{m} \) at the upper side, are about unity for filling percentages in the range of 0% to 50%. A filling percentage of 75% reduces the hydraulic coefficient to about 0.75. The hydraulic
coefficient of an impermeable plastic bag with a patch of 500 μm at the upper side, simulating a blocked nylon bag, is about 0.8 for a filling percentage in the range of 0% to 50%. A filling percentage of 70% reduces the hydraulic coefficient to about 0.70. Based on these results, a maximum filling percentage of 50% is advised to be used. Blocking of the nylon material by fine sediments will result in a hydraulic coefficient of about 0.8.

Finally, it is noted that the importance of a hydraulic coefficient equal to unity should not be overemphasized, because actually the sampling efficiency is the most important parameter. A hydraulic coefficient of unity does not necessarily give a sampling efficiency of unity, because other factors are involved.

The sampling efficiency is defined as the ratio of the bed-load transport measured by the sampler at a certain location during a certain period and the true bed-load transport at the same location during the same period (if the sampler had not been there). The sampling efficiency expresses sampling errors related to the initial and scooping effect, the gap effect and the loss of sediment particles through the patch of the bag.

The bed-load transport (in kg/sm) is determined as:

\[ s_b = \frac{\alpha (G_s - G_o)}{bT} \]  

(13.6.3)

in which:

- \( G_s \) = dry mass of sediment catch (kg)
- \( G_o \) = dry mass of sediment catch related to initial and scooping effect determined by 'zero'-sampling (sampler is lowered to bed and immediately raised) (kg)
- \( \alpha \) = calibration factor (1 to 1.5) (-)
- \( b \) = width of intake opening (m)
- \( T \) = sampling period (s)

The \( G_o \)-values should be determined by taking 10 zero-samplings at each location. Practical experience sofar shows a value of about 0.03 kg.

The sampling efficiency (\( \alpha \)-factor) was determined by means of tests in a flume at the Hydraulics and Sediment Research Institute in Deltabarrage, Egypt. Different velocity and sand size ranges were considered. The \( \alpha \)-factor was found to be in the range of 1 to 1.5.

The technical specifications are:

- Intake opening = width 0.096 m
  height 0.055 m
- dimension frame = 0.5 x 0.55 x 1.1 m
- mesh size bag = 150 and 250 μm
- weight of sampler = 60 kg

13.6.3 Bed-form tracking

The basic principle is the computation of the bed-load transport from bed-form profiles measured at successive time intervals under similar flow conditions (Figure 13.6.4).

Assuming steady flow conditions and undisturbed bed-form migration, the bed-load transport rate can be computed as (Engel and Lau, 1980, 1981):

\[ s_b = \alpha (1-p) \rho_s \Delta \]  

(13.6.4)
in which:

\( \text{s}_b = \) bed-load transport (kg/sm)
\( \omega_s = \) shape factor (0.5 - 0.6)
\( \text{p} = \) porosity factor (≈ 0.4)
\( \rho_s = \) density of sediment (≈ 2650 kg/m³)
\( \text{a} = \) average migration velocity (m/s)
\( \Delta = \) average bed-form height (m)

To apply this equation, the migration velocity and the bed-form height must be determined from the bed profiles. The bed-load transport rate can also be computed directly from the (successive) profile data using all data instead of selecting the characteristic parameters such as the average migration velocity and the bed-form height (see Havinga, 1982).

To collect the bed profile data along a prefixed course, an accurate three-dimensional positioning system must be available consisting of a two-dimensional horizontal positioning system and a one-dimensional vertical sounding system.

In (isolated) field conditions, where an accurate positioning system is too complicated, a much simpler method can be used. By means of an analogue echo sounder two or more successive bed-profile registrations can be made in a longitudinal section between two well-defined cross-sections (bank markers). Using a simple hand method, the average migration velocity and bed-form height can be determined quite easily, as shown in Figure 13.6.4.

![Figure 13.6.4 Bed-form profiles in IJssel river, The Netherlands (Havinga, 1982)](image)

13.7 Instruments for suspended load transport

13.7.1 Introduction

A wide range of instruments is available from simple mechanical samplers to sophisticated optical, acoustical and nuclear samplers. Most samplers are used as point-integrating samplers which means that the relevant parameters are measured in a specific point above the bed as
a function of time. Some instruments are used as depth-integrating samplers, which means continuous sampling over the depth by lowering and raising the instrument at a constant transit rate. The direct and the indirect measurement methods are explained in Section 13.3.1.

The trap and bottle-type instruments can only be used in (quasi) steady flow conditions. The other samplers can also be used in oscillatory flow conditions.

Table 13.11 shows a classification of instruments according to measuring principles.

<table>
<thead>
<tr>
<th>Suspended load samplers</th>
<th>Point-integrating</th>
<th>Depth-integrating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct method</td>
<td>Delft-Bottle sampler</td>
<td>USD-49 sampler</td>
</tr>
<tr>
<td></td>
<td>Acoustic sampler</td>
<td>Collapsible-Bag sampler</td>
</tr>
<tr>
<td>Indirect method</td>
<td>Trap sampler</td>
<td>USD-49 sampler</td>
</tr>
<tr>
<td></td>
<td>Bottle sampler</td>
<td>Collapsible-Bag sampler</td>
</tr>
<tr>
<td></td>
<td>USP-61 sampler</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pump sampler</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Optical sampler</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Impact sampler</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nuclear sampler</td>
<td></td>
</tr>
</tbody>
</table>

Table 13.11 Classification of suspended load samplers

13.7.2 Bottle and Trap Samplers

1. General aspects

The basic principle of all mechanical bottle and trap samplers is the collection of a water-sediment sample to determine the local sediment concentration, transport and/or particle size. Optimal sampling of a water-sediment volume by means of a mechanical instrument requires an intake velocity equal to the local flow velocity (iso-kinetic sampling) or a hydraulic coefficient, defined as the ratio of the intake velocity and local flow velocity, equal to unity (hc = 1).

Differences between the intake velocity and local flow velocity result in sampling errors. Sampling at a lower velocity than that of the ambient flow would result in a higher sediment concentration than present in the flow due to diverging flow lines which cannot be followed by the sediment particles, being of higher density than the water particles (Figure 13.7.1A). Conversely, sampling at a higher velocity than that of the ambient flow would result in a lower sediment concentration (Figure 13.7.1B). The magnitude of error due to incorrect intake velocities in suspensions of various concentrations has been determined experimentally by Nelson and Benedict (1950), whose results are shown in Figure 13.7.2. The results show a decreasing effect with a decreasing particle size. It is also clear that a hydraulic coefficient smaller than 1 results in a relatively larger error. Nelson and Benedict found no significant change of the sampling error in relation to the flow velocity (range 0.9 to 1.5 m/s), the size of the intake nozzle (range of 4 to 7 mm), the angle between flow direction and nozzle axis (range 0° to 20°), for a hydraulic coefficient in the range of 0.5 to 2 in all tests.

Crickmore and Aked (1975) report that no systematic trends can be distinguished for a hydraulic coefficient in the range 0.5 to 4 and particle sizes from 60 µm to 250 µm. The maximum experimental error found was about 10%.
Laboratory tests using an intake nozzle normal to the flow direction show errors of about 15% for particle sizes of 150 μm and 450 μm, while for 60 μm-sediment no errors were found (Nelson and Benedict, 1950). Crickmore and Aked (1975) found a maximum error of 20% for a misalignment of 180° using a hydraulic coefficient larger than 1.

Figure 13.7.1 Influence of intake velocity on sediment paths
2. Bottle Sampler

The method is based on the filling of a bottle to determine the silt and/or sand concentration at a specific point in the flow.

Usually, the bottle is placed vertically in a container (Figure 13.7.3) and lowered to the sampling point, where the bottle is opened (mechanical or electrical). A cork ball should be present to close the bottle after filling. The exact filling time is unknown but may vary from about 20 to 400 sec., depending on the bottle orientation, flow velocity and sampling height (Delft Hydraulics, 1980).
Rapid profile measurement can be achieved by using a rack with five bottles (or more), which are opened at prefixed depths.

The silt and sand concentrations are determined as:

\[ c = \frac{G_s}{V} \]  \hspace{1cm} (13.7.1)

in which:
\( G_s \) = dry mass of sediment (kg)
\( V \) = volume of water sample (m³)

![Figure 13.7.3 Bottle sampler](image)

Measurements in a laboratory flume (Delft Hydraulics, 1980) have shown that the efficiency of a bottle in collecting the sand particles (> 50 μm) is strongly dependent on the orientation of the bottle opening to the main flow direction. The optimum angle (between vertical axis and bottle axis) was found to be about 35°. For \( \alpha < 35° \) the measured concentration is too small, while for \( \alpha > 35° \) the measured concentration is too large.

Using a bottle in a vertical position, the inaccuracy of the measured sand concentration may be as large as 50%. In field conditions the inaccuracy of the measured sand concentration may even be larger when the filling time is small compared with the characteristic time scale of the fluctuating concentrations.

The inaccuracy of the measured silt concentrations may be about 10%.

**Advantages**

1. simple and reliable,
2. for silt and sand particles,
3. usable in rough weather conditions,
4. no electricity.
Disadvantages

1. many bottles for laboratory analysis,
2. short sampling periods (statistically unreliable),
3. small sediment catch (size analysis),
4. not for sampling close to the bed,
5. inaccurate sampling of sand particles.

3. Instantaneous trap sampler

The instantaneous trap sampler consists of a horizontal cylinder equipped with end valves which can be closed suddenly (by a messenger system) to trap a sample instantaneously, as shown in Figure 13.7.4. The water is allowed to flow through the open horizontal cylinder while the sampler is lowered to the desired point. The silt and sand concentration are determined as

\[ c = \frac{G_s}{V} \]  

(13.7.2)

in which:

- \( G_s \) = dry mass of sediment particles (kg)
- \( V \) = volume of water sample (m³)

As the trap sampler yields an instantaneously measured concentration, many samples are necessary to obtain a statistically reliable average value.

Advantages

1. simple and reliable,
2. for silt and sand particles,
3. usable in rough weather conditions.

Disadvantages

1. many bottles for laboratory analysis,
2. short sampling period,
3. small sediment catch (size analysis).

Figure 13.7.4 Instantaneous Trap Sampler
4. **Time-integrating trap sampler**

This trap sampler (used in lakes and oceans) consists of a cylinder-shaped or tunnel-shaped box with a closed bottom and placed vertically in the water column, see Figure 13.7.5. These traps are used to determine:

- the deposition rate at a certain elevation,
- the sediment concentration at a certain elevation.

In still water the trapping efficiency is unity. In flowing water the trapping efficiency depends on many factors: turbulence generated inside trap, trap geometry, concentration and fall velocity of particles. Cylinders were found to give the most accurate results. The trapping efficiency was found to be 0.8 to 1.2 for mud suspensions in current velocities of 0.05 to 0.1 m/s (Bloesch and Burns, 1980; Gardner, 1980). The ratio of the height and diameter of the trap should be in the range of 3 to 5. Funnel-shaped traps were found to give an understimation of the deposition rate in flowing water. Traps with a small mouth and a wide body (bottle) were found to give a large overestimation of the deposition rate.

Antsyferov et al. (1990) used tripods and poles equipped with traps to measure the sediment concentrations in coastal conditions (surf zone). The traps consisted of cylinder-shaped boxes (height = 0.1 m, diameter = 0.075 m) with six openings of 7.5 mm at the upper part of the boxes, see Figure 13.7.5. The traps were set into operation (by divers) by aligning corresponding openings in the trap body and in the cover.

The concentration of size fraction i at height z is given as:

$$c_i(z) = \frac{M_i}{k_e F \Delta T |U|}$$  \hspace{1cm} (13.7.3)

where:

- $M_i$ = sediment mass of fraction i in trap (kg)
- $F$ = area of intake openings projected normal to wave direction
- $\Delta T$ = sampling period
- $|U|$ = time-averaged value of the absolute horizontal orbital velocity at height $z$
- $k_e$ = trapping coefficient.

The $k_e$-value was obtained from field and laboratory calibrations. Concentrations were measured by means of a pump sampler close to the sand traps. Velocity measurements were also performed close to the trap (0.5 to 0.8 m/s).

Antsyferov et al. (1990) found:

- $k_e = 0.26$ for field conditions,
- $k_e = 0.21$ for laboratory conditions.

The $k_e$-values are only valid for steady wave conditions (during period $\Delta T$). In storm conditions with growth, stabilization and decay of the waves the $k_e$-value is different. It was found that 70% of the sediment mass was trapped during the growth and decay phase of the waves (inside breaker zone).

Katori (1983) and Kraus (1987) used portable streamer traps to measure transport rates in the surf zone, see Figure 13.7.6. The traps consist of long rectangular bags of polyester sieve cloth material (100 $\mu$m), vertically mounted on a stainless steel rack (Kraus, 1987). An operator standing downstream attends the trap during a sampling interval of 10 min. The use of these traps is restricted to shallow water (1 m) with wave heights less than about 0.5 m.
Figure 13.7.5 Time-integrating trap samplers for concentration and deposition rate
Figure 13.7.6 Time-integrating trap samplers for transport rate
Katori (1983) used a similar trap to measure cross-shore transport at the bed. The trap was mounted on a rubber mat resting on the bed (to prevent scour), see Figure 13.7.6.

Advantages of the streamer traps are:

- absolute measurement of transport rate.
- vertical distribution can be measured,
- simultaneous deployment at many locations.
- simple, robust and cheap.

Disadvantages are:

- disturbance of flow fields,
- scour around the traps,
- many operators involved,
- analysis of many samples,
- restricted to shallow water.

5. USP-61 Point-Integrating Sampler

The sampler consists of a streamlined bronze casting (≈ 50 kg), which encloses a small bottle (∼ 500 ml), as shown in Figure 13.7.7. The sampler head is hinged to provide access to the bottle. The intake nozzle, which can be opened or closed by means of an electrically operated valve, points directly into the approaching flow. To eliminate a sudden inrush after opening of the intake nozzle, the air pressure in the bottle is balanced with the hydrostatic pressure prior to opening of the valve. This is accomplished by means of an air bell in a body cavity connecting the bottle and the surrounding stream. After opening of the valve, the air in the bottle can escape through a special air-exhaust tube pointing downstream on the side of the sampler head. As a result the hydraulic coefficient is nearly unity during sampling. The filling time varies from 10 to 50 seconds, as shown in Figure 13.7.7. To avoid a circulation flow, the bottle should only be filled for about 75%. The USP-63 is a heavier version (∼ 100 kg) of the USP-61.

The silt and sand concentration can be determined as:

\[
c = \frac{G_s}{V} \quad (13.7.4)
\]

in which:
\(G_s\) = dry mass of sediment (kg)
\(V\) = volume of water sample (m³)

The silt and sand transport be determined as:

\[
s = \frac{G_s}{F \cdot T} \quad (13.7.5)
\]

in which:
\(F\) = area of nozzle (m²)
\(T\) = sampling period (s)
\(G_s\) = dry mass of sediment (kg)

13.42
A. DIMENSIONS OF INSTRUMENT

B. FILLING PERIOD

Figure 13.7.7 USP-61 sampler
The sampling efficiency of the USP-61 is strongly dependent on the ratio of the intake velocity and the local flow velocity (hydraulic coefficient). Extensive laboratory measurements, summarized by Dijkman (1978), have shown that the hydraulic coefficient varies from about 0.8 to 1.3 depending on the water temperature, sample height above the bed and the nozzle orientation (maximum deviation with flow direction of 20°). For this range a maximum sampling error in the concentration of about 10% may be expected in case of a steady concentration. In field conditions with fluctuating concentrations the inaccuracy of individual samples may be as large as 50%. To obtain a reliable average value in a statistical sense, a large number of samples (say 10) should be collected at each sampling point.

**Advantages**

1. simple,
2. for sand and silt particles,
3. direct determination of sand transport.

**Disadvantages**

1. many bottles for laboratory analysis,
2. relatively short sampling period,
3. small sediment catch (size analysis),
4. fragile intake nozzle.

6. **Delft bottle sampler**

The Delft Bottle (Figure 13.7.8) is based on the flow-through principle, which means that the water entering the intake nozzle leaves the bottle at the backside. As a result of a strong reduction of the flow velocity due to the bottle geometry, the sand particles larger than about 100 \( \mu m \) settle inside the bottle. Using this instrument, the local average sand transport is measured directly.

The Delft Bottle (DB) can be used as follows:
- suspended at a wire using a straight nozzle,
- suspended in a frame resting on the bottom using a bended or straight nozzle for measurements close to the bed.

Depending on the flow conditions, a small nozzle (velocities < 1 m/s) with an internal diameter of 15.5 mm or a big nozzle with an internal diameter of 22 mm can be used.

The local average sediment transport (in kg/m's) is determined as:

\[
s = \frac{\alpha (1 - p) \rho_s V_s}{F T} \quad \text{or} \quad s = \frac{\alpha G_s}{F T} \quad (13.7.6)
\]

where:
- \( \alpha \) = calibration factor
- \( p \) = porosity factor (≈ 0.4)
- \( \rho_s \) = density of sediment (≈ 2650 kg/m³)
- \( V_s \) = volume of sediment sample (inclusive pores) (m³)
- \( F \) = cross-section of nozzle (m²)
- \( T \) = sampling period (s)
- \( G_s \) = dry sand mass (kg)
A. DELFT BOTTLE, ARRANGEMENT FOR SAMPLING FROM WATER-SURFACE TILL 0.5 m FROM RIVER-BED

B. LONGITUDINAL SECTION

Figure 13.7.8 Delft Bottle sampler

Sampling errors are introduced by:
- incorrect intake velocity compared with local flow velocity; the hydraulic coefficient (ratio of intake velocity and local flow velocity) varies from 1 to 1.5 (Dijkman, 1978, 1981),
- inefficiency of the sampler to collect relatively fine sediment material (particles finer than 100 μm),
- additional sampling during raising and lowering of the instrument,
- sediment losses during removal of the sand catch.

The first two errors can be corrected using a calibration factor \( \alpha \) (Dijkman, 1981). The \( \alpha \)-factor varies from 0.7 to 2.5 depending on the nozzle type, particle size and local flow velocity.
The sample error due to the collection of sediment particles during lowering and raising of the instrument can be reduced by using a relatively large sampling period (15 minutes). Otherwise, an additional calibration factor is necessary (Dijkman, 1981). The minimum sampling time is about 5 minutes to obtain a statistically reliable result. An additional advantage of a long sampling period is the collection of a large sediment catch enabling an accurate determination of particle size (by sieving). Field measurements show sampling errors up to 50% for individual samples, even after the application of the calibration factor. Considering these large errors, the Delft Bottle can only be used to obtain a rough estimate of the local sand transport.

7. **USD-49 Depth-Integrating Sampler**

The USD-49 is a depth-integrating sampler. The sampler is lowered at a uniform rate from the water surface to the streambed, instantly reversed, and then raised again to the water surface. At least one sample should be taken at each vertical selected in the cross-section of the stream. A clean bottle is used for each sample. The USD-49 sampler has a cast bronze streamlined body in which a pint-bottle sample container is enclosed. The head of the sampler is hinged to provide access to the sample container. The head of the sampler is drilled and tapped to receive the 1/4 inch, 3/16 inch or 1/8 inch intake nozzle which points into the current for collecting the sample. The transit rate depends on the mean velocity in the vertical, the water depth and the nozzle diameter. The USD-49 is suitable for depth integration of streams less than about 5 m in which the velocities do not exceed 2 m/s.

The concentration can be determined as

\[
 c = \frac{G_s}{V} \quad (13.7.7)
\]

in which:
- \( G_s \) = dry mass of sediment (kg)
- \( V \) = volume of water sample (m³)

The depth-integrated suspended sediment transport can be determined as:

\[
 s_s = \frac{G_s h_s}{\alpha F T} \quad \text{or as } s_s = c \cdot q \quad (13.7.8)
\]

in which:
- \( s_s \) = depth-integrated suspended load (kg/sm)
- \( G_s \) = dry mass of sediment (kg)
- \( F \) = nozzle area (m²)
- \( T \) = total sampling period (s)
- \( h_s \) = height of sampled zone (m)
- \( q \) = specific discharge (m²/s)
- \( \alpha \) = calibration factor (-)

The sampler cannot sample down to the stream bed surface. When the sampler touches the bed, the distance between the sample nozzle and the bed is about 0.1 m. Thus, the depth of the sampled zone is about equal to the water depth minus 0.1 m (\( h_s = h - 0.1 \) in m). Another problem is the short sampling period at each specific point in the vertical. As a result concentration fluctuations are not averaged out and repeat samples are necessary. Information of the vertical concentration distribution cannot be obtained.
8. **Collapsible-Bag Depth-Integrating Sampler**

The Collapsible-Bag sampler is based on the principle that the static pressure acting on the outside surface of the flexible bag (devoid of air) creates a pressure equal to the hydrostatic pressure at the nozzle entrance. Using this method, samples can be collected throughout any depth.

The sampler consists of a wide-mouth, perforated, rigid plastic container enclosed in a cage-like metal frame. The head of the frame supports a plastic intake nozzle (6 or 13 mm) and swings open to permit the plastic container to be removed. When the head is closed, the end of the nozzle extends slightly into the mouth of the container. Perforations in the container allow the air in the container to escape during submergence. For sampling, a collapsed flexible bag is placed inside the rigid container. The neck of the flexible bag is stretched over the neck of the rigid container and this unit is placed into the sampler. When the sampler is lowered into the flow, water enters the perforations in the rigid container, surrounds the collapsed bag and equalizes the hydrostatic pressure at the nozzle entrance. The velocity head forces water through the intake nozzle into the collapsed bag, which unfolds to conform to the rigid container. This sampling action eliminates any possibility for flow to rush in due to unequal pressures and insures that water-sediment mixture is collected at stream velocity regardless of velocity distribution and depth, provided that the sample container does not overfill and the vertical transit rate is smaller than 0.4 of the depth-averaged velocity.

Nordin et al. (1983) have used a large-volume (6 l) bag sampler in combination with an Ott-type current meter in water depths up to 30 m. By lowering and raising the current meter over the depth, the instantaneous velocity profile can be recorded.

The constant vertical transit rate of the bag sampler should be fast enough so that the sample container is not completely filled, but not greater than 0.4 \( \bar{\nu} \) (\( \bar{\nu} \) = depth-averaged velocity). Nordin et al. tried to keep the transit rate smaller than 0.2 \( \bar{\nu} \).

A large sampler (length = 3 m, weight = 200 kg) was used by Christiansen (1985) in Germany, see Figure 13.7.9.

The following variables can be computed:

\[
\text{Depth-averaged velocity (m/s)} = \frac{V}{\alpha \ F \ T} \\
\text{Mean concentration (kg/m}^3\text{)} = \frac{G_s}{V} \\
\text{Specific discharge (m}^2\text{/s)} = \frac{V \ h_s}{\alpha \ F \ T} \\
\text{Suspended sediment discharge (kg/sm)} = \frac{G_s \ h_s}{\alpha \ F \ T}
\]

in which:

\( V \) = volume of water
\( F \) = nozzle area
\( T \) = sampling period
\( G_s \) = sediment mass
\( h_s \) = depth of sampled zone
\( \alpha \) = calibration factor
13.7.3 Pump-samplers

1. General aspects

To obtain a reliable average sediment concentration, the sampling or measuring period should be rather large (about 300 seconds). Furthermore, the collection of a large sediment sample for size-determination by sieving or settling tests requires the sampling of a relatively large water volume (about 25 to 50 litres). Both requirements can be satisfied by collecting water samples by means of a pump in combination with an in-situ separation of water and sediment particles. Usually, a pump sampler consists of a submersible carrier (with intake nozzle, current meter and echo-sounder), a deck-mounted pump and a flexible hose connecting the intake nozzle and the pump. The hose diameter should be as small as possible to reduce the stream drag on the hose. Using a hose diameter (bore) in the range of 0.003 to 0.016 m, the pump discharge must be in the range of 0.3 to 30 litres per minute. In case a deck-mounted pump is used, the maximum suction lift will be about 7 m. Assuming a static lift (= height of pump above water level) of about 2 m, the suction lift available for operation of the pump will be about 5 m resulting in a maximum hose length of about 50 m. In extreme deep waters an underwater pump must be used. Operation of a pump sampler is limited to flow conditions with velocities smaller than 2 m/s because of excessive stream drag on the pump hose and carrier.

2. Type of pump

Peristaltic pumps or propeller type pumps can be used. Peristaltic pumps (24 volt/220 volt) as used in the medical industry have proven to be very efficient for pump sampling in river and coastal conditions. At Delft Hydraulics the Ismatec Vp 380 (CH 8152 Glattbrugg, Zürich) is used. The discharge is relatively small (0.5 l/min) yielding a relatively small water-sediment sample which can be handled easily. The hose diameter is extremely small (0.006 m), which reduces the fluid drag forces on the hose. The pump direction can be easily changed to remove small objects (shell fragments, organic materials, etc.) blocking the intake nozzle.

Propeller type pumps produce a relatively large discharge (10 l/min) resulting in the handling of a large water-sediment volume. The hose diameter is in the range of 0.01 to 0.016 m.
3. Intake velocity and accuracy

Unidirectional flow

Ideally, the intake velocity of the water-sediment sample should be equal to the local flow velocity (see Figure 13.7.2). In conditions with varying velocities this would mean a continuous adjustment of the intake velocity. For practical reasons it is preferable to operate the pump system as much as possible with a fixed discharge and hence a fixed intake velocity. This can be done by using a fixed intake velocity for each class of flow velocities (see Table 13.12).

Using these values, the hydraulic coefficient will be in the range of 0.8 to 2.0 during pump sampling, resulting in a maximum error in the concentration of about 20%, which is quite acceptable for concentration measurements (see Figure 13.7.2).

<table>
<thead>
<tr>
<th>local flow velocity (m/s)</th>
<th>intake velocity (m/s)</th>
<th>pump discharge (l/min)</th>
<th>diameter intake nozzle (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.003</td>
<td>0.010</td>
</tr>
<tr>
<td>0.5 - 1.0</td>
<td>1</td>
<td>0.45</td>
<td>5</td>
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<tr>
<td>1 - 1.5</td>
<td>1.25</td>
<td>0.55</td>
<td>6.25</td>
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<tr>
<td>1.5 - 2.0</td>
<td>1.75</td>
<td>0.80</td>
<td>8.75</td>
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<tr>
<td>2.0 - 2.5</td>
<td>2.25</td>
<td>1.0</td>
<td>11.25</td>
</tr>
</tbody>
</table>

Table 13.12 Intake velocity and pump discharge

Oscillatory flow

Pump sampling is an attractive method for concentration measurements in coastal conditions because a relatively long sampling period can be used which is of essential importance to obtain a reliable time-averaged value. The sampling period should be rather long (15 min) in irregular wave conditions (at least 100 waves).

A problem of sampling in conditions with irregular waves is that the magnitude and direction of the fluid velocity is changing continuously. This complicates the principle of isokinetic sampling in the flow direction. A workable alternative is the method of normal (or transverse) sampling, which means that the intake nozzle of the sampler is situated normal to the plane of fluid velocity.

Bosman et al. (1987) studied the sampling error related to the orientation of the intake nozzle, because they were interested in pump sampling under wave conditions. They found that a transverse pumping direction yields good results. The intake nozzle is directed downward or normal to the plane of orbital motion. Figure 13.7.10 shows the ratio c/c₀ as a function of the ratio u/u₀ and the nozzle orientation for 170 µm-sediment (c = measured concentration, c₀ = original concentration, u = intake velocity, u₀ = local ambient velocity). For a transverse orientation (90°) and a ratio of u/u₀ ≥ 2, the c/c₀ ratio is about α = 0.7 to 0.8, which means a systematic error of 20% to 30% in the measured concentration. Similar results were obtained for 220 µm, 280 µm, 360 µm and 450 µm-sediment. This systematic error can be eliminated by multiplying the measured concentrations with a factor 1/α.

Peristaltic pumps have proven to be very efficient in coastal conditions.
Figure 13.7.10 Sampling error of concentration related to intake velocity and nozzle direction (Bosman et al., 1987).

4. Determination of sampling elevation

To determine the actual position of the intake nozzle above the bed, an echo-sounder attached to the carrier should be used. Usually it is desirable to have a sampling position close to the bed. This can be achieved by placing the carrier on the bed. In that case the position of the intake nozzle is equal to the distance between the nozzle and the underside of the carrier. When measurements at a few centimetres above the bed are necessary, special equipment should be used which allows vertical adjustment of the intake nozzle over a certain range (remote controlled).
5. Handling of water volume after sampling

When a peristaltic pump is used, a relatively small quantity of water and sediment is obtained (2 litres in 5 minutes). This small sample can be stored in a bottle to be returned to the laboratory for analysis.

When a propeller-type pump is used, a large water-sediment volume is collected, (say 30 to 50 litres). The handling of a large water-sediment volume requires the in-situ separation of water and sediment particles. A practical solution can be obtained by using:

- filtration method (pump-filter),
- sedimentation method (pump-sedimentation),
- subsampling method (pump-bottle).

Pump-filter method

The water-sediment sample is pumped through a filter which separates all particles larger than the mesh size of the applied filter material. The method is shown schematically in Figure 13.7.11. To separate the sand fraction, nylon filter material with a mesh size of 50 μm can be used. The water volume is recorded by means of a volume meter. After taking a sample, the filter system is opened and the filter material with the sand catch is removed and returned to the laboratory for drying, weighing and size analysis. During removal of the filter, the pumping is continued using a bypass system. The filtration method cannot be used in a silty environment with silt concentrations larger than about 50 mg/l, because of rapid filter blocking by the fine silt particles.

![A. Sketch Pump-Filter Method](image)

Figure 13.7.11 Pump-filter method

Pump-sedimentation method

The method is based on the filling of a large calibrated container (≈ 50 liters), in which the sand particles can settle (bottle 1), as shown schematically in Fig. 13.7.12. Using a settling height of about 0.75 m, the sand particles larger than 50 to 60 μm can be separated in about 5 minutes. A high separation efficiency can be obtained by using a conical container and a vibrator to avoid settlement of the sand particles on the inside of the container. To determine the silt concentration (particles smaller than 50 μm), a small water sample (bottle 2) can be taken during emptying of the container.
Pump-bottle method

This method is based on the continuous pumping of water and sediment. On board of the survey vessel a small part of the pump discharge (subsampling) is used to fill a 1 or 2 liter-bottle in 3 to 5 minutes by using a small tap, see Figure 13.7.13. The tap discharge \( Q_s \) should be 0.3 to 0.5 liters per minute for a velocity of 1 to 1.5 m/s in the pump line to obtain a sampling error less than 10% (Van Rijn, 1993). Using this method, a relatively long sampling period and hence a (statistically) reliable sand concentration is obtained. An optical sensor can be used to determine the silt concentration in the bottle after settling of the sand particles.

When a peristaltic pump is used (discharge = 0.5 to 1 l/min) the bottle can be filled directly.
6. Automatic pump samplers

Site location, flow conditions, frequency of collection and operational costs sometimes make collection of sediment data by manual methods impractical. For these reasons automatic pumping type samplers have been developed.

Such a sampler consists of:
- intake nozzle and tube system,
- pump to draw water-sediment samples from the flow (tube flushing after sampling is necessary),
- sample container unit to hold sample bottles in position for filling,
- sample distribution system to divert a pumped sample to the correct bottle,
- activation system that starts and stops the sampling cycle.

13.7.4 Optical and acoustical samplers

1. General principles

Optical and acoustical sampling methods enable the continuous and contactless measurement of sediment concentrations, which is an important advantage compared to the mechanical sampling methods. Although based on different physical phenomena, optical and acoustical sampling methods are very similar in a macroscopic sense. For both methods the measuring principles can be classified in (see Figure 13.7.14):
- transmission,
- scattering,
- transmission-scattering.

Transmission

The source and detector are placed in opposite direction of each other at a distance \( L \). The sediment particles in the measuring volume reduce the beam intensity resulting in a reduced detector signal. The relationship between the detector signal \( I_0 \) and the sediment concentration \( c \) is:

\[
I_\text{r} = k_1 e^{-k_2 c} \tag{13.7.13}
\]

in which:
- \( k_1 = \) calibration constant depending on instrument characteristics, fluid properties and travel distance \( L \).
- \( k_2 = \) calibration constant depending on particle properties (size, shape), wave length and travel distance \( L \).

Scattering

The source and detector are placed at an angle \( \phi \) relative to each other (see Figure 13.7.14). The detector receives a part of the radiation scattered by the sediment particles in the measuring volume.

The relationship between detector signal \( I_s \) and sediment concentration \( c \) is:

\[
I_s = k_3 c e^{-k_2 c} \tag{13.7.14}
\]
A. TRANSMISSION

B. SCATTERING

C. TRANSMISSION - SCATTERING

Figure 13.7.14 Optical and acoustical sampling principles
in which:
\[ k_3 = \text{calibration constant depending on instrument characteristics, fluid properties, particle properties, wave length and travel distance (L)}. \]

An important disadvantage of the scattering method is the strong non-linearity of the relation between the detector signal and sediment concentration for large concentrations.

**Transmission-scattering**

This method is based on the combination of transmission and scattering, as shown in Figure 13.7.14. If the travel distance for transmission and scattering is equal, a linear relationship for the ratio of both signals is obtained.

\[ \frac{I}{I_t} = k_4 c \]  \hspace{1cm} (13.7.15)

in which:
\[ k_4 = \text{calibration constant depending on instrument characteristics and particle properties}. \]

Important advantages are the absolute linearity between the output signal \( I \) and the sediment concentration, the independence of water colour and the reduced influence of fouling.

**Calibration**

For all measuring principles an in-situ calibration for determining the constants is necessary, if possible under representative flow conditions covering the whole range of flow velocities and measuring positions (close to bed and water-surface). Regular calibration is required because the constants may change in time due to variations in temperature, salinity and pollution.

In practice, the optical and acoustical sampling methods can only be used in combination with a mechanical sampling method to collect water-sediment samples for calibration. Usually, about 10% of the measurements should be used for calibration.

The inaccuracy of field measurements may sometimes be rather large because of calibration problems, particularly for optical samplers. The main problem is the lack of synchronity between the optical and mechanical sample collection. To minimize synchronity errors, the optical samplers should be calibrated by measuring the silt concentration on board of the ship using a pre-collected water-silt sample.

**Measurement range**

For an optimum sampling resolution the wave length and particle size must be of the same order of magnitude. Therefore the optical method is most suitable for silt particles \( (< 50 \, \mu m) \). Laboratory experiments using the optical sampler, have shown that the addition of sand particles with a concentration equal to the silt concentration increased the output signal with about 10% only. The upper concentration limit for optical samplers is about 30 kg/m². The acoustical method is most suitable for sand particles \( (\geq 50 \, \mu m) \). The upper concentration limit is about 10 kg/m³.
Advantages

An important advantage of optical and acoustical samplers is the continuous measurement of the suspended sediment concentration. In combination with a chart recorder for data collection a relatively long period (one month) can be sampled continuously and automatically. When there is very little variation of the silt concentrations in lateral direction of the cross-section, measurements at one point can be considered as representative for the whole cross-section. In that case the sensor can be fixed to a bridge pier or river side installation. The measuring location must be easily accessible for regular cleaning of the sensor and changing of batteries and chart records. Energy consumption and recorder maintenance can be minimized by using a switch system activating the sensor and recorder only for short periods (5 min) at preset intervals (1 hour).

Another advantage of the continuous monitoring is the possibility of determining continuous concentration profiles by raising the optical or acoustical sensor from the bed to the water surface (rapid profile method). Using this latter method a complete concentration profile can be determined in one minute. To check the representativeness of these profiles, occasionally the concentration profile should also be determined by means of point-integrated measurements. The horizontal variability can be determined by towing the sensor at a (monitored) depth below the water surface.

Finally, it is remarked that both sampling methods can also be used to measure the instantaneous sediment concentration under wave conditions, provided the response period is small enough.

2. Optical BTG sensor RD 20/10 (submersible)

The RD sensor is constructed of an epoxy material (diameter = 0.038 m, height = 0.114 m). The optical components are premolded into a translucent resin filled with a light diffusing material. The premolded optics assembly is cast into the sensor body. Electrical connections are made through a poly-urethane jacketed cable with a quick-disconnect style connector which connects the sensor to the electronics transmitter. The optical components consist of two infrared emitting diodes (LD) and two silicon photodetectors. The electronics transmitter alternates the LED’s on and off, while continuously measuring the amplitudes of light transmitted through the process media to the detectors. The alternating light principle (using two lightbeams of different length) automatically compensates for fouling and component aging.

The overall inaccuracy of the measured concentration is determined by the scatter of the calibration curve. Optical sensors are very sensitive to particle size, especially silt and clay particles. The sensor cannot be used in conditions with a wide particle range (clay, silt and sand).

Software is available for selection of desired sampling interval, averaging period and sampling duration.

The technical specifications are:

- dimensions sensor = 0.038 x 0.114 m
- weight sensor = 0.5 kg
- energy = 12/24 volt DC and 220 volt AC
- measurement range mud = 10 - 10,000 mg/l
- response period = 2 s
- light path lengths = 0.01 m and 0.02 m
3. *Optical OBS (submersible sensor)*

The Optical Back Scatterance sensor measures infrared radiation scattered by particles in the water at angles ranging from 140° to 165°. Infrared radiation from the sensor is strongly attenuated in clear water (more than 98% after travelling just 0.2 m). Therefore, even bright sunlight does not interfere with measurements made in shallow water (D A instruments, 1989).

The OBS-sensor consists of a high-intensity infrared emitting diode (IRED), a detector comprised of four silicon photodiodes and a linear solid state temperature transducer. The IRED produces a beam with half-power points at 50° in the axial plane of the sensor and at 30° in the radial plane. The sensor integrates radiations scattered through angles greater than 140°. The IRED irradiates a 1.3 cm³ sample volume through a 5.6 mm aperture in the center of the detector. The detector is shielded from visible light by a gelatin filter with transmittances less than 1% in the visible range (400-700 nm), see Figure 13.7.15. The sensor temperature compensated from 0-30 °C with a solid state temperature transducer molded in the sensor.

The sensor components are potted in a glass-filled polycarbonate housing with optical-grade epoxy.

The performance of the OBS-sensor is claimed to be superior to most other in-situ turbidity sensors, because of:

- small size and sample volume,
- linear response and wide dynamic range,
- insensitivity to bubbles and phytoplankton
- ambient light rejection and low temperature coefficient,
- low costs.

The overall inaccuracy of the measured concentration is determined by the inaccuracy (scatter) of the calibration curve. Optical sensors are very sensitive to particle size, especially silt and clay particles. Sample calibrations are shown in Fig. 13.7.15.

The sensor cannot be used in conditions with a wide particle size range (clay, silt and sand). The threshold concentrations are 5 mg/l for mud and 100 mg/l for sand.

The technical specifications are:

- dimensions sensor = 0.018 x 0.05 m
- dimensions housing = 0.06 x 0.23 m
- weight = 1.3 kg
- energy = 8 - 35 V/70 mA
- measuring range mud = 5 - 5,000 mg/l
- measuring range sand = 100 - 100,000 mg/l
- response period = 10 hz
- temperature drift = 0.05% per °C

*Advantages*

1. small size and small sample volume,
2. linear response,
3. insensitive to air bubbles and ambient light,
4. large measurement range,
5. high frequency response,
6. low cost.
Disadvantages

1. regular calibration,
2. not usable in conditions with combined clay, silt and sand.

Figure 13.7.15 Optical OBS sampler

4. Acoustic sampler (AZTM)

The Ultrasonic Sand Transport Monitor (AZTM) is based on the transmission and scattering of ultrasound waves by the suspended sand particles in the measuring volume (point sampling), as shown schematically in Figure 13.7.16. Using the amplitude and frequency shift of the scattered signal, the concentration and velocity and hence the transport of the sand particles can be determined simultaneously and continuously.

The AZTM consists of a sensor with a pre-amplifier unit mounted on a submersible carrier and a separate converter with panel instruments and switches. The velocity measurement is one or two dimensional and related to the carrier orientation, which is measured by means of a magnetic compass. The vertical position is measured by a pressure gauge (height beneath water surface) and an echosounder (height above bed) mounted on the carrier.

A transmitting frequency of 4.5 MHz has been chosen to minimize the particle size dependency and to make the instrument insensitive to silt particles (< 50 µm). The influence of temperature and salinity variations is also negligible.

The instrument has been designed at the Delft Hydraulics (Jansen, 1978, 1979, 1981; Schaafsma and Der Kinderen, 1985).
A. MEASURING SYSTEM

B. LABORATORY CALIBRATION CURVE

C. FIELD CALIBRATION CURVE

Figure 13.7.16 Acoustic sampler (AZTM)
The technical specifications are:

sound paths = 0.150 m
dimensions carrier = 1.5 x 0.5 x 0.5 m
dimensions converter = 0.5 x 0.4 x 0.3 m
weight carrier = 200 kg
weight converter = 20 kg
weight cable = 30 kg
energy = 200 Volt (AC), 50-60 Hz, 40 Watt
output signal = 0 - 10 Volt (analog)
measurement range (velocity) = 0 - 5 m/s
measurement range (concentration) = 10-5000 mg/l (linear)
response period = 0.1 s

5. Acoustic Profiling System

Acoustic Profiling Systems consist of a downward directed acoustic concentration meter (Thorne et al., 1991). A short pulse (10 μs) of high frequency (1 to 5 MHz) is emitted from a transducer and some of the acoustic energy is scattered back from the suspended sediment particles to the transducer. The magnitude of the backscattered signal is related to the concentration, particle size and the time delay between transmission and reception. The acoustic backscatter intensity from a uniform field of particles of constant concentration is assumed to be an inverse function of the distance from the source with corrections for attenuation due to water and particles. Calibration in uniform suspensions is required to find this relationship.

Vertical resolution is limited by the length of the acoustic pulse and by the speed at which the signal is digitized and recorded. Vincent et al. (1991) obtained a vertical resolution of 0.8 cm. Temporal resolution depends on the pulse repetition rate and on the number of pulses which must be averaged to produce a statistically meaningful backscatter profile. Vincent et al. (1991) used a pulse repetition rate of 10 Hz and four profiles were averaged before storing the data on disc. One average profile was recorded every 0.58 s; 1250 average profiles were recorded during each run (12 min).

The translation from the acoustic measurements to suspended sediment concentrations at different levels in the water column has considerable uncertainty associated with it and this imposes limits on the accuracy and reliability of the measured concentrations.

The Acoustic Profiling Systems are in a developing stage of research, but offer very promising possibilities for measuring instantaneous concentration profiles.

6. Nuclear radiation samplers

Nuclear samplers for suspended sediment concentrations have been used in Russia, Hungary, Poland (Basinski, 1989) and in China (Liu Yu-Ren, 1987). The principle is based on the absorption of radio-active energy by the sediment particles. In China the nuclear sampler consists of a Pu238 X-ray source. Other sources are Cs137, Am241 and Cd109. The radio-activity is measured by (radiation) counters. Calibration is required. The concentration range is 0.3 to 1000 kg/m³ with an inaccuracy of 20% for low concentrations and 5% for high concentrations.

13.7.5 Comparison of suspended load samplers

1. USP-61, Delft Bottle and Pump-Filter sampler

In May 1979 a field investigation using the USP-61, the Delft Bottle and the Pump-Filter Sampler was carried out in the Danube River near Ilók, Yugoslavia (Dijkman, 1982). At this location the measuring section is straight; the cross-section is rather wide and regular. The
mean width was about 560 m and the mean depth was about 8 m. The water temperature varied from 14 to 18°C. The three instruments were used at the same level above the bed within a few metres from each other. The (vertical) positioning was done by means of an echo-sounder present on each instrument. Figure 13.7.17 shows typical results of the sediment transport per grain-size fraction measured by each instrument. The results can be summarized as follows:

![Graphs showing sediment transport per grain-size fraction](image)

**Figure 13.7.17** Comparison of USP-61, Delft Bottle and Pump-Filter Sampler

**USP-61**

The measured sediment transport shows rather large fluctuations, even for the silt particles (d < 70 μm). The fluctuations are caused by the relatively small sampling time of about 30 seconds. For individual samples the accuracy may be as large as ± 50%. To obtain a reliable average value in a statistical sense, at least 10 samples must be collected.

**Delft Bottle**

The measured values for each grain-size fraction were corrected using standard calibration curves. Although the sampling time was rather long (600 to 900 s), the fluctuations in the measured transport rates are still rather large. Probably, these are caused by sediment losses during hoisting of the instrument.
Compared with the USP-61, the results of the Delft Bottle are systematically smaller, particularly for the 70-150 μm grain-size fraction. For all measurements the average difference between the Delft Bottle and the USP-61 Bottle varies from about 10% to 100%, depending on the grain-size fraction and the nozzle type.

**Pump-Filter Sampler**

The water-sediment mixture was pumped through 50 μm filters. Laboratory analysis showed that almost all particles smaller than 70 μm were lost. Figure 13.7.17 shows relatively small fluctuations, which means that a sampling period of about 300 s is sufficiently large to obtain a reliable average value. The mean value of the sand transport (d > 70 μm) measured by means of the Pump-Filter Sampler agrees rather well with that of the USP-61 Bottle. For all measurements the average difference between the USP-61 and the Pump Filter Sampler varies from 5% to 15%.

2. **Pump-Filter and Acoustical sampler**

The measurements were carried out in the Eastern Scheldt, a wide tidal estuary in the southwest part of the Netherlands. The local water depth was about 10 m, the flow velocities ranged from 0.4 to 1.0 m/s. The local bed material consisted of sand with $d_{50} = 200$ μm. The intake nozzle of the pump-filter sampler was installed next to the sensor of the AZTM. The flow velocities and sediment concentrations were measured at a level of 1 m above the bed during flood tide. The results, shown in Figure 13.7.18, show reasonable good agreement. The maximum deviation is about 30% for small concentrations (< 50 mg/l).

![Figure 13.7.18 Comparison of Pump-Filter and Acoustical Sampler](image-url)
13.7.6 Selection of suspended load sampler

1. Criteria

To select the most appropriate instrument, quantitative information of the physical parameters to be measured should be available prior to the actual field survey. It is important to have information of the various transport modes at the sampling site such as the (relative) value of the wash and bed material load, the bed and suspended load, velocities (small or large), concentrations (low or high), etc. To get this information, existing data (if available) should be analyzed or a reconnaissance study should be carried out. Another important criterion is the purpose and the accuracy required for the morphological study. For example, the predicted sedimentation in a planned shipping channel should be rather accurate when the maintenance dredging costs of the channel are critical with respect to the economic feasibility of the project. In that case the most accurate instruments and the most sophisticated facilities (boat, winches, electricity) should be used. In the case of river regulation for flood control, measurement of the wash load will usually be unnecessary but accurate measurements of the bed material load are of essential importance.

Summarizing, the actual selection of the most appropriate instrument for a specific sampling site (rivers, estuaries, coasts) depends on the:

- physical parameters to be measured,
- purpose and required accuracy of the morphological study,
- instrument characteristics,
- measuring facilities (boat, pier, platform, tripod, sledge, winches, electricity, etc.).

2. Instrument characteristics

The most important characteristics of the instruments are summarized in Table 13.13.

Sampling period

The sampling period is the time period during which a sample is collected. For the bottle-type and trap-type samplers the sampling period is equal to the filling period of the bottle or trap. The sampling period of the Delft Bottle is restricted by the size of the sediment catch which should be small compared with the size of the sedimentation chamber of the instrument. The sampling period of the pump-filter sampler is restricted by the filter characteristics. A small 50 \( \mu \text{m} \) filter may be blocked rather easily, especially in a silty environment. The sampling period of the optical and acoustic samplers is free. For time-averaging, additional equipment should be used allowing a digital reading.

Minimum cycle period

The minimum cycle period is the minimum time period between two successive measurements at adjacent points in a vertical. In case of a free sampling period a minimum sampling period of about five minutes is used to obtain the minimum cycle period, as given in Table 13.12. The cycle period can be used to evaluate the time period needed to cover a full concentration profile measurement. In case of tidal flow conditions this latter period should be small in relation to the tidal period.
average velocity = 0.85 - 1.25 m/s
average depth = 2.4 - 4.5 m
median particle size (bed) = 135 - 195 μm

Figure 13.7.19 Variability of suspended sediment concentrations
(Eyster and Mahmood, 1976)
Overall accuracy

The overall accuracy is an estimate of the overall error of a single measurement due to (systematic and random) measuring errors and stochastic (fluctuation) errors related to the physical process. The latter errors are introduced by the stochastic fluctuations of the physical parameters to be measured. Figure 13.7.19 presents some information of sediment concentration fluctuations, as observed by Eyster and Mahmood (1976) in Pakistan irrigation channels. They used a pint-bottle sampler yielding a time-averaged value over a five-second period. Based on these results, each single bottle-measurement may have an error of about 100%. This error can be reduced by using a larger sampling period or by collecting more samples. The overall accuracy of the samplers with a free sampling period, as presented in Table 13.13, is based on a relatively long sampling period (say 5 minutes). The accuracy of the optical and acoustical samplers is largely dependent on the number and accuracy of the calibration samples.

<table>
<thead>
<tr>
<th>Suspended load samplers (point-integrating)</th>
<th>Suspended load parameter</th>
<th>Measured parameter</th>
<th>Measurement range (mg/l)</th>
<th>Response time (s)</th>
<th>Sampling period (min)</th>
<th>Minimum cycle period (min)</th>
<th>Overall accuracy (one measurement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottle</td>
<td>silt (&lt; 50 µm)</td>
<td>yes</td>
<td>concentration</td>
<td>&gt; 1</td>
<td>-</td>
<td>1</td>
<td>100%</td>
</tr>
<tr>
<td>Trap</td>
<td>sand (&gt; 50 µm)</td>
<td>yes</td>
<td>concentration</td>
<td>&gt; 1</td>
<td>1</td>
<td>variable</td>
<td>100%</td>
</tr>
<tr>
<td>USP-61 Bottle</td>
<td>silt (&lt; 50 µm)</td>
<td>yes</td>
<td>concentration</td>
<td>&gt; 1</td>
<td>-</td>
<td>= 1</td>
<td>100%</td>
</tr>
<tr>
<td>Delft Bottle</td>
<td>sand (&gt; 100 µm)</td>
<td>yes</td>
<td>concentration</td>
<td>&gt; 10</td>
<td>-</td>
<td>5-10</td>
<td>50%</td>
</tr>
<tr>
<td>Pump Filter</td>
<td>silt (&lt; 50 µm)</td>
<td>yes</td>
<td>concentration</td>
<td>&gt; 10</td>
<td>-</td>
<td>5-10</td>
<td>50%</td>
</tr>
<tr>
<td>Pump-sedimentation</td>
<td>sand (&gt; 100 µm)</td>
<td>yes</td>
<td>concentration</td>
<td>&gt; 10</td>
<td>-</td>
<td>5-10</td>
<td>50%</td>
</tr>
<tr>
<td>Pump-Bottle</td>
<td>silt (&lt; 50 µm)</td>
<td>yes</td>
<td>concentration</td>
<td>&gt; 10</td>
<td>-</td>
<td>5-10</td>
<td>50%</td>
</tr>
<tr>
<td>Optical-BTG</td>
<td>silt (&lt; 50 µm)</td>
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<td>concentration</td>
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<tr>
<td>Optical-OBS</td>
<td>sand (&gt; 50 µm)</td>
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<td>concentration</td>
<td>100-100000</td>
<td>0.1</td>
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<td>50%</td>
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<tr>
<td>Acoustical-AZTM</td>
<td>silt (&lt; 50 µm)</td>
<td>yes</td>
<td>velocity concentration</td>
<td>10-50000</td>
<td>0.1</td>
<td>free</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 13.13 Instrument characteristics of suspended load sampler

13.8 Instruments for particle size and settling velocity

13.8.1 General aspects

Particle size and fall velocity are of essential importance for the estimation of erosion, transport and deposition rates.

As the sizes of sediment particles vary over extremely wide ranges, sediments are therefore measured in very large numbers and grouped into specific, but arbitrary size classes according to various analysis methods and definitions. Sediment particles not only vary widely with respect to size, but also with respect to specific weight and shape.

Because of the wide range of particle characteristics, particle size usually needs to be defined in terms of the method of analysis. Large sizes including boulders and cobbles can be measured directly by immersion and weighing. Intermediate sizes of gravel and sand are
measured semi-directly by sieving resulting in sieve diameters. Small sizes of silts and clays are measured hydraulically by sedimentation or settling methods resulting in the particle fall velocity and the standard fall diameter. The relationship between the median sieve diameter and the standard fall diameter is a measure of the effect of shape, roughness and specific gravity on the settling velocity of a particle.

This leads to the fact that there are essentially two types of measurements:
- size- or volume-measurements,
- fall velocity measurements

The size- or volume-measurements include the determination of the:
- diameter by means of photographs, sieves or the diffraction of coherent light beams (Laser granulometer),
- volume by means of immersion or conductivity (Coulter Counter).

The fall velocity measurements, usually, consists of the determination of sediment accumulation as a function of time using:
- dispersed suspensions for silt particles (pipet-withdrawal tube, bottom-withdrawal tube, balance-accumulation tube),
- stratified suspensions for sand particles (visual accumulation tube, manual accumulation tube, balance accumulation tube).

In a dispersed suspension the (silt) particles begin to settle from an initially uniform suspension. The lower size limit is approximately 5 \( \mu m \) because the settling of smaller particles is hindered by the Brownian motion.

In a stratified suspension the particles start from a common source at the upper end of the tube and become stratified according to their settling velocities. As settling medium, water is generally preferred because it is the usual environment of (fluvial and marine) sediments.

Greatest consideration must be given to the effect of proximity of particles to each other. Concentrations larger than 5000 mg/l result in hindered settling and hence smaller settling velocities than for individual particles. It is also conceivable that a group of particles creates a region of high concentration and will act as one 'large' particle resulting in a relatively large settling velocity.

It may be noted that a sample containing silt, clay and coarse material will require analysis by two or more methods because of the limitation on the range of sizes that can be analyzed by each specific method.

The following instruments are described:
- sieve instruments
- sedimentation instruments
- Coulter Counter
- Laser diffraction
- video camera

13.8.2 Sieve instruments

Sieve analysis (dry sieving, wet sieving, air-jet sieving) is one of the simplest, most widely used methods of particle size analysis, that covers the approximate size range from 10 \( \mu m \) to 50000 \( \mu m \) using standard woven wire sieves. Micromesh sieves extend the range down to 10 \( \mu m \) and punched-plate sieves extend the upper range.
Sieve results can be highly reproducible (within 5%). Inaccuracies may be caused by:
1. size of total sample and size of particle fractions on each sieve,
2. presence of aggregated lumps of particles,
3. inaccuracies in size and shape of the sieve openings,
4. the duration of the sieving operation.

13.8.3 Sedimentation instruments

1. General aspects

Basically, two methods are used for particle size and fall velocity analysis:
1. stratified suspensions,
2. dispersed suspensions.

Stratified suspension

In a stratified system the particles start from a common source and become stratified at the bottom of the tube according to the settling velocities. This method is only used for sand particles. The stratified sediment layers at the bottom of the tube can be measured by means of a small capillary tube (Visual Accumulation Tube, VAT). Another possibility is to weigh the settled sediment particles by means of an under-water balance or to extract the settled sediment particles at pre-fixed time intervals by means of a mechanical method. The latter two methods produce the accumulated sediment weight as a function of time. Using the known settling height (L), the weight percentage of the particles with a fall velocity larger than \( W_1 (= L/T_1) \) can be determined (Figure 13.8.1).

![Figure 13.8.1 Sedimentation of stratified suspension](image)

**Figure 13.8.1 Sedimentation of stratified suspension**

Dispersed suspension

In a dispersed system the particles begin to settle from an initially uniform dispersion (equal concentration). Generally, this method is used for silt or fine sand particles (5 to 150 \( \mu m \)). The sediment weight can be determined as a function of time by means of an under-water balance (Figure 13.8.2).
At time \( T \), being the ratio of the settling length \( L \) and the fall velocity \( W \), all particles are settling at a weight rate per unit time: \((dG/dt)_T\). During the period 0 to \( T \) all particles with a fall velocity smaller than \( W \) have settled (as a group) at the same constant rate \((dG/dt)_T\). Consequently, the total weight of all particles on the balance can be represented by \( T(dG/dt)_T \). However, the total weight of all particles on the balance at time \( T \) is \( G_T \). Hence, the difference represents all particles of the whole sample with a fall velocity larger than \( W \):

\[
G = G_T - T \left( \frac{dP}{dt} \right)_T
\]

(13.8.1)

In terms of weight percentages:

\[
P = P_T - T \left( \frac{dP}{dt} \right)_T = P_T - (\log e) \left( \frac{dP}{d\log t} \right)_T
\]

(13.8.2)

in which:

\( P = \) weight percentage of particles with a fall velocity larger than \( W \) (oversize).

Equation (13.8.2) is known as the Oden-equation.

![Sedimentation of dispersed suspension](image)

**Figure 13.8.2 Sedimentation of dispersed suspension**

2. **Accumulation tubes**

An accumulation tube can be operated as a stratified system for sand particles in the range 50 to 2000 \( \mu \)m or as a dispersed system for silt and fine sand particles smaller than 150 \( \mu \)m.

Typical examples of the accumulation tube method are:
- Visual Accumulation Tube (VAT),
- Balance Accumulation Tube (BAT).

The VAT, which operates as a stratified system, consists of a settling tube with a length of about 2 m and a diameter of about 0.03 m (Figure 1A). The sample can be released on top
of the tube by means of a simple clamp device or by means of rotating lamellae (Figure 13.8.3). Under the settling tube a small capillary tube is suspended in which the deposit height can be determined as a function of time. Based on this information, the fall velocity distribution can be determined. The capillary tube has a length of about 150 mm and an internal diameter of 4 mm for particles of 50 to 500 μm and 10 mm for particles of 500 to 2000 μm.

The method may not be very accurate due to hindered settling of the particles in the contracted section and the capillary tube. To minimize this effect, the length of the settling tube should be relatively large compared with that of the capillary. The overall inaccuracy of the fall velocity distribution is about 10% for particles in the range 50 to 500 μm.

The major advantage of the VAT-method is the rapid determination of the fall velocity distribution of a sand sample. A routine analysis can be easily done in about 5 minutes (included elaboration of the measuring results). Gibbs (1972) found that the tube diameter should be larger than 0.1 m and the sample mass should be 1 to 2 grams to obtain accurate results (< 5%).

The BAT is based on the weighing of the settled particles by means of an under-water balance. The method can be operated as a stratified system using a long tube of about 2 m or as a dispersed system using a short tube of about 0.5 m for silt particles. In the latter case the tube should be equipped with a temperature control system because the measuring period may be rather large (24 hours) to determine the weight of all particles.

![Diagram of Visual Accumulation Tube (VAT)](image)

**Figure 13.8.3 Visual Accumulation Tube (VAT)**

3. **Bottom withdrawal tube (BWT)**

The instrument is based on the sedimentation of sediment particles from a uniform suspension (dispersed system).

The bottom withdrawal tube method can be used for the fall velocity analysis in the laboratory, but also for the in-situ determination of the fall velocity distribution. This latter
possibility offers the advantage of using an undisturbed suspension sample and native water as settling medium, which is essential for floculated sediments. The laboratory instrument consists of a tube with a length of about 1 m and an internal diameter of 0.05 m (or 0.025 m). The lower end of the tube is contracted into a nozzle (Figure 13.8.4). A pinch clamp on a short piece of rubber is used to enable quick withdrawals. The field instrument consists of a stainless steel tube with a length of about 1 m and an internal diameter of 0.05 m.

![Image of laboratory and field instruments](Image)

Figure 13.8.4 Bottom withdrawal tube (BWT)

The tube is used for the collection of the sample as well as for the determination of the fall velocity distribution by means of a settling test. Therefore, the tube is equipped with two valves on both ends (Figure 13.8.4) and a double wall for temperature control. The tube is
lowered to the sample location in a horizontal position with opened valves. After closing the valves, the tube is placed in an upright position (start of settling process) and hoisted on board of the survey vessel.

Usually, eight equal volume samples of about 0.2 litre are withdrawn at prefixed time intervals chosen in such a way as to best define the accumulative weight curve. A suitable schedule for particles in the range of 5 to 100 μm is withdrawals at 3, 6, 10, 20, 40, 60 and 120 minutes. The initial concentration should not be smaller than about 200 mg/l. Computation of the fall velocity curve is given by Van Rijn (1986, 1993).

4. Pipet-Withdrawal Tube (PWT)

The fundamental principle of the pipet method is to determine the sediment concentrations of an initially uniform suspension (dispersed system) at a pre-fixed depth below the water surface as a function of settling time (Figure 13.8.5). Particles having a settling velocity greater than the ratio of the depth and the elapsed time period will settle below the point of withdrawal after the elapsed time period.

The sediment concentration at a certain depth can be determined by withdrawing samples at that height. Usually, eight or nine samples are withdrawn.

The pipet method can be used for the laboratory analysis of a silt sample, but also for the in-situ analysis of a silt suspension. This latter possibility offers the advantage of using an undisturbed suspension sample and native water as settling medium, which is essential for flocculated sediments.

The laboratory instrument consists of a 1 litre-cylinder with an internal diameter of 0.075 m for suspensions with an initial concentration larger than 1000 mg/l, while a 25 ml-pipet is used for withdrawing the samples (Figure 13.8.5A). A 3 litre-cylinder (internal diameter of 0.1 m) in combination with 200 ml-withdrawal samples should be used for suspensions with an initial concentration in the range of 100 to 1000 mg/l. In the latter case the side-withdrawal method can be used, as shown in figure 13.8.5B. For accurate results the initial settling height should be 0.2 m. The analysis period is about 2 hours for separation to about 5 μm.

For suspensions with an initial concentration larger than about 1000 mg/l, the Andreasen-Eisenwein pipet can be used for routine analysis.

Figure 13.8.5 Pipet-withdrawal tube (PWT)
The field instrument consists of a stainless steel tube with a length of about 0.3 m and an internal diameter of 0.12 m. The tube is used for sample collection as well as for the determination of the fall velocity distribution by means of a settling test. Therefore, the tube is equipped with two valves on both ends and a double wall for temperature control. The tube is lowered to the sampling location in horizontal position with opened valves. After closing the valves, the tube is put in a vertical position (start of settling process, \( t = 0 \)) and raised. On board of the survey vessel withdrawals are taken at pre-fixed times. Computation of the fall velocity curve is given by Van Rijn (1986, 1993).

13.8.4 Coulter Counter

The method is based on the electric conductivity difference between the particles and the common diluent. Particles act as insulators and diluents as good conductors. The particles suspended in an electrolyte are made to pass through a small aperture through which an electrical current path has been established. As each particle displaces electrolyte in the aperture, a pulse essentially proportional to the particle volume is produced. Particles in the range of 1 to 500 \( \mu \)m can be counted and measured volumetrically. The results are presented in terms of a cumulative volume distribution and the percentage of particles in each volume class.

13.8.5 In-situ Laser diffraction

A coherent light beam passing at right angles through a planar collection of spherical, opaque particles of only one size produces on a screen ahead a series of coaxial rings of light about the primary beam, the rings constituting what is known as a Fraunhofer diffraction pattern. The angular position and intensity of the light and dark rings is a function of particle diameter. When the particles are of different diameters (irregular rather than spherical, not entirely in one plane or not completely opaque) well defined rings are not formed and deducing particle size information becomes more complex.

Recently developed instruments carry out this latter analysis by virtue of built-in minicomputers and approximate solutions to the governing equations. A diffraction pattern is formed by coherent light passing through a cloud of particles circulating within a parallel walled, glass sample cell. The primary laser beam is blocked once through the cell and the diffracted light at small forward angles falls on one or more patterned disks which allow selected portions to pass to a photo detector. Some instrument types employ a multi-element photo detector in stead of a disk and one photo detector.

It should be noted that the detected signal is a composite arising from the diffraction created by many particles at one instant and a another set of particles at the next instant. A schematic diagram is shown in Figure 13.8.6. The Laser Diffraction method can be used to determine particle sizes in the range of 1 to 1000 \( \mu \)m.

According to observations of McCave et al. (1986) on natural sediment, the instrument does not give an accurate indication of the amount of particles with a size in the range of 0.5 \( \mu \)m (= wave length of applied light) and 2 \( \mu \)m. Only 20% of the amount of particles with a size less than 2 \( \mu \)m was detected by the instrument.

Bale and Morris (1987) have used a submersible Laser diffraction instrument. Measurements of the size distributions of suspended particle populations in the Tamar Estuary (England) using this apparatus were compared with measurements carried out on discrete samples obtained from the same depth by pumping.
13.8.6 In-situ video camera

Van Lcussen and Cornclisse (1991) and others used an in-situ video camera system and an in-situ settling tube to determine floc sizes and settling velocities in the Ems estuary in the Netherlands. The in-situ video system consists of a small vertical tube with a closed end at the bottom in which particles are settling down in still water. Two small windows are present in the tube for enlightening (light beam) and for videorecordings (camera). The instrument was connected by a signal cable to the survey vessel which floated with the flow during the sampling period. Floc sizes and settling velocities were obtained from the recordings by computer analysis.

Computer software for rapid image-analysis is not generally available, but is under development at various institutes.

The in-situ video camera is a very promising instrument for determination of particle fall velocity distributions and will become the standard instrument for this type of measurements.

13.8.7 Selection of instruments

The characteristics of the instruments for determination of particle size and fall velocity are summarized in Table 13.14.

It is emphasized that the settling velocity of clay and silt particles should be determined by means of in-situ instruments using the pipet-withdrawal tube, the bottom-withdrawal tube or the in-situ video camera.
<table>
<thead>
<tr>
<th>Methods</th>
<th>Size range (µm)</th>
<th>Required quantity</th>
<th>Analysis period</th>
<th>Inaccuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photographic</td>
<td>100 - 100000</td>
<td>-</td>
<td>1 hour</td>
<td>5%</td>
</tr>
<tr>
<td>sieves</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dry</td>
<td>50 - 50000</td>
<td>1 - 1000 gram</td>
<td>30 min</td>
<td>10%</td>
</tr>
<tr>
<td>wet</td>
<td>10 - 100</td>
<td>0.1 - 1 gram</td>
<td>60 min</td>
<td>10%</td>
</tr>
<tr>
<td>air jet</td>
<td>10 - 1000</td>
<td>10 - 100 gram</td>
<td>30 min</td>
<td>10%</td>
</tr>
<tr>
<td>sedimentation methods</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V.A. tubes</td>
<td>50 - 2000</td>
<td>1 - 10 gram</td>
<td>5 min</td>
<td>10%</td>
</tr>
<tr>
<td>BA-tubes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>large</td>
<td>50 - 2000</td>
<td>0.1 - 10 gram</td>
<td>15 min</td>
<td>5%</td>
</tr>
<tr>
<td>small</td>
<td>5 - 100</td>
<td>500 - 5000 mg/l</td>
<td>3 hours</td>
<td>5%</td>
</tr>
<tr>
<td>Bottom-withdrawal tube</td>
<td>5 - 100</td>
<td>500 - 5000 mg/l</td>
<td>3 hours</td>
<td>20%</td>
</tr>
<tr>
<td>Pipet-withdrawal tube</td>
<td>5 - 100</td>
<td>100 - 5000 mg/l</td>
<td>3 hours</td>
<td>10%</td>
</tr>
<tr>
<td>conductivity coulter counter</td>
<td>1 - 500</td>
<td>10 - 100 mg/l</td>
<td>30 min</td>
<td>10%</td>
</tr>
<tr>
<td>(in situ) Laser diffraction</td>
<td>5 - 1000</td>
<td>10 - 1000 mg</td>
<td>15 min</td>
<td>5%</td>
</tr>
<tr>
<td>(in situ) video camera</td>
<td>10 - 1000</td>
<td>100 - 1000 mg/l</td>
<td>15 min</td>
<td>10%</td>
</tr>
</tbody>
</table>

**Table 13.14 Instrument characteristics for particle size and fall velocity**

13.9 Instruments for bed material sampling

13.9.1 Introduction

Broadly, there are four methods of bed material sampling:
- **grab samplers**
- **dredge samplers**
- **scoop samplers**
- **core samplers**

Grab, dredge and scoop-type samplers can be used to collect a surface sample of the bed material. Grabs are applicable when the bed material consists of non-cohesive sandy material, while dredges should be used in the case of coarse and/or firm bed material. Corers generally produce the least disturbed samples. For stratified bed material or deposits only corers should be used.

Core sampling consists of driving a tube into the bed material through the use of gravity, hydrostatic pressure and/or vibration. Free-fall corers can cause compaction of the vertical structure of the sediment samples, while shock waves generated ahead of the descending sampler may wash away the fine fraction of the sediment bed. This latter effect can be minimized by using samplers with openings to create a flow-through system during descent. Free-fall corers recommended are: the PHLEGGER corer for samples up to 0.5 m in all sediment layers, the ALPINE gravity corer for samples up to 2.0 m and the BENTHOS gravity corer suitable for samples up to 3.0 m in soft clays, muds or sandy silts.

If the vertical stratification of a core sample is of essential importance, a piston and/or vibration corer should be used. These devices utilize special equipment to prevent sediment compaction during sampling.

The actual choice between corers and grabs should be based on the type of project being evaluated. If the project requires new work or channel deepening, samples should be collected with a corer. If the estimation of maintenance dredging of an existing channel or harbour is considered, a grab sampler will usually be sufficient because vertical stratification may be relatively small. The number of samples can be reduced substantially by using sub-bottom profiling systems (seismic surveying).
13.9.2 Grab, dredge and scoop samplers

Grab, dredge and scoop-type samplers are used to collect a bed-surface sample. A grab sampler consists of two buckets or jaws which are in an open position during lowering of the sampler. After contact with the bed the buckets are closed by using a messenger system or by pulling the hoisting cable. For coarse and/or firm bed material a dredge-type sampler should be used. Various grab samplers were evaluated by Sly (1969). Simple and good samplers are the SHIPEK grab sampler and the VAN VEEN grab and dredge samplers. A scoop-type sampler consists of a single scoop-type bucket which swings out of the lower side of the sampler body. The bucket surrounds and encloses the bed material sample. An advantage of the US BM-54 scoop sampler is its streamlined body enabling sample collection in high-velocity conditions.

13.9.3 Core samplers

Core sampling consists of driving a tube into the bed material through the use of manpower, gravity, hydrostatic pressure or vibration. A simple hand corer can be used in shallow streams, which can be waded or on tidal flats. The lower end of the sampler contains a cylinder which is pressed into the bed. A piston with a handle on its upper end passes through the sampler frame. The piston is retracted when the cylinder is pressed into the bed material. The suction created by the piston holds the sample in the cylinder. The gravity (or free-fall) corer is allowed to fall freely through the water and is driven into the bed by its weight. A one-way valve at the top end of the tube permits the passage of water during the descent and prevents flushing of the sample during retrieval and raising of the sampler. A core-catcher generally is present at the inside of the tube just above the cutting edge. Plastic liners generally are used to minimize the problem of sample extrusion and storage. The core length is limited to 10 core diameters in sand and 20 core diameters in firm clay. A major disadvantage of gravity cores is the compaction of the vertical structure of the bed material during sampling.

A piston corer can be used to reduce the compaction during sampling. This sampler is essentially a gravity corer, but it has an internal piston which remains at the level of the water-sediment interface when the corer penetrates into the bed (Figure 13.9.1). The corer is attached to a trip mechanism which is released when a counter weight hits the bed. The piston creates a slight vacuum above the sample and is supposed to reduce friction and to prevent compaction.

Figure 13.9.1 Piston core sampler

Vibration cores are used when core samples with a length up to 10 metres are required in all types of bed material with the exception of rock and stiff clay. The corer is driven into the bed by vibration equipment mounted on top of the corer. Some vibrocorers use pistons,
valves and compressed air to assist sample retrieval. Sample disturbance is inevitable but this can be restricted to the outer part of the core. Vibrocorers require a relatively large survey vessel and skilled personnel.

13.10 Instruments for in-situ measurement of wet bulk density

13.10.1 General aspects

In deposition and navigation depth studies of muddy areas the wet (bulk) density defined as the mass of the water-sediment mixture per unit volume is an important parameter. The position of the surface of consolidated mud layers can be determined by means of echo-sounding instruments. Good penetration can be obtained with 30 kHz-instruments, see Fig. 13.10.1. Higher frequencies (210 kHz) do not have sufficient energy to penetrate into the bed.

Various methods are available to determine the wet bulk density:
- mechanical core samplers,
- acoustic probe,
- nuclear radiation probe,
- electric conductivity probe,
- vibration transducer probe,
- pressure transducer probe.

Electric conductivity probes and pressure transducer probes are not generally applicable. Electric conductivity probes are very sensitive to the fluid salinity which should be known beforehand. Pressure and vibration transducer probes can only be used in unconsolidated fluid muds (low density < 1200 kg/m³).

13.10.2 Mechanical core sampler

A basic requirement is undisturbed sampling of bed material. Various mechanical core samplers (see section 13.9.3) are available to take undisturbed bed material samples of the surface layers (upper 0.5 m of the bed). Most samplers can only be used during low velocity conditions to ensure vertical penetration of the bed.

After sampling, it is common practice to make slices by a machined ring of the same internal diameter as the core. The core content is extruded into the ring until it is full of the water-sediment mixture. A thin plate is then introduced between the ring and the core to isolate the sample.

As the core diameter is known and fixed and the slice thickness is fixed by the ring, the volume can be calculated. After weighing (and drying) of the sample, the wet and dry density can be determined. The wet density is defined as:

\[
\rho_{\text{wet}} = \frac{M_w + M_s}{V} - \rho_f + (1-p)\rho_s
\]  

(13.10.1)

in which:
- \(\rho_{\text{wet}}\) = wet (bulk) density (kg/m³)
- \(M_w\) = water mass (kg)
- \(M_s\) = sediment mass (kg)
- \(V\) = sample volume (m³)
- \(\rho\) = fluid density (kg/m³)
- \(\rho_s\) = sediment density (kg/m³)
- \(p\) = porosity factor (kg/m³)

13.76
Hilton et al. (1986) tested several samples from a 75 mm core which were sliced at either 5 or 10 mm intervals. Each slice was transferred to a calibrated bottle to determine the sample volume. The measured volumes were found to be systematically larger (10% to 20%) than the volumes calculated from the ring dimensions. This difference is caused by the centre of the sediment surface bowing upwards slightly when the edges are just level with the top of the ring.

This error was reduced substantially when a sheet of perspex was put on top of the ring. A small gap was left at one side to allow air to escape until the sediment almost touched the lid, after which it was moved to completely cover the ring. The slice was then treated similarly (determination of volume, wet and dry mass).

The wet (bulk) density of dredged material in a hopper dredger can be determined as:

$$\rho_{\text{wet}} = \frac{\rho \ V_d}{V_h}$$  \hspace{1cm} (13.10.2)

in which:
- $V_d$ = displaced volume of hopper vessel (m$^3$)
- $V_h$ = volume of hopper
- $\rho$ = fluid density (kg/m$^3$).

13.10.3 Acoustic probe

The principle is based on measuring the attenuation of the intensity of monochromatic ultrasonic waves through the (fluid) mud layer.

The basic equation is:

$$I = I_o \exp(-\alpha L \rho_{\text{wet}})$$  \hspace{1cm} (13.10.3)

in which:
- $I$ = acoustic intensity measured through water-sediment mixture
- $I_o$ = acoustic intensity measured in clear water
- $L$ = path length
- $\alpha$ = absorption coefficient of water-sediment mixture (calibration)
- $\rho_{\text{wet}}$ = wet (bulk) density.

Granboulan et al. (1987) presented an instrument composed of two piezo-electric transducers each 50 mm in diameter and placed 150 mm apart. A series of waves (860 kHz) are transmitted between the two transducers. An envelop sound detector connected to a microprocessor is used to process the received signal, which is compared with the signal in clear water. Calibration of the instrument is required. The linear dry density range was from 100 to 500 kg/m$^3$ (wet density of 1050 to 1300 kg/m$^3$) with an inaccuracy of ± 30 kg/m$^3$.

The probe (mass 10 kg) was lowered and raised by a winch for vertical measurements. The probe was also attached to a submarine vehicle which was towed horizontally through fluid mud layers to determine the navigation depth.

13.10.4 Nuclear radiation probe

The principle is based on measuring the attenuation or scattering of the radiation intensity through a water-sediment mixture.

The basic equation reads as:
\[ I = I_0 \exp(-\mu L \rho_{\text{wet}}) \quad (13.10.4) \]

in which:
- \( I \) = intensity measured in water-sediment mixture
- \( I_0 \) = intensity measured in clear water
- \( L \) = path length of radiation
- \( \mu \) = absorption coefficient (calibration)
- \( \rho_{\text{wet}} \) = wet (bulk) density of water-sediment mixture.

The attenuation principle is preferred above the scattering principle because this latter method is more sensitive to sediment properties, requires more input energy and has a larger vertical measuring volume.

The radiation sources usually are gamma radiation sources: Cs\(^{137}\), Am\(^{241}\) and Cd\(^{109}\). The detector usually is a scintillation (crystal) probe.

Calibration of the probe is required. The calibration coefficient depends on the distance (\( L \)) between the source and the detector, the radiation source and the absorption coefficient of the sediment particles. The measurement range (dry density) is 50 to 1000 kg/m\(^3\) with an inaccuracy of \( \pm 20 \) kg/m\(^3\). The wet (bulk) density range is 1020 to 1500 kg/m\(^3\), see Fig. 13.10.1 and 13.10.2.

The Public Works Department of Rotterdam in the Netherlands uses a nuclear backscatter probe to determine in-situ density profiles in the harbour basins. The needle-shaped probe (= 70 kg) has a spatial resolution of 0.15 m being the distance between the source and the detector. The probe is calibrated in a large container using artificial mud. In field conditions the probe measures the local density of the bed while it penetrates into the bed (free fall). A complete density profile over 2 to 3 meter can be obtained in approximately 1 minute. The measurement is stopped when the inclination of the probe is larger than 10°. Some results are shown in Figure 13.10.1B.

The GKSS research center Geesthacht in Germany (Elbe estuary) uses a gamma radiation attenuation probe, see Figure 13.10.2. The instrument consists of two needle-shaped tubes (diameter = 0.07 m, length = 3 m, weight = 70 kg). The radiation source is 0.37-Gbq-Cs\(^{137}\) with a gamma-energy of 662 keV. The detector is a scintillation crystal placed at a distance of \( L = 0.25 \) m from the source. The vertical position of the instrument is determined by means of a pressure sensor. Inclinometers are used to determine the vertical inclination of the instrument. The maximum penetration depth in soft mud is approximately 3 to 5 m (see Figure 13.10.2B).

Reproducibility tests show variations of the order of 20 to 50 kg/m\(^3\), see Figure 13.10.2C.
A. IN-SITU NUCLEAR BULK DENSITY PROFILE THROUGH STATIONARY SUSPENSION SHOWING LAYERED STRUCTURE AND COMPARISON WITH 30 KHZ ECHOSOUNDER

B. BULK DENSITY PROFILE BASED ON NUCLEAR BACKSCATTER METHOD

*Figure 13.10.1 Wet bulk density measurements*
Figure 13.10.2 Nuclear radiation probe and wet bulk density measurements
REFERENCES


REFERENCES (continued)


REFERENCES (continued)

Federal Inter-Agency Sedimentation Project. Instructions for USP-61, Suspended Sediment Sampler. Minnesota 5541K, USA.


REFERENCES (continued)


REFERENCES (continued)


REFERENCES (continued)


APPENDIX A

TRANSPORE-program, computation of sediment transport in current and in wave direction
1. Input

- $h$ = water depth (m)
- $\bar{v}_R$ = depth-averaged velocity vector in main current direction, see Fig. A.1 (m/s)
- $\bar{u}_s$ = time-averaged and depth-averaged return velocity below wave trough compensating the mass transport between wave crest and trough
  (— in backward or offshore direction), see Fig. A.1 (m/s)
- $u_b$ = time-averaged near-bed velocity due to waves, wind or density-gradient (+ in forward direction, — in backward direction), see Fig. A.1 (m/s)
- $H_s$ = significant wave height (m)
- $T_p$ = (absolute) wave period of peak of spectrum (s)
- $\phi$ = angle between wave and main current direction (0-360°) (-)
- $d_{50}$ = median diameter of bed material (m)
- $d_{90}$ = 90% diameter of bed material (m)
- $d_s$ = representative diameter of suspended material (m)
- $k_{s,c}$ = current-related bed roughness height (minimum $k_{s,c} = 0.01$ m) (m)
- $k_{s,w}$ = wave-related bed roughness height (minimum $k_{s,w} = 0.01$ m) (m)
- $T_e$ = fluid temperature (°C)
- $SA$ = fluid salinity (%)

![Diagram](image)

**Figure A.1** Schematic presentation of current and wave direction

**Remarks:**

A. The representative particle size ($d_s$) of the suspended sediment will be in the range of:

\[ d_s = (0.6 \text{ to } 1) \times d_{50,\text{bed}}, \text{ see Section 8.4.3.} \]

A reasonable estimate is $d_s = 0.8 \times d_{50,\text{bed}}$. 

A — 1
B. The wave-related bed roughness height in the ripple regime will be in the range $k_{b,w} = (1 \text{ to } 3) \Delta_s$ with values from 0.01 to 0.1 m. The wave-related bed roughness height in the sheet flow regime will be: $k_{b,w} = 0.01$ m. The current-related bed roughness height will be in the range $k_{c,c} = 0.01$ to 1 m.

C. The constant of Von Karman is assumed to be $\kappa = 0.4$. The sediment density is $\rho_s = 2650$ kg/m$^3$.

2. Compute general parameters

Chlorinity: $CL = (SA - 0.03)/1.805$

Fluid density: $\rho = 1000 + 1.455 CL - 0.0065 (Te - 4 + 0.4 CL)^2$

Kinematic viscosity: $\nu = (4/(20 + Te)) \times 10^{-5}$

Fall velocity: see Equations (3.2.21), (3.2.22) and (3.2.23)

3. Compute sediment characteristics

Relative density: $s = \rho_s/\rho$

Particle parameter: $D_* = \frac{d_{50}}{(s-1)g/\nu^2}]^{1/3}$

Shields parameter:
- $1 < D_s \leq 4$: $\theta_s = 0.24 D_s^{-0.64}$
- $4 < D_s \leq 10$: $\theta_s = 0.14 D_s^{-0.1}$
- $10 < D_s \leq 20$: $\theta_s = 0.04 D_s^{-0.29}$
- $20 < D_s \leq 150$: $\theta_s = 0.013 D_s$
- $D_s > 150$: $\theta_s = 0.055$

Critical bed-shear stress: $\tau_s = (\rho_s - \rho)g d_{50} \theta_s$

Critical depth-averaged velocity: $\overline{u}_s = 5.75(s-1)g d_{50}^{0.5} (\theta_s)^{0.5} \log(4h/d_{50})$

Critical peak orbital velocity (Komar):
- $d_{50} < 0.0005$ m: $\hat{U}_s = \left[ 0.12(s-1)g(d_{50})^{0.5} (T_p)^{0.5} \right]^{2/3}$
- $d_{50} \geq 0.0005$ m: $\hat{U}_s = \left[ 1.09(s-1)g(d_{50})^{0.75} (T_p)^{0.25} \right]^{0.371}$

4. Compute wave length

Wave length modified by currents: $\left[ \frac{L'}{T_p} - \hat{v}_R \cos \phi \right]^2 = \left[ \frac{gL'}{2\pi} \right] \tanh \left[ \frac{2\pi h}{L'} \right]$

5. Compute relative wave period

The relative wave period is: $T_p' = \frac{T_p}{1 - (\hat{v}_R T_p \cos \phi)/L'}$

6. Compute wave parameters

Near-bed peak orbital excursion: $\hat{A}_\delta = \frac{H_s}{2 \sinh(2\pi h/L')}$

Near-bed peak orbital velocity: $\hat{U}_\delta = \frac{\pi H_s}{T_p' \sinh(2\pi h/L')}$

Wave-boundary layer thickness: $\delta_w = 0.072 \hat{A}_\delta (\hat{A}_\delta/k_{b,w})^{0.25}$
Near-bed peak orbital velocity in forward direction
\[
\mathbf{h} < 0.01 \frac{g(T_p)^2}{T_p} : \quad \hat{U}_{b,f} = \frac{3 \pi^2 (H_s)^2}{4\left(T_p / L\right)^4 (\sinh(2\pi h/L))} \hat{U}_\delta
\]
\[
\mathbf{h} < 0.01 \frac{g(T_p)^2}{T_p} : \quad \hat{U}_{b,f} = \alpha \hat{U}_\delta
\]
\[
\alpha = 1 + 0.3 \left( \frac{H_s}{h} \right)
\]
Near-bed orbital velocity in backward direction
\[
\mathbf{h} > 0.01 \frac{g(T_p)^2}{T_p} : \quad \hat{U}_{b,b} = \hat{U}_\delta - \frac{3 \pi^2 (H_s)^2}{4\left(T_p / L\right)^4 (\sinh(2\pi h/L))} \hat{U}_\delta
\]
\[
\mathbf{h} < 0.01 \frac{g(T_p)^2}{T_p} : \quad \hat{U}_{b,b} = (2 - \alpha) \hat{U}_\delta
\]

Return velocity mass transport
\[
\bar{u}_r = -\frac{0.125 \frac{g^{0.5} (H_s)^2}{h^{1.5} h_t}}{h_t = (0.95 - 0.35 \left( \frac{H_s}{h} \right)) h}
\]

Near-bed wave-induced velocity
\[
u_b = (0.05 - (\alpha - 0.5) \hat{U}_\delta
\]
\[
\alpha = \hat{U}_{b,f}/(\hat{U}_{b,f} + \hat{U}_{b,b})
\]

7. Compute apparent bed roughness
\[
k_a = k_{s,c} \exp\left[ \gamma \frac{\hat{U}_\delta}{((\bar{V}_R)^2 + (\bar{u}_r)^2)^{0.5}} \right], \quad k_{s,max} = 10 k_{s,c}
\]
\[
\gamma = 0.8 + \beta - 0.3 \beta^2
\]
\[
\beta = \left( \frac{\phi}{360^\circ} \right) 2\pi
\]

8. Compute friction factors
Current:
\[
C' = 18 \log(12h/3d_{90})
\]
\[
C = 18 \log(12h/k_{s,c})
\]
\[
f'_c = 0.24 \log^{-2}(12h/3d_{90})
\]
\[
f_c = 0.24 \log^{-2}(12h/k_{s,c})
\]
\[
f_a = 0.24 \log^{-2}(12h/k_a)
\]

Waves:
\[
f_{w} = \exp[-6 + 5.2(\hat{A}_s/3d_{90})^{-0.19}]
\]
\[
f_w = \exp[-6 + 5.2(\hat{A}_s/k_{s,\ast})^{-0.19}]
\]
\[
f_{w,max} = 0.3
\]
9. Compute effective time-averaged bed-shear stresses

Efficiency factor current : \( \mu_c = \frac{f'}{f_c} \)

Efficiency factor waves : \( \mu_w = \frac{f'}{f_w} \)

\( \mu_{w,a} = 0.6/D_* \)

Wave-current interaction coefficient : \( \alpha_{cw} = \frac{\ln(90\delta_w/k_a)^2}{\left[ \ln(90\delta_w/k_{a,c}) \right]^2} \left[ \frac{1 + \ln(30h/k_{a,c})}{1 + \ln(30h/k_a)} \right] \)

\( \alpha_{cw,max} = 1 \)

Bed-shear stress current : \( \tau_c = \frac{1}{8} \rho \frac{f_c}{f_c} \left[ (\bar{v})^2 + (\bar{u}_r)^2 \right]^{0.5} \)

Bed-shear stress waves : \( \tau_w = \frac{1}{4} \rho \frac{f_w}{f_w} \left( \bar{U}_w \right)^2 \)

Bed-shear stress current-waves : \( \tau_{cw} = \tau_c + \tau_w \)

Effective bed-shear velocity current : \( u_{r,c}' = \left[ (\alpha_{cw} \mu_c \tau_c / \rho)^{0.5} \right] \)

10. Compute bed-shear stress parameters

Dimensionless bed-shear stress for bed load transport : \( T = \frac{(\alpha_{cw} \mu_c \tau_c + \mu_w \tau_w)}{\tau_{cr}} - \tau_{cr} \)

Dimensionless bed-shear stress for reference concentration at \( z=a \) : \( T_a = \frac{(\alpha_{cw} \mu_c \tau_c + \mu_{w,a} \tau_w)}{\tau_{cr}} - \tau_{cr} \)

\( (T = 0 \text{ if } T < 0) \)

11. Compute velocity distribution over the depth

Outside wave-boundary layer, \( z \geq 3\delta_w \) : \( v_{R,Z} = \frac{\bar{v}_R \ln(30z/k_a)}{-1 + \ln(30h/k_a)} \)

Inside wave-boundary layer, \( z < 3\delta_w \) : \( v_{R,Z} = \frac{\bar{v}_\delta \ln(30z/k_{a,c})}{\ln(90\delta_w/k_{a,c})} \)

\( v_\delta = \frac{\bar{v}_R \ln(90\delta_w/k_a)}{-1 + \ln(30h/k_a)} \)
12. Compute sediment mixing coefficient distribution over the depth

Current, \( z < 0.5 \ h \): \( \varepsilon_{s,c} = \kappa \beta u_{*,c} (1-z/h) \)
\( z \geq 0.5 \ h \): \( \varepsilon_{s,c} = 0.25 \kappa \beta u_{*,c} h \)
\( u_{*,c} = (g^{0.5}/C) \left[ \left( \overline{v_R} \right)^2 + (\overline{u_z})^2 \right]^{0.5} \)
\( \beta = 1 + 2 \left( \frac{w_s}{u_{*,c}} \right)^2 \)
\( \beta_{\text{max}} = 1.5 \)

Waves, \( z \leq \delta_s \): \( \varepsilon_{s,w} = \varepsilon_{s,\text{bed}} = 0.004 D_s \delta_s \dot{U}_s \)
\( z \geq 0.5 \ h \): \( \varepsilon_{s,w} = \varepsilon_{s,\text{max}} = 0.035 h H_s/T_p \)
\( \delta_s < z < 0.5 \ h \): \( \varepsilon_{s,w} = \varepsilon_{s,\text{bed}} + \left[ \varepsilon_{s,\text{max}} - \varepsilon_{s,\text{bed}} \right] \frac{z - \delta_s}{0.5h - \delta_s} \)
\( \delta_s = 0.3 h (H_s/h)^{0.5} \)
\( \delta_{s,\text{max}} = 0.05 \text{ m}, \delta_{s,\text{max}} = 0.2 \text{ m} \)

Current and waves \( \varepsilon_{s,cw} = \left[ (\varepsilon_{s,c})^2 + (\varepsilon_{s,w})^2 \right]^{0.5} \)

13. Compute concentration distribution over the depth by numerical integration

Reference level \( a = \text{maximum}(k_{s,c}, k_{s,w}) \)

Concentration gradient \( (z > a) \): \( \frac{dc}{dz} = -\frac{(1-c)^5 c w_s}{\varepsilon_{s,cw} (1 + (c/c_o)^{0.8} - 2(c/c_o)^{0.4})} \)

Bed concentration \( (z \leq a) \): \( c = 0.015 \frac{d_{50}^{1.5}}{a} \frac{T_a}{D_s^{0.3}} \)
\( c_o = 0.65 = \text{maximum volume concentration} \)
\( w_s = \text{fall velocity of suspended sediment} \)

14. Compute time-averaged suspended load transport rates

Current direction \( u_s = \rho_s \int_a^h \overline{v_R} c \ dz \)

Wave direction \( q_s = \rho_s \int_a^h u_z c \ dz \)
15. Compute instantaneous and time-averaged bed-load transport

x-axis along current velocity vector (see Fig. A.2)
y axis normal to current velocity vector (see Fig. A.2)

Current velocities at \( z = \delta \)
above bed
\[
\delta = \max(3\delta_w, k_{sc})
\]
\[
\nu_{r,\delta} = \frac{\overline{v}_R \ln(30\delta/k_s)}{-1 + \ln(30h/k_s)}
\]
\[
u_{r,\delta} = \left(\overline{u}_r/\overline{v}_R\right)\nu_{R,\delta}
\]

Orbital velocities (asymm.)
\[
U_{0,r} \text{ and } U_{0,p}
\]

Instantaneous velocity \( x \)
\[
\sum U_{\delta,x} = U_\delta \cos\phi + v_{R,\delta} + (u_b + u_{r,\delta}) \cos\phi
\]

Instantaneous velocity \( y \)
\[
\sum U_{\delta,y} = U_\delta \sin\phi + (u_b + u_{r,\delta}) \sin\phi
\]

Instantaneous velocity
\[
U_{\delta,R} = \sqrt{\left(\sum U_{\delta,x}\right)^2 + \left(\sum U_{\delta,y}\right)^2}
\]

Instantaneous friction coefficient
\[
\alpha = \frac{|v_{R,\delta}|}{|v_{R,\delta}| + |\overline{U}_R|}
\]
\[
\beta = 0.25 \left[ \frac{-1 + \ln(30h/k_{sc})}{\ln(30h/k_{sc})} \right]^2
\]
\[
f_{c_w}' = \alpha \beta f_c' + (1 - \alpha)f_w'
\]

Instantaneous bed-shear stress
\[
\tau_{b,cw}' = 0.5 \rho f_{c_w}'(U_{\delta,R})^2
\]

Instantaneous bed-load transport
\[
\gamma = 1 - \left(\frac{H_s}{h}\right)^{0.5}, \gamma_{\min} = 0.3
\]
\[
a_b = 0.25 \gamma \rho_s d_{so} D_s^{-0.3} \left[ \frac{\tau_{b,cw}'}{\rho} \right]^{0.5} \left[ \frac{\tau_{b,cw} - \tau_{b,cr}}{\tau_{b,cr}} \right]^{1.5}
\]
\[
a_{b,x} = (\sum U_{\delta,x}/U_{\delta,R}) q_b
\]
\[
a_{b,y} = (\sum U_{\delta,y}/U_{\delta,R}) q_b
\]

Time-averaged values are obtained by averaging over the wave period.

16. Compute bed form dimensions

Bed form dimensions are computed according to formulae given in Chapter 5.
Figure A.2 Instantaneous velocity vector near bed ($z = \delta$)
17. Examples

The method of Van Rijn and that of Engelund-Hansen (Eq. 7.4.9) have been used to compute the total load transport in the current direction.

The basic data are:
- water depth, $h = 5 \text{ m}$
- depth-averaged return current, $\bar{u}_c = 0 \text{ m/s}$
- time-averaged near-bed velocity, $u_b = 0 \text{ m/s}$
- angle between current and waves, $\phi = 90^\circ$
- bed material characteristics, $d_{50} = 0.00025 \text{ m}$, $d_{90} = 0.0005 \text{ m}$
- fluid temperature, $T_c = 15^\circ\text{C}$
- fluid salinity, $SA = 0\%$

The other input parameters are given in the Table A.1.

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<th>$H_s = 0 \text{ m}$</th>
<th>$H_s = 0.5 \text{ m}, T_p = 5 \text{ s}$</th>
<th>$H_s = 2 \text{ m}, T_p = 7 \text{ s}$</th>
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**Table A.1 Input parameters TRANSPO**

The computed total load transport rates (in kg/sm) are shown in Figure A.3. The methods of Van Rijn and Engelund-Hansen show good agreement for the current alone case ($H_s = 0 \text{ m}$). The method of Engelund and Hansen overpredicts the transport rates at low velocities (0.3-0.6 m/s), because initiation of motion is not taken into account.
Figure A.3 Computed total load transport in current direction
APPENDIX B

Sand transport in closed conduits
1. Introduction

Transport of water-sand mixtures in pipelines for dredging and in sewers is herein discussed.

Dredging and transportation of sand through pipelines are important aspects of land reclamation projects and sand bypass projects (near coastal breakwaters). The water-sediment mixture usually is transported at high velocities to prevent deposition.

The water in sewer systems in urban areas usually contains sediment. If the transport capacity of the sewer flow is insufficient to transport these sediments (during relatively dry weather), solids will be deposited. Deposition may also be caused by changing of operational practices and inadequate maintenance.

2. Transport modes

Three transport modes can be distinguished (Breusers, 1968):
- homogeneous suspension for \( v > 200 w_s \)
  (concentration approximately constant over cross-section).
- heterogeneous suspension for \( 30 w_s < v < 200 w_s \)
  (strongly non-uniform concentrations).
- bed load regime for \( v < 30 w_s \)
  (sliding, rolling or saltating particles).

in which:
\( v \) = cross-section averaged velocity = \( Q/A \),
\( w_s \) = particle fall velocity.

Homogeneous suspension will generally occur in dredging-related pipelines. Transport of solids as heterogeneous suspensions or as bed load will occur in sewers. The sediment transportation in sewers may be continuous or discontinuous, because of lack of supply of sediment material. Transport at low velocities is accompanied by the formation of bed forms giving a considerable increase of the friction and hence the head loss.

3. Critical velocity for deposition

The critical velocity for deposition (= transition from heterogeneous suspension to bed load regime) is defined as the velocity at which stationary deposits (bed forms) are formed at the bottom of the pipe. This condition is critical for operation of the pipeline because a considerable increase of bottom friction and hence head loss will occur for velocities below the critical velocity.

Based on the experimental data (\( d = 0.2 \) to 25 mm, \( D = 40 \) to 580 mm, \( c = 2 \) to 23\%) of Durand (1953), the critical velocity can be represented as:

\[
V_{crit} = \alpha \left[2g(s-1)D\right]^{0.5} \tag{B.1}
\]

in which:
\( D \) = pipe diameter (m)
\( s \) = relative density \( \rho_s/\rho \)
\( g \) = acceleration of gravity (m/s\(^2\))
\( \alpha \) = coefficient
\( c \) = volume concentration (%)
The $\alpha$-coefficient is:

\[
\begin{align*}
\alpha &= 0.6 \quad \text{for} \quad d = 100-200 \ \mu m \\
\alpha &= 0.7 \quad \text{for} \quad d = 200-300 \ \mu m \\
\alpha &= 0.8 \quad \text{for} \quad d > 300 \ \mu m \quad \text{and} \quad c = 2\% \\
\alpha &= 1.0 \quad \text{for} \quad d > 300 \ \mu m \quad \text{and} \quad c = 15\%
\end{align*}
\]

Similar results were obtained by Führböter (1961) with sand sizes ranging from 190 to 880 $\mu m$ in a pipe with diameter of 0.3 m.

4. Head loss

4.1 Heterogeneous suspension regime

Empirical relationships are available to determine the energy gradient ($I_m$) required to transport the fluid-sediment mixture for a given velocity $v$, concentration $c$, particle fall velocity $w_s$, pipe diameter $D$ and friction factor $f$.

Durand (1953) proposed the following relationship:

\[
\frac{I_m - I}{c \ I} = \gamma \left[ \frac{g \ D}{v^2} \frac{w_s}{(gd)^{0.5}} \right]^{1.5}
\]  

(B.2)

in which:

$I_m$ = energy gradient for water-sand mixture  
$I$ = energy gradient for clear water = $((f/D)(v^2/2g)$  
$c$ = volumetric sediment concentration  
$D$ = pipe diameter  
$v$ = mean velocity ($= Q/A$)  
$w_s$ = particle fall velocity  
$d$ = particle diameter  
$f$ = friction factor of smooth pipe

The $\gamma$-coefficient was found to be (based on data of Durand):

$\gamma = 180$ by Gibert (1960)  
$\gamma = 180$ by Zandi et al (1967)

Führböter (1961) proposed:

\[
\frac{I_m - I}{c \ I} = \frac{\beta}{v}
\]  

(B.3)

with:

\[
\begin{align*}
\beta &= 0.5 \ \text{m/s} \quad \text{for} \quad d = 100-500 \ \mu m \\
\beta &= 1 \ \text{m/s} \quad \text{for} \quad d = 500-1000 \ \mu m \\
\beta &= 2 \ \text{m/s} \quad \text{for} \quad d = 1000-3000 \ \mu m \\
\beta &= 3 \ \text{m/s} \quad \text{for} \quad d \geq 3000 \ \mu m
\end{align*}
\]

Equation (B3) yields reasonable results for fine sand particles and large pipe diameters (dredging pipe lines).
4.2 Bed load regime

Graf et al (1968) proposed:

\[ \phi_t = 10.4 \theta^{2.5} \quad (B.4) \]

in which:

\[ \phi_t = \frac{c v R}{(s-1)^{0.5} g^{0.5} d^{1.5}} = \text{dimensionsless transport rate} \]

\[ \theta = \frac{I_m R}{(s-1) d} = \text{dimensionless mobility parameter} \]

\[ R = \frac{A}{P} = \text{hydraulic radius} \]

\[ A = \text{area in which flow takes place} \]

\[ P = \text{wetted perimeter} \]

Flow resistance in the bed load regime is primarily related to the bed form characteristics. Prediction methods for flow resistance and sediment transport were developed by Kleijwegt (1992).

References


Durand, R., 1953. Basic Relations of the Transportation of Solids in Pipes, IAHR Congress, Minneapolis, USA, pp. 89-103.


APPENDIX C

Side-wall roughness correction method of Vanoni-Brooks
To determine the shear stress related to the bed in the case of unequal bed and sidewall roughness, a correction method must be used, when the width-depth ratio of the flow is less than about 5.

A method which is frequently used, is that of Vanoni-Brooks (1957):

\[
\begin{align*}
\nu_{\ast, b} &= (g \, R_b \, I)^{0.5} = \text{shear stress related to the bed (m/s)} \\
R_b &= \frac{f_b \, R}{f} = \text{hydraulic radius related to the bed (m)} \\
R &= \frac{b \, h}{b + 2h} = \text{hydraulic radius (m)} \\
f &= 8 \left[ \frac{\nu_{\ast, b}^2}{v^2} \right] = \text{friction coefficient (-)} \\
\nu_{\ast} &= (g \, R \, I)^{0.5} = \text{shear velocity (m/s)} \\
v &= \frac{Q}{bh} = \text{mean flow velocity in cross-section (m/s)} \\
f_b &= f + \frac{2h}{b} (f - f_w) = \text{friction coefficient related to the bed (-)} \\
Re &= \frac{4 \, v \, R}{\nu} = \text{Reynolds' number (-)} \\
f_w &= 0.0026 [\log(Re/f)]^2 - 0.0428 \log(Re/f) + 0.1884 = \text{friction coefficient related to smooth side-walls according to Vanoni-Brooks (10^5 \leq Re/f < 10^8)}
\end{align*}
\]

\( h \) = depth (m)
\( Q \) = discharge (m\(^3\)/s)
\( b \) = width (m)
\( I \) = slope (-)
\( \nu \) = kinematic viscosity coefficient (m\(^2\)/s)

References

APPENDIX D

Pollution aspects of sediments
1. Introduction

Pollutants can be transported in dissolved form or in solid-associated form. Solids are herein defined as materials with a size larger than 0.45 μm obtained through membrane filtration. The solids are composed of a mixture of inputs from different sources:
- eroded rock and sediments,
- sewage and solid waste particles,
- atmospheric fall-out,
- autochthonous formations (inorganic precipitates, biogenic matter, complexed and colloidal matter, adsorbates or particles).

Sediment deposits and dredged materials in fluvial, marine and estuarine conditions are becoming increasingly polluted with trace (heavy) metals, phosphorus, nutrients (dissolved chemical components vital to the health of plants and animals; nitrogen, phosphorus, organic carbon) and other contaminants. For example, the cadmium concentrations in the sediments from the Rhine river have increased more than 100-fold from 1900 to 1980. Human activities which have intensified the problem of polluted sediments are:
- channelization of rivers,
- closing of channels and lagoons,
- extension and deepening of navigation channels and harbour basins.

The resulting increased maintenance dredging yields enormous quantities of polluted sediments for which safe disposal areas on land or in the aquatic system have to be found. Questions with regard to the presence of contaminated sediments that should be addressed, are:
- What is the behaviour of the contaminants in recent sediment deposits (is remobilization possible)?
- What is the bioavailability of the contaminants for aquatic life?
- What is the behaviour of contaminants in dredged materials after dumping on the land or in the water?

2. Dissolved and solid-associated materials

2.1 Dissolved materials

Surface waters receive a substantial part of their dissolved load from atmospheric fall-out (blown soil and dust, biological emissions from living vegetation, burning of organic matter, volcanic emissions) and anthropogenic emissions. The largest contribution of the dissolved load is caused by chemical reactions between water, rock and soil minerals (chemical weathering). The dissolved load is also affected by biotic processes associated with the soil and vegetation cover of the drainage basin. Biological processes in the aquatic system will also affect the dissolved load (uptake of nutrients and silica by aquatic organisms). Stream bed biota play a role in the (im)mobilization of dissolved components. Dissolved loads exhibit variations in time related to the occurrence of storm events, diurnal variations (day and night effects) and seasonal events (snow melt, rain fall).

2.2 Solid-associated materials

Solids (suspended, bottom) are important carriers of substances attached to or incorporated into the solids. Interactions between dissolved and solid-associated materials can take place. Solid-associated substances can be released into solution and conversely dissolved substances may be sorbed by the solids.
The solids consist of inorganic materials and organic materials. A distinction can be made between:

- **allochthonous materials**
  Materials washed into the aquatic system from the surrounding drainage basin (clay minerals, rock fragments, precipitates).

- **autochthonous materials**
  Materials formed in the aquatic system itself (mostly organic micro and macro-organisms; bacteria, phytoplankton, zooplankton).

Understanding of the mobility of substances (pollutants) requires detailed knowledge of the mechanisms for bonding of substances on sediment particles. The five major physical (electrostatic attraction) and chemical bonding mechanisms are:

- adsorptive bonding on fine-grained solids (clay, silt, mud),
- precipitation of discrete metal compounds,
- precipitation of metals with hydrous oxides (Fe and Mn) and Carbonates,
- associations with organic compounds,
- incorporation in crystalline material.

These processes are affected by temperature, pH-value, the type of clay minerals, the presence of coatings (oxides) around the solids. The chemical activity of the sediment particles increases with decreasing sediment size due to the increased specific area of the fine particles. These particles exhibit greatly increased levels of cation exchange capacity.

Adsorption of substances on sediments often is a reversible process. Biological activity and other effects may cause release from the solid phase to the dissolved phase.

The solid phase accounts for most (> 90%) of transport of phosphorus and trace (heavy) metals like Fe, Al, Ti and Mn.

3. Contaminants

The aquatic system in densely populated areas is mostly polluted with:

- nutrient elements,
- heavy metals,
- organic contaminants.

The pollutants originate from waste water discharges into the aquatic system and are mainly carried by the fine-grained sediment fractions (silt, clay, mud; particles smaller than 50 μm), but also, in dissolved form by the fluid phase (surface water, pore water). The trace metals in recent sediment deposits or in dredged materials can generally be divided into two categories in accordance with their predominant source of origin: lithogenic or anthropogenic. These categories are often simply referred to as: geochemical and man made.

The first category (lithogenic) of metals consists of:

- zirconium,
- rubidium,
- strontium.

These metals are derived from rock material by natural weathering processes.

The second category (anthropogenic) consists of metals introduced by human activities (emissions and disposal wastes of metal mines, smelters and refineries) and includes:

- chromium,
- cobalt,
- nickel,
copper,
zinc,
cadmium,
mercury,
lead.

Sediment deposits and dredged materials may also contain numerous **synthetic organic components** such as:
- chlorinated hydrocarbons (including pesticide residues and PCB’s),
- polycyclic aromatic hydrocarbons (PAH’s).

Most abundant usually is the composite group of oil, grease comprising hydrocarbons, fatty acids, soaps, fats, waxes and mineral oils.

To establish the degree of pollution of sediment deposits and dredged materials, it is essential first to define the natural uncontaminated levels of the components in the sediments (pre-civilization level) and then subtract it from existing values to derive the total effect caused by anthropogenic influences. The pre-civilization levels can be obtained by analyzing:
- fossil lake and river sediments,
- recent deposits in uncontaminated areas,
- core profiles of deposits providing a historical record of events.

For determination of pollutant concentrations, samples should be taken from:
- bottom surface
- bottom core (for historical sequence),
- suspended sediments,
- pore water.

Storage of samples under oxygen-free conditions is necessary.

A great variety of analysis methods to determine the type and quantity of the contaminants is available. Standard methods should be used as much as possible.

A quantitative measure of the metal pollution in aquatic sediments has been introduced by Müller (1979) referred to as the index of geo accumulation:

\[ I_{geo} = \log\left(\frac{C_n}{1.5 \ B_n}\right) \]

where \( C_n \) = measured concentration of element \( n \) in the sediment fraction smaller than 2 \( \mu m \) and \( B_n \) = geochemical background value in fossil argillaceous sediments.

This index reflects the relative enrichment of a certain metal in a given system. A comparison of these index values (\( I_{geo} \)) with IAWR-water quality index values for trace metals in sediments of the Rhine river is given in Table D.1.

The IAWR-index is a classification of the International Association of Waterworks in the Rhine Catchment.
<table>
<thead>
<tr>
<th>IAWR classification</th>
<th>IAWR index</th>
<th>$I_{geo}$</th>
<th>Metals</th>
</tr>
</thead>
<tbody>
<tr>
<td>very strong polluted</td>
<td>4</td>
<td>&gt; 5</td>
<td>Cd</td>
</tr>
<tr>
<td>strongly to strongly polluted</td>
<td>3 - 4</td>
<td>4 - 5</td>
<td>Pb, Zn</td>
</tr>
<tr>
<td>strongly polluted</td>
<td>3</td>
<td>3 - 4</td>
<td>Cd, Pb, Hg</td>
</tr>
<tr>
<td>moderately to strongly polluted</td>
<td>2 - 3</td>
<td>2 - 3</td>
<td>Zn, Hg, Cu</td>
</tr>
<tr>
<td>moderately polluted</td>
<td>2</td>
<td>1 - 2</td>
<td>Cr, Co</td>
</tr>
<tr>
<td>unpolluted to moderately</td>
<td>1 - 2</td>
<td>0 - 1</td>
<td>Cu</td>
</tr>
<tr>
<td>practically unpolluted</td>
<td>1</td>
<td>&lt; 0</td>
<td>Cr, Co</td>
</tr>
</tbody>
</table>

Table D.1 Comparison of IAWR-index and geoaccumulation index for Rhine sediment

4. Processes in aquatic systems

4.1 Rivers

The particulate pollutants are carried by the suspended fine-gradient substances. At high discharges the particulate metal concentrations in the suspended sediments may decrease (in a relative sense) due to the contribution of other contaminated eroded soil particles coming into the river by surface run-off. At high discharge also the residence time of the sediments in the river is reduced because of the presence of higher velocities. Consequently, the particles have less time to pick-up trace metals from polluted sources. Moreover, dissolved metal concentrations are much lower because of dilution by relatively uncontaminated water and hence less metal will be absorbed. However, the sediment (mud) concentrations usually increase at high discharges. Polluted fine sediments may be deposited in the flood plains at high discharges.

The concentration of dissolved material usually decreases at increasing discharges (dilution effect), because the water is transported much more rapidly to the river and there is less time for pick-up of materials.

As long as the river is outside the influence of tidal action (near the sea), the metal pollutants remain fixed to the suspended matter. In the tidal areas some metals are mobilized, going into solution (dissolved matter).

4.2 Estuaries

In estuaries the fresh river water and the saline sea water are mixed by tidal energy and by wind energy. Consequently, the relatively contaminated fluvial sediments and the relatively uncontaminated marine sediments are also mixed. These two sediment sources differ in mineral, chemical and isotopic composition.

The percentage of fluvial mud and hence the pollutant (metal) concentrations in suspended sediments usually is high at the landward end of the estuary (where the salinity is low) and low at the seaward end of the estuary (where the salinity is high). The percentage of fluvial mud and pollutant concentrations in the bottom sediment may be quite different.

In a stratified estuary (salt wedge) the fresh water discharge from the river generates a residual seaward current near the water surface and a landward current near the bottom. Since the suspended sediment concentrations near the bottom generally are greater than near the water surface, this causes a residual landward sediment transport in the transition zone between salt and fresh water. Accumulation of mud will occur until a state of dynamic equilibrium is obtained between sediment supply along the bed, removal of sediment from the lower layer to the upper layer by mixing, seaward transport in the upper layer and settling from upper to lower layers (see Chapters 10 and 11). This accumulation zone of sediments is termed the maximum turbidity zone, see Figure D.1.
Superimposed on physical mixing are other processes, such as: adsorption, precipitation and mobilization of pollutants (from the bottom) and will thus affect the composition of the pollutants. Adsorption of dissolved trace metals on suspended matter in the turbidity zone results in removal of trace metals from the fluid phase. Factors stimulating this process are: low salinity, high turbidity, high pH-values and flocculation (larger flocs). Usually, the physical mixing process is the dominant process in estuaries.

4.3 Lakes

Usually, the water in lakes is stagnant resulting in long residence times. River inflow of water and contaminated sediments cause high concentrations of trace metals which are deposited in the bed and mixed with relatively uncontaminated bottom sediments (dilution). The deposited sediments of the bottom are easily stirred up by wave action (storm periods) in shallow water and transported by wind-induced drift currents. The metal concentrations related to these processes are considerably smaller than the river input values. Lakes are sinks for suspended sediments and associated pollutants.

Other processes affecting pollution in lakes are:
- High phosphorus input causing massive algae blooms resulting in large quantities of organic material part of which accumulates in bottom deposits and part of which are attached to suspended sediments. Metals are also accumulated by the algae giving removal of metals from the fluid and sediment phases;
- Uptake of carbon dioxide (CO₂) by algae and carbon dioxide exchange between surface water and atmosphere resulting in an increase of pH-values. At high pH-values the production of calcium carbonates increases giving more accumulation of trace metals;
- Higher pH-values causes more adsorption of some dissolved metals on suspended matter.

5. Dredged materials

Extension and deepening of navigation channels and harbour basins below the natural bottom levels require continuous maintenance dredging to maintain the new bed levels, because the channels and basins act as sediment traps.

The silting rates depend on:
- horizontal and vertical dimensions of the dredged areas,
- sediment supply (concentration) to the area,
- water volume entering the basins (tidal motion),
- exchange water volume through entrance of basin (eddy motion and density-induced motion),
- settling velocity of sediments (flocculation).
The dredged materials are dumped on land or in other parts of the aquatic system. Classification of the dredged material is required. Heavy metals are mainly fixed to the fine sediment fractions. The uncontaminated sand fraction should be separated from the contaminated fine silt and clay (mud) fraction. Separation can be obtained by use of hydrocyclones with subsequent washing of the underflow. The fine material can be dewatered by centrifugation.

The most severely contaminated materials (fluvial origin) usually are dumped on land. Special care is required because the contaminated levels give rise to concern about their potential effects on biota (plants, crops). Plants differ in their ability to accumulate heavy metals. Cadmium is considered to be the most critical element because it is readily transferred to vegetation grown on dredged material disposal sites. Special areas should be preserved for onland disposal of dredged materials. An example is the "sludge island" near Rotterdam where about 150 million m$^3$ of moderately to strongly polluted dredged sludge from the harbour areas will be stored. Problems related to onland disposal sites of dredged materials are discussed by Van Driel and Nijssen (1988).

The less contaminated dredged materials (marine origin) usually are dumped in other parts of the aquatic system (near or in the sea). During the dumping process in the water a large part of the fine sediments is washed out and kept in suspension by local currents. The deposited materials (in the bed) may also contaminate the overlying water by diffusion, compaction (expelling pore water), erosion and bioturbation.

References


